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ANTI-FUZZY TRANSLATION AND ANTI-FUZZY MULTIPLICATION IN INK - ALGEBRAS

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ABSTRACT

In this paper, we define a anti fuzzy translation and anti fuzzy multiplication on INK algebras and discussed some of their properties in detail by using the concepts of anti fuzzy INK-ideal and anti fuzzy INK-sub algebra.

Keywords: Anti -Fuzzy-α-Translation, anti -Fuzzy-α-Multiplication of anti Fuzzy INK-Algebra, anti Fuzzy INK-Ideal, anti Fuzzy INK-Sub Algebra, anti Fuzzy

INTRODUCTION

The concept of fuzzy set was initiated by L .A .Zadeh in1965 [4]. It has opened up keen insights and applications in a wide range of scientific fields. Since its inception, the theory of fuzzy subsets has developed in many directions and found applications in a wide variety of fields. The study of fuzzy subsets and its applications to various mathematical contexts has given rise to what is now commonly called fuzzy mathematics. Fuzzy algebra is an important branch of fuzzy mathematics. Fuzzy ideas have been applied to other algebraic structures such as groups ,rings, modules, vector spaces and topologies. In this way ,K.Iseki and S.Tanaka [1] introduced the concept of BCK-algebras in 1978. K. Iseki[2] introduced the concept of BCI-algebras in 1980. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.T.Priya and T.Ramachandran [6][7] introduced the class ofPS-algebras , which is a generalization of BCI / BCK/Q /KU/ dalgebras.

In this paper, we introduce theconcept ofanti fuzzy-α-translation ,antifuzzy-α-multiplication of fuzzy INK-algebras and anti fuzzy extensions and established some of its properties in detail.

II.PRELIMINARIES

In this section we site the fundamental definitions thatwill be used in the sequel.

Definition:2.1[1] A BCK-algebraisanalgebra(X,∗,0)∗ of type(2,0)satisfying the following conditions:

i) $(x * y) * (x * z) \leq (z * y)$

ii) $x * (x * y) \leq y$

iii) $x \leq x$

iv) $x \leq y$ and $y \leq x \Rightarrow x = y$

v) $0 \leq x \Rightarrow x = 0$, where $x \leq y$ is defined by $x * y = 0$,forall $x, y, z \in X$.

Definition2.2 [2]A BCI-algebra is an algebra $(X, *, 0)$ of type(2,0)satisfyi2ng the following conditions:

i) $(x * y) * (x * z) \leq (z * y)$

ii) $x * (x * y) \leq y$

iii) $x \leq x$

iv) $x \leq y$and $y \leq x \Leftrightarrow x = y$

v) $x \leq 0 \Rightarrow x = 0$, where $x \leq y$ is defined by $x * y = 0$,forall $x, y, z \in X$.

Definition 2.3:[7]A nonempty set X with a constant 0 and a binaryoperation '*' is called INKalgebra if it satisfies thefollowing axioms.

1. $x * x = 0$

2. $x * 0 = 0$

3. $x * y = 0$ and $y * x = 0 \Rightarrow x = y$, $\forall x , y \in X$.

Definition 2.4:[7]Let S be a non empty subset of a INK-algebra X , then Sis called a INK-sub algebra of X if$x * y \in S$,for all $x , y \in S$.

Definition 2.5:[7]Let X be a INK-algebra and I be a subset of X, then I is called a INK-ideal of X if it satisfies following conditions:

1. $0 \in I$

2. $y * x \in I$ and$y \in I \Rightarrow x \in I$

Definition 2.6:[6]Let X be a INK-algebra. A fuzzy setμin X is called a fuzzy INK-ideal of X if it satisfies the followingconditions.

i) μ(0) ≤ μ(x)

ii) μ(x) ≤ max{μ(y ∗x), μ(y)}, for all x, y∈X

Definition 2.7: [6] A fuzzy set μ in a INK-algebra X is called a fuzzy INK-sub algebra of X if μ(x ∗ y) ≤ max{μ(x), μ(y)}, for all x, y∈X.

II. ANTI-FUZZYTRANSLATION AND ANTI –FUZZYMULTIPLICATION IN INKALGEBRA

Let X be a INK-algebra. For any fuzzy set μ of X, we define

T = 1–inf{μ(x) / x∈X }, unless otherwise wespecified.

Definition 3.1: ([3][5]) Let μ be a anti fuzzy subset of X and α∈[0,T]. A mapping

$\mu_\alpha{}^T$: X→[0, 1] is said to be a anti fuzzy-α-translation of μ if itsatisfies

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

$\mu_\alpha{}^T$(x) = μ(x) + α, ∀x∈X.

Definition 3.2: ([3][5]) Let μ be a fuzzy subset of X and α∈[0,1]. A mapping

μ_δ^M: X→[0, 1] is said to be a anti fuzzy-α-multiplication of μ if it satisfies

μ_δ^M (x) = αμ(x) , ∀x∈X.

Example 3.3: Let X = { 0, 1, 2 ,3} be the set with thefollowing table.

Then (X,*, 0) is a INK–algebra. Define a fuzzy set μ of X by $\mu(x) = \begin{cases} 0.5 & if\ x \neq 1 \\ 0.4 & if\ x = 1 \end{cases}$. Then μ is a fuzzy INK-sub algebra of X. Here T = 1–inf{μ(x) /x∈X} = 1–0.5= 0.5, Choose

α=0.2∈[0, T] and β= 0.5∈[0, 1]. Then the mapping μ0.4T: X→[0, 1] is defined by $\mu_{0.2}{}^T = \begin{cases} 0.5 + 0.2 = 0.7 & if\ x \neq 1 \\ 0.4 + 0.2 = 0.6 & if\ x = 1 \end{cases}$

which satisfies μ0.4T(x) = μ(x) + 0.4 , ∀x∈X, is a anti fuzzy 0.2-translation and the mapping $\mu_{0.3}{}^M(x) = \begin{cases} (0.5)(0.3) = 0.15 & if\ x \neq 1 \\ (0.4)(0.3) = 0.12 & if\ x = 1 \end{cases}$

which satisfies $\mu_{0.3}{}^{M}$ (x) = (0.3)μ(x), ∀x∈X, is a anti fuzzy 0.3-multiplication.

Theorem 3.4:If μ of X is a anti fuzzy INK-sub algebra and α∈[0,T] ,then the anti fuzzy-α-translation $\mu_\alpha{}^T$ (x) of μ is also a anti fuzzy INK-sub algebra of X.

Proof :Let x, y∈X and α∈[0,T].Then

μ(x * y) ≤ max{μ(x),μ(y)}

Now $\mu_\alpha{}^T$ (x * y) =μ(x * y) +α≤max{μ(x),μ(y)} +α

=max{μ(x) +α,μ(y) +α}

= max{$\mu_\alpha{}^T$ (x),$\mu_\alpha{}^T$ (y)}

Theorem 3.5:Let μ be a anti fuzzy subset of X such that the anti fuzzy-α-translation $\mu_\alpha{}^T$ (x) of μ is a anti fuzzy sub algebra of X for some subalgebra of X for some α∈[0,T].then μ is a anti fuzzy sub algebra of X.

Proof:

Assume that ,$\mu_\alpha{}^T$ is a anti fuzzy subalgebra for some α∈[0,T].

Let x , y∈X. We have μ(x * y) +α=$\mu_\alpha{}^T$ (x * y)

≤max{$\mu_\alpha{}^T$ (x),$\mu_\alpha{}^T$ (y)}

=max{μ(x) +α,μ(y) +α}

=max{μ(x),μ(y)} +α⇒μ(x * y)

≤max{μ(x),μ(y)} for all x, y∈X.Hence μ is anti fuzzy sub algebra of X.

Theorem 3.6:For any anti fuzzy INK-sub algebra μ of X and α∈[0,T], if the anti fuzzy-α-multiplication $\mu_\alpha{}^M$ (x) of μ is a anti fuzzy INK-sub algebra of X.

Proof: Let x, y∈X and α∈[0,T].Then

μ(x * y) ≤max{μ(x),μ(y)}

Now,$\mu_\alpha{}^M$ (x * y) =αμ(x * y)

≤αmax{μ(x),μ(y)}

=max{αμ(x),αμ(y) }

= max{$\mu_\alpha{}^M$ (x),$\mu_\alpha{}^M$ (y)}

Theorem 3.7:For any fuzzy subsetμof X andα∈[0,T],if theanti fuzzy-α-multiplication$\mu_\alpha{}^M$(x) ofμis a anti fuzzy INK-sub algebra ofX, then so isμ.

Proof :Assume that$\mu_\alpha{}^M$(x) ofμis a anti fuzzy INK-sub algebra ofX for someα∈[0,T].Let x , y∈X. We have

αμ(x * y)=$\mu_\alpha{}^M$(x * y)

≤max{$\mu_\alpha{}^M$(x)$\mu_\alpha{}^M$(y)}

=max{αμ(x),αμ(y)}

=αmax {μ(x),μ(y)}

⇒μ(x * y) ≤ max{μ(x),μ(y)}.

Henceμis aanti fuzzy INK-sub algebra of X.

Theorem 3.8:If theanti fuzzy-α-translationμαT(x) ofμis a anti fuzzy INK-ideal, then it satisfies the conditionμαT(x * (y * x)) ≤ $\mu_\alpha{}^T$(y).

Proof:$\mu_\alpha{}^T$(x * (y * x)) = μ(x * (y * x)) +α

≤max{ μ(y *(x * (y * x))) +α,μ (y) +α}

= max{ μ(0) +α, μ (y) +α}

≤max{$\mu_\alpha{}^T$(0),$\mu_\alpha{}^T$(y) }

=$\mu_\alpha{}^T$(y).

Theorem 3.9: Ifμis a anti fuzzy INK-ideal of X, then the anti fuzzyα-translation$\mu_\alpha{}^T$ofμis a anti fuzzy INK-ideal of X, for allα∈[0,T].

Proof : Letμ be a fuzzy INK-ideal of X and letα∈[0,T].

Then$\mu_\alpha{}^T$(0) =μ(0) +α≥μ(x) +α

=$\mu_\alpha{}^T$(x)

And$\mu_\alpha{}^T$(x) =μ(x) +α≤max{μ(y*x),μ(y)} +α

= max{μ((y*x) +α,μ(y) +α}

= max{$\mu_\alpha{}^T$(y*x) ,$\mu_\alpha{}^T$(y) }

Hence$\mu_\alpha{}^T$ofμis a anti fuzzy INK-ideal of X ,∀α∈[0, T]

Theorem 3.10:Letμbe a fuzzy subset of X such that theanti fuzzyα-translation$\mu_\alpha{}^T$ofμis a anti fuzzy INK-ideal of X for someα∈[0,T], thenμis a anti fuzzy INK-ideal of X.

Also, $\mu(x) + \alpha = \mu_\alpha{}^T(x) \leq \max\{\mu_\alpha{}^T(y*x), \mu_\alpha{}^T(y)\}$

$= \max\{\mu(y*x) + \alpha, \mu(y) + \alpha\}$

$= \max\{\mu((y*x), \mu(y)\} + \alpha$ and so

$\mu(x) \leq \max\{\mu(y*x), \mu(y)\}$ Hence μ is a anti fuzzy INK-ideal of X.

Theorem 3.11: Let $\alpha \in [0,T]$ and let μ be a anti fuzzy INK-ideal of X. If X isa INK-algebra, then the anti fuzzy α-translation $\mu_\alpha{}^T$ of μ is aanti fuzzy INK-sub algebra of X.

Proof : Let x, $y \in X$ .Now, we have

$\mu_\alpha{}^T(x*y) = \mu(x*y) + \alpha$

$\leq \max\{\mu(y * (x * y)), \mu(y)\} + \alpha$

$\leq \max\{\mu(0), \mu(y)\} + \alpha \geq \min\{\mu(x), \mu(y)\} + \alpha$

$= \max\{\mu(x) + \alpha, \mu(y) + \alpha\}$

$= \max\{\mu_\alpha{}^T(x), \mu_\alpha{}^T(y)\}$

Hence $\mu_\alpha{}^T$ is aanti fuzzy INK-sub algebra of X.

Theorem 3.12:Iftheanti fuzzy α-translation $\mu_\alpha{}^T$ of μ is a anti fuzzy INK-idealof X, $\alpha \in [0,T]$, then μ is a anti fuzzy INK-sub algebra of X.

Proof: Let us assume that $\mu_\alpha{}^T$ of μ is aanti fuzzy INK-ideal of X.Then

$\mu(x * y) + \alpha = \mu_\alpha{}^T(x * y)$

$\leq \max\{\mu_\alpha{}^T(y * (x * y)), \mu_\alpha{}^T(y)\}$

$\leq \max\{\mu_\alpha{}^T(0), \mu_\alpha{}^T(y)\}$

$\leq \max\{\mu_\alpha{}^T(x), \mu_\alpha{}^T(y)\}$

$= \max\{\mu(x) + \alpha, \mu(y) + \alpha\}$

$= \max\{\mu(x), \mu(y)\} + \alpha$

$\Rightarrow \mu(x * y) \leq \max\{\mu(x), \mu(y)\}$

Hence μ is aanti fuzzy INK-sub algebra of X.

Theorem 3.13:Let μ be a fuzzy subset of X such that theanti fuzzy α-multiplication $\mu_\alpha{}^M$ of μ is a anti fuzzy INK-ideal of X for some $\alpha \in (0,1]$, then μ is a anti fuzzy INK-ideal of X.

Proof: Assume that $\mu_\alpha{}^M$ is a anti fuzzy INK-ideal of X for some $\alpha \in (0,1]$.

Let x, y$\in$X.Then$\alpha\mu(0)=\mu_\alpha{}^M(0)$

$\leq \mu_\alpha{}^M(x)$

$=\mu(x)$

Also,$\alpha\mu(x)=\mu_\alpha{}^M(x)$

$\leq\max\{\mu_\alpha{}^M(y*x), \mu_\alpha{}^M(y)\}$

$= \max\{\alpha\mu(y*x),\alpha\mu(y)\}$

$=\alpha\max\{\mu(y*x),\mu(y)\}$

and so$\mu(x) \leq\max\{\mu(y*x),\mu(y)\}$Hence$\mu$is a anti fuzzy INK-ideal of X.

Theorem 3.14:If$\mu$is a anti fuzzy INK-ideal of X, then the anti fuzzyα-multiplication$\mu_\alpha{}^M$of$\mu$is a anti fuzzy INK-ideal of X, for all$\alpha\in(0,1]$.

Proof :Letμbe a anti fuzzy INK-ideal of X and let$\alpha\in(0,1]$.

Then$\mu_\alpha{}^M(0) =\alpha\mu(0)$

$\leq\alpha\mu(x)$

$=\mu_\alpha{}^M(x)$

And$\mu_\alpha{}^M(x) =\alpha\mu(x)$

$\leq\alpha\max\{\mu(y*x),\mu(y)\}$

$= \max\{\alpha\mu(y*x),\alpha\mu(y)\}$

$= \max\{\mu_\alpha{}^M(y*x), \mu_\alpha{}^M(y)\}$

Hence$\mu_\alpha{}^M$of$\mu$is aanti fuzzy INK-ideal of X ,$\forall\alpha\in(0,1]$.

Theorem 3.15:Let$\alpha\in[0,1]$ and letμbe a anti fuzzy INK-ideal of a INK-algebraX. Then the anti fuzzyα-multiplication$\mu_\alpha{}^M$ofμisa anti fuzzy INK-sub algebra of X.

Proof : Let x, y$\in$X .Now, we have

$\mu_\alpha{}^M(x*y) =\alpha\mu(x*y)$

$\leq\alpha\max\{\mu(y * (x * y)),\mu(y)\}$

$\leq\alpha\max\{\mu(0),\mu(y)\}$

$\leq \alpha \max\{\mu(x), \mu(y)\}$

$= \max\{\alpha\mu(x), \alpha\mu(y)\}$

$= \max\{\mu_\alpha{}^M(x), \mu_\alpha{}^M(y)\}$

Hence $\mu_\alpha{}^M$ is aanti fuzzy INK-sub algebra of X.

Theorem 3.16: Ifthe anti fuzzy α-multiplication $\mu_\alpha{}^M$ of μ is a anti fuzzy INK-ideal of X, $\alpha \in [0,1]$, then μ is a anti fuzzy INK-sub algebra ofX.

Proof: Let us assume that $\mu_\alpha{}^M$ of μ is aanti fuzzy INK-ideal of X.Then

$\alpha\mu(x * y) = \mu_\alpha{}^M(x * y)$

$\leq \max\{\mu_\alpha{}^M(y * (x * y)), \mu_\alpha{}^M(y)\}$

$\leq \max\{\mu_\alpha{}^M(0), \mu_\alpha{}^M(y)\}$

$\leq \max\{\mu_\alpha{}^M(x), \mu_\alpha{}^M(y)\}$

$= \max\{\alpha\mu(x), \alpha\mu(y)\}$

$= \alpha \max\{\mu(x), \mu(y)\}$

Hence μ is aanti fuzzy INK-sub algebra of X.

Theorem 3.17: Intersection and union of any two anti fuzzy translations of a anti fuzzy INKideal μ of X is also a anti fuzzy INK–ideal ofX.

Proof: Let $\mu_\alpha{}^T$ and $\mu_\nu{}^T$ be two anti fuzzy translations of a anti fuzzy INK-ideal μ of X, where $\alpha, \nu \in [0, T]$.Assume that $\alpha \leq \nu$.Then by theorem 3.14,$\mu\alpha$Tand$\mu\nu$Tareanti fuzzy INK-ideals of X.

Now, $(\mu_\alpha{}^T \cup \mu_\alpha{}^T)(x) = \max\{\mu_\alpha{}^T(x), \mu_\nu{}^T(x)\}$

$= \max\{\mu(x) + \alpha, \mu(x) + \nu\}$

$= \mu(x) + \alpha$

$= \mu_\alpha{}^T(x)$ And $(\mu_\alpha{}^T \cup \mu_\nu{}^T)(x)$

$= \max\{\mu_\alpha{}^T(x), \mu_\nu{}^T(x)\}$

$= \max\{\mu(x) + \alpha, \mu(x) + \nu\}$

$= \mu(x) + \nu = \mu_\nu{}^T(x)$

Hence $\mu_\alpha{}^T \cup \mu_\nu{}^T$ and $\mu_\alpha{}^T \cup \mu_\nu{}^T$ are anti fuzzy INK-ideals of X.

IV. ANTI FUZZYEXTENSIONS OF INK-IDEALS OF INK-ALGEBRAS

In this section, we introduced the ofanti fuzzy extensions of INK-ideals of INK-algebras and proved some standard results.

Definition 4.1: Le μ_1 an μ_2 be two fuzzy sets of X such tha μ_2 is a anti fuzzy extension o μ_1. I $\mu 1$ is a fuzzy INK-ideal of Ximplies tha μ_2 is a anti fuzzy INK-idealof X, the μ_2 is called as anti fuzzy INK-ideal extension o μ_1

Theorem4.3: Intersection of any two anti fuzzy INK-ideal extensions of a anti fuzzy INKidealμof X is a anti fuzzy INK-ideal extension ofμ.

Proof :Let μ_1 and μ_2 be two anti fuzzy INK-ideal extensions of a anti fuzzy INK-idealμof X. Then μ_1 (x) ≤μ(x) and μ_2 (x) ≤μ(x),for all x∈X. Sinceμis aanti fuzzy INK-ideal of X, μ_1 and μ_2 are anti fuzzy INK-ideals of X. Then $\mu_1 \cup \mu_2$ is also a anti fuzzy INK-ideal of X (By theorem 3.4[6]).Now

$(\mu_1 \cup \mu_2)$(x)= max{μ_1(x),μ_2(x) }≤max {μ(x),μ(x)}=μ(x). Hence $\mu_1 \cup \mu_2$ is a anti fuzzy INK-ideal extension of μ.

Theorem 4.4 :Letμ be a anti fuzzy INK-ideal of X. The anti fuzzyα-translation $\mu_\alpha{}^T$ is a anti fuzzy INK-ideal extension ofμ, for allα∈[0,T].

Proof :Ifμis a anti fuzzy INK-ideal of X, then by theorem3.11,theanti fuzzyα-translation $\mu_\alpha{}^T$ ofμis also a anti fuzzy INK-ideal of X ,for allα∈[0,T]. Now $\mu_\alpha{}^T$ (x) =μ(x)+α≥μ(x), for all x∈X. Hence, the anti fuzzyα-translation $\mu_\alpha{}^T$ is a anti fuzzy INK-idealextension ofμ.

Theorem 4.5 :Letμbe a anti fuzzy INK-ideal of X. Ifα≥δ, withα,δ∈[0,T],then theanti fuzzyα-translation $\mu_\alpha{}^T$ ofμis a anti fuzzy INK-idealextension of the anti fuzzyδ-translationμδTofμ.

Proof:Letμbe a anti fuzzy INK-ideal of X.Then by theorem 3.11,theanti fuzzyα-translation $\mu_\alpha{}^T$ ofμand the anti fuzzyδ-translation $\mu_\alpha{}^T$ ofμare anti fuzzy INK-ideals of X, for allα,δ∈[0,T]. Sinceα≥δ,μ(x)+α≤μ(x)+δ, for all x∈X.Therefore $\mu_\alpha{}^T$ (x)≤ $\mu_\alpha{}^T$ (x). Hence $\mu_\alpha{}^T$ is a anti fuzzy INK-idealextension of $\mu_\alpha{}^T$.

Theorem 4.6 :Letμbe a fuzzy setof X,α∈[0,T] andδ∈(0,1]. Iftheanti fuzzy-δ-multiplicationμ_δ^M(x) ofμ is a fuzzy INK-ideal ofX, then the anti fuzzy-α-translation $\mu_\alpha{}^T$(x) ofμ is a fuzzy INK-ideal extension ofμ_δ^M

Proof: Letα∈[0,T],δ∈(0,1] andμ_δ^M(x) ofμis a anti fuzzy INK-ideal of X. Then by theorem3.13,μis aanti fuzzy INK-ideal ofX. By theorem 3.9, $\mu_\alpha{}^T$(x) ofμis a anti fuzzy INK-ideal of X.Now,μ_δ^M(x)=μ(x) +α≥μ(x)≥μ(x)δ=μ_δ^M(x).Therefore, $\mu_\alpha{}^T$(x) ofμis a anti fuzzy INK-ideal extension ofμ_δ^M.

V. CONCLUSION

In this article authors have been discussed anti fuzzytranslation and anti fuzzy multiplication on INK-algebras through INK-sub algebras and INK-ideals.It has been observed that INKalgebras as an another generalization of BCK/BCI/Q/d/TM/KU-algebras.Interestingly, anti fuzzy extensions of INK-ideals of INK-algebras has been studied,whichadds an another dimension to the defined INK-algebras. This conceptcan further be generalized toIntuitionistic fuzzy set, interval valued fuzzy sets for newresults in our future work.

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EMERGING TRENDS IN INDIAN BANKING INDUSTRY

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A B S T R A C T

The banking industry plays a crucial job for the improvement of any country's economy. The development of banking industry depends upon the facilities granted by them to the clients in different perspectives. The developing drift of banking industry is found noteworthy after the financial and economical betterment in India. Nowadays, India contains a reasonably well created banking framework with diverse classes of banks –public segment banks, private segment banks, foreign banks, – both the traditional and modern era, co-operative banks and regional rural banks with the Reserve Bank of India as the wellspring starting of the framework. These days banking segment acts as a spine of Indian economy which revolved as a supporter amid the period of boom and recession. From 1991 different patterns and advancements in banking segment are credited. It moreover showed the different changes were caused to move forward their facilities to convince the clients.

KEY WORDS: Current trends, financial segment, developments, security, banking industry etc.

INTRODUCTION

The banking framework in India is altogether distinctive from other Asian countries since of the country's interesting social, geographic, financial and economical characteristics. India includes a huge population, a different culture, huge area and extraordinary inequality in wage, which are stamped among its areas. There are giant levels of lack of education among a huge rate of its people but simultaneously, the nation encompasses a huge supply of administrative and mechanically progressed endowments. Between around 35 and 40 percent of the population of India dwells in metro and urban cities and the rest is spread in a few semi-urban and rustic areas.

The country's financial and economical arrangement system combines communist and capitalistic highlights with an overwhelming predisposition towards public segment venture. India has taken after the way of growth driven exports instead of the export driven growth of other Asian economies, with accentuation on self-reliance through import substitution. These characteristics are showed within the system, area, structure and differing qualities of the country's banking and financial division. The banking framework had to serve the objectives of financial and economical arrangements articulated in progressive five year advancement schemes, especially focusing on the evenhanded wage dissemination, adjusted territorial financial and economical development and the decrease and disposal of private division monopolies business models in exchange and industry.

In arrange for the banking industry to act as a tool of state arrangement, it was exposed to different nationalization plans completely different stages (1955, 1969, and 1980). As an outcome, banking prevailed globally confined and simultaneously few Indian banks had opened their branches in overseas in worldwide monetary areas, since of distractions with household needs, particularly gigantic department extension and pulling in more individuals to the structure. Additionally, the segment had doled out the part of giving assistance to other financial and economical divisions such as small-scale businesses, trades and agribusiness and banking exercises within the developed commercial areas like urban and a constrained number of semi-urban areas.

The banking structure‘s universal segregation was too because of severe department authorizing controls on foreign banks previously working within the nation as well as section confinements confronting modern foreign banks. A measure of correspondence was needed for any Indian bank to open a branch of their bank in foreign countries. These highlights had cleared out the confusion of Indian banking division‘s shortcomings and qualities. An enormous challenge confronting Indian banks was how beneath the current proprietorship system, to achieve operational proficiency appropriate for contemporary financial intermediation. On the other side, it had generally simple for the public segment banks to recapitalize, provided the increments in nonperforming resources because their Government overwhelmed proprietorship system had diminished the clashes of intrigued that private banks would confront.

OBJECTIVES:

- ✓ To study about the rising trends in banking sector.
- ✓ To explore the current patterns and improvements in banking segment.
- ✓ To show the innovative and technological improvements in Indian banking segment.

RESEARCH METHODOLOGY:

The nature of study is conceptual. Data was collected from secondary sources like reports of RBI, journal, magazines and online websites.

RECENT TRENDS IN INDIAN BANKING SEGMENT:

Nowadays, we are having a reasonably well created banking framework with diverse classes of banks – private segment banks, public segment banks, regional rural banks, foreign banks and co-operative banks. The Reserve Bank of India is the predominant of all the banks.The RBI's (Reserve Bank of India) most vital objective is to preserve money related steadiness in India and for this purpose the RBI utilizes monetary policy to preserve cost steadiness and a satisfactory flow of credit. The rates utilized by RBI to realize the repo rate, bank rate, cash reserve ratio and reverse repo rate. Decreasing inflation has become one of the foremost critical objectives of RBI at a few point of time.

Development and enhancement in banking segment had risen above limits all over the globe. In 1991, the Government unlocked the entryways for the foreign banks to begin their activities in India and impart their wide extent of prerequisites, subsequently giving a solid competition to the household banks, and making a difference for the clients in providing the leading facilities of the banks. The Reserve Bank offered to move towards the leading universal banking facilities will advance hone the prudential standards and fortify its administrator component.

There had been impressive advancement and expansion within the trade of prime commercial banks. A few of them have locked in within the ranges of leasing, credit cards, customer credit, internet and phone banking, merchant banking, mutual funds etc. A number of banks had already established their branches for leasing, merchant banking and mutual funds and numerous more are within the preparation of doing so. A few banks had started factoring trade. So, following are the current trends which we can see in the functioning of each and every bank in India.

- **BANK NET:**

Bank net could be to begin with national level framework in India, which was contracted in February 1991. It is communication framework built up by Reserve Bank of India on the premise of proposal of the committee scheduled by it beneath the chairmanship of the official chief T.N.A. Lyre. Bank net had two stages: Bank net-I and Bank net- II.

- **INTERNET:**

Web may be a mesh of computers. In this promoting technique, any message and any information can be exchanged and accepted around the globe. In no time, internet can do numerous works for us, like the net can work as electronic mailing framework. It had eruption of remote database, which may be a daily newspaper, magazines of another nation. Clients can transmit their thoughts through Web and can make link with anybody who could be connected with internet.

On web, one can transmit any kind of data like numerals, letters, diagrams, picture, video and any kind of audio like music recording. Internet could be a quick creating net and is of most extreme imperative for public division undertaking, Research Institutions, financial Institutions and educational organization etc.

• TELE-BANKING:

Tele banking is a major development in banking sector, which given the service of 24 hour banking to the client. Tele banking depends on the voice handling service accessible on bank computers. The caller more often than not a client calls the bank anytime and can ask about the balance in his account or other exchange details happened previously. In this framework, the computers at bank are associated to a phone interface with the assistance of a modem. Voice processing service given within the software. This computer program recognizes the voice of caller and gives him appropriate reply. Some banks utilize telephonic replying machine but usually constrained to a few brief capacities. This can be as it were phone replying framework and presently known as Tele-banking. Tele banking is getting to be well known since inquiries at Automated Teller Machines are presently getting to be as well long.

• AUTOMATED TELLER MACHINE:

Automated Teller Machine (ATM) is an electronic machine, which accessed by the client himself to operate the functions of withdrawals, deposits and other financial exchanges. ATM could be a step of advancement in providing facilities to clients. ATM service is accessible to the clients 24 hours a day. The banks provided an ATM card to clients. It is usually a plastic card, coded magnetically which can be easily recognized by the ATM and it had the client‘s name.

Each cardholder is given with a confidential personal identification number (PIN). When the client needs to utilize the card, he needs to embed his plastic card within the opening of the machine. After the card may be acknowledged by the ATM, the client enters his PIN. After setting up the verification of the clients, the ATM takes after the client to enter the sum to be pulled back by him. After preparing that exchange and finding adequate equalizations in his account, the yield space of ATM grant the specified cash to client. When the exchange is completed, the ATM expels the customer‘s card.

• PHONE BANKING:

Clients can presently dial up the bank‘s outlined phone number and he by dialing his ID number will be accomplished to induce network to bank‘s designated computer. The computer program given within the machine associated with the computer inquiring him to dial the code number of facility needed by him and appropriately answers him. By utilizing Automatic voice recorder (AVR) for straightforward inquiries and exchanges and staff phone terminals for complicated questions and exchanges, the client

• **MOBILE BANKING:**

Mobile banking service is an expansion of web banking. The bank is in affiliation with the cellular facility suppliers provide this service. For this facility, mobile phone ought to either be SMS or WAP empowered. These services are accessible indeed to those clients who had credit card accounts with the bank.

• **INTERNET BANKING:**

Web Banking empowers a client to do banking exchanges functions through the bank's website with the help of internet. It could be a framework of retrieving accounts and common data on bank services, products and different facilities by a computer or mobile phone while the client sits in his home or office. Moreover, it is typically called virtual banking. It is just like escorting the bank in to your home whether by computer or by mobile phone. In conventional banking one should approach the department in individual, to deposit money, to deposit a cheque, to withdraw cash or ask an articulation of accounts etc. but now internet banking had modified the way of banking.

Presently everybody can easily work out all these sort of exchanges through website of bank on his mobile phone or computer with the help of internet. All these sort of exchanges are encoded utilizing advanced multi-layered security design, consisting firewalls and filters. The client can be rest guaranteed that his exchanges are assured and secret.

• **CASH DISPENSERS:**

Taking out your deposited money is the fundamental facility provided by the branches of banks. The money given to client by cashier or teller of money dispensers is an interchange to save the time. The functions performed by this machine are so economical as compared to manual functions and this machine is inexpensive and quick as compared to Automated Teller Machine. A plastic card issued to client, which had coated magnetically. After fulfilling the conventions, software permits the machine to perform the exchanges functions for needed sum.

• **CHIP CARD:**

The client of the bank is given an uncommon sort of credit card which had a special code and client's name etc. The credit sum of the client account is recorded on the card with magnetic strategies. The software in the bank's machines or bank's computer can easily recognize these magnetic spots. At the time, when client utilizes this credit card, the credit sum recorded on the card begins diminishing. After utilize of number of times, at one organize, the total amount gets to be nil on the card. At that point of time, the card lost its utility. The client should deposit money in his bank account for availing the service of again utilizing the card. Once more the credit sum is recorded on the card with the help of magnetic strategies.

• ELECTRONIC CLEARING SERVICE:

In 1994, RBI scheduled a committee to investigate the computerization within the banks additionally to evaluate the electronic clearing Service. The committee prescribed in its final report that electronic clearing service or credit clearing service ought to be mould accessible to all Government organizations or corporate institutions for doing monotonous little worth remittance like discount, compensation, interest, monthly installments, salary, pension, commission and dividend.

it was too prescribed by the committee that one more function of electronic clearing service which is debit clearing may be launched for pre-authorized charges for remittance of insurance premium, segments of leasing, utility bills, and financing institutions. RBI had taken vital movements to launch these plans, at first in Chennai, Mumbai, Calcutta and New Delhi.

• SOCIETY FOR WORLDWIDE INTER-BANK FINANCIAL TELECOMMUNICATIONS :

SWIFT is an acronym of society for worldwide inter-bank financial telecommunications. It is a co-operative society was established in May 1973 with 239 engaging banks from 15 nations with its central command offices at Brussels. It had begun working in May 1977. Reserve Bank of India and 27 other public segment banks along with 8 foreign banks in India had gotten the association of the SWIFT. SWIFT gives fast, reliable, dependable and fetched compelling mode of communicating the financial information around the globe. Nowadays, more than 3000 banks are the participants of this system. To serve to the improvement in information, SWIFT was overhaul within the 80s and this form is pronounced as SWIFT-II.

Indian banks are snared to SWIFT-II framework. SWIFT can be a strategy of the advanced information communication of universal notoriety. This technique is super economical, dependable and secure method of money exchange. It encourages the communication of information regarding the interest installment, fixed deposit, debit-credit articulations, foreign trade etc. This facility is accessible all through the year, 24 hours a day. It guarantee against any misfortune of disfigurement regarding exchange. It is evident from the above stated advantages of SWIFT that it is exceptionally advantageous in viable client facility. SWIFT had expanded its scope to clients like dealers, institutions and other organizations.

• REAL TIME GROSS SETTLEMENT :

Real time gross settlement system acts as a money exchange framework. Settlement in real time implies the exchanges occurred nearly promptly and gross settlement implies exchange is cleared one to one premise not at all like national electronic fund transfer (NEFT), where the exchange occurred in large size at a given point of time amid the whole day. It is typically primarily utilized for exchange which is huge in value and ought to be cleared promptly. In this procedure the bank that gets cash must credit the sum within the account with in 30 min after accepting it. Facilities of RTG‘s window for exchange are accessible to banks from 9.00 am to 4.30 pm in a week and 9.00 am to 4.00 pm in Saturday‘s for clearance at from RBI‘s segment.

CONCLUSION:

Within the days to come, banks are supposed to romp a really valuable part within the financial and economical improvement and the rising market will produce the trading openings to saddle. As banking in India will gotten to be increasingly information backed, capital will appear as the finest resources of the banking segment. Eventually banking is individuals and not fair numerals. Conclusively, the banking segment in India is advancing with the expanded development in client base, because of the recently enhanced and inventive services provided by banks.

The financial and economical development of the nation is a pointer for the development of the banking segment. The Indian economy is anticipated to develop at a rate of 7-8 % and the nation‘s banking system is anticipated to contemplate this development. The

responsibility of it resides within the potential of the Reserve Bank of India as a central administrative specialist, whose approaches had protected Indian banks from intemperate clouting and creating major risk capitations. By the government assistance and a cautious re-examination of subsisting trade techniques can made the platform for Indian banks to gotten to be greater and more powerful, subsequently making the platform for developments into a worldwide client foundation.

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E-COMMERCE: MEANING, BENEFITS AND GROWTH IN INDIA

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A B S T R A C T

The E-Commerce market is thriving and poised for robust growth in Asia.E-commerce involves an online transaction. Electronic commerce is a term for any type of business, or commercial transaction that involves the transfer of information across an electronic network, primarily the Internet. EC has expanded rapidly over the past decade and is predicted to continue at this rate, or even accelerate because it allows consumers to exchange goods and services with no barriers of time or distance and it is often faster, cheaper and more convenient than the traditional methods of commerce. Electronic commerce as part of the information technology revolution became widely used in the world trade in general and Indian economy in particular.

This paper is outcome of a review of various research studiescarried out on Impact of E commerce on Indian Commerce.The purpose the study is to explore and bring about the benefits of e-commerce in Indian context. E-commerce has seen unprecedented growth in India in the last decade.450

Though the e-commerce is benefiting the business and society at large there are some challenges and limitations also the main being that of financial security, trust, delivery and human less transaction.

Key words: E-commerce, India, internet, globe,impact,benefits,online.

INTRODUCTION

E-commerce has so many advantages in our life because it makes convenient in daily life of the people. E-commerce stands for electronic commerce and pertains to trading in goods and services through the electronic medium, i.e. the Internet or phone. It can be basically defined as the production, promotion, selling and distribution of products and services in an online environment. As with e-commerce, e-business also has a number of different definitions and is used in a number of different contexts. E-commerce evolved in various means of relationship within the business processes. It can be in the form of electronic advertising, electronic payment system, electronic marketing, electronic customer support service and electronic order and delivery.

Today, major corporations are rethinking their businesses in terms of the Internet and its new culture and capabilities and this is what some see as e-business. E-commerce has an impact on three major stakeholders, namely society, organizations and customers. The cutting edge for business today is e-Commerce. The effects of e-commerce are already appearing in all areas of business, from customer service to new product design. It facilitates new types of information based business processes for reaching and interacting with customers like online advertising and marketing, online order taking and online customer service etc. It can also reduce cost in managing orders and interacting with a wide range of suppliers and trading partners, areas that typically add significant overheads to the cost of products and services

The objectives of this article are to present a snapshot of the evolution of e-commerce business indicating the chronological order, category of e commerce business, description of organizations involved in e-businesses in India, key characteristics of the firms engaged in e-commerce application, to examine the growth of e-commerce in both physical and financial terms, to evaluate the benefits obtained from e-business and to develop a framework for effective dissemination of e-commerce in India.

Distinct categories of e-commerce:-

Four distinct categories of electronic commerce can be identified as follows:

• *Business-to-business (B2B):*

Business-to-business (B2B) is commerce transactions between businesses, such as between a manufacturer and a wholesaler, or between a wholesaler and a retailer. Pricing is based on quantity of order and is often negotiable.B2B transactions are largely between industrial manufacturers, partners, and retailers or between companies. Business-to-Business refers to the full spectrum of e-commerce that can occur between two organizations. Among other activities, B2B ecommerce includes purchasing and procurement, supplier management, inventory management, channel management, sales activities, payment management, and service and support.

• *Business-to-Consumer (B2C):*

Business or transactions conducted directly between a company and consumers who are the end-users of its products or services. B2C transactions take place directly between business establishments and consumers. Although business-to-business transactions play an important part in e-commerce market, a share of e-commerce revenues in developing countries like India is generated from business to consumer transactions.Business-toConsumer e-commerce refers to exchanges between businesses and consumers. Similar transactions that occur in business-to businesse-commerce also take place in the businessto-consumer context. No doubt, the total value of the B2B transactions is muchlarger than that of the B2C transactions, because typically B2B transactions are of much greater value thanB2C transactions.

• *Consumer-to-Consumer (C2C):*

Customer to Customer (C2C) markets are innovative ways to allow customers to interact with each other. While traditional markets require business to customer relationships, in which a customer goes to the business in order to purchase a product or service. In customer to customer markets the business facilitates an environment where customers can sell these goods and or services to each other.C2C sites don't form a very high portion of web-based commerce. Most visible examples are the auction sites. Basically, if someone has something to sell, then he gets it listed at an auction sites and others can bidfor it. Consumer-to-Consumer exchanges involve transactions between and among consumers. Theseexchanges may or may not include third-party involvement as in the case of the auction-exchange eBay.

• *Consumer-to-Business (C2B):*

Consumer-to-business (C2B) is a business model in which consumers individuals create value and businesses consume that value. C2B model, also called a reverse auction or demand collection model, enables buyers to name or demand their own price, which is often binding, for a specific good or service. The website collects the demand bids then offers the bids to participating sellers. Consumers can band together to form and present themselves as a buyer group to businesses in aconsumer-to-business relationship.

Benefits of e-commerce:-

The benefits of e-commerce include it's the speed of access, a wider selection of goods and services, accessibility, and international reach. E-commerce can have good effects on society which are enables more individuals to work at home, and to do less traveling for shopping, resulting in less traffic on the roads, and lower air pollution, allows some merchandise to be sold at lower prices benefiting the poor ones, enables people in Third World countries and rural areas to enjoy products and services which otherwise are not available to them, facilitates delivery of public services at a reduced cost, increases effectiveness, and/or improves quality. Today, in every aspect of our day to day life internet has become undivided part of our life.

One of the ecommerce benefits is that it has a lower startup cost. Physical retail stores have to pay up to thousands of dollars to rent one of their store locations. Another advantage is that online stores are always open for business. Moreover it's easy to scale the business quickly. You can increase your ad budget when ads are performing well without having to worry too much about keeping up with the demand, especially if you dropship. It's easy to create retargeting ads to retarget customers in your area when running an online business making it one of the most profitable ecommerce benefits.

Another one of the ecommerce benefits is that getting your customers to become impulse buyers is possible. If you have an attractive product photography, one with vibrant color or human emotion, you can create ads that drive impulse buys. Ecommerce benefits like being able to easily display best-sellers makes it easier to show off products to customers. While you can design a brick and mortar store to sway people to buy certain products, it's easier for a customer to find the best-sellers in an online store. Next on the list of ecommerce benefits is that a new brand can sell to customers around the world easily.

E-commerce: growth and prospects in India:-

E-commerce in India is still in budding stage but it offers extensive opportunity in developing countries likeIndia. India's ecommerce industry is on the growth curve and experiencing a surge in growth.Increasing internet and mobile penetration, growing acceptability of online payments and favorable demographics has provided the ecommerce sector in India the unique opportunity to companies connect with their customers. There would be over a five to seven fold increase in revenue generated through e-commerce as compared to last year with all branded apparel, accessories, jewellery, gifts, footwear are available at a cheaper rates and delivered at the doorstep. Many sites are now selling a diverse range of products and services from flowers, greeting cards, and movietickets to groceries, electronic gadgets, and computers. With stock exchanges coming online the time fortrue e-commerce in India has finally arrived.

On the negative side, there are many challenges faced bye-commerce sites in India. The relatively small credit card population and lack of uniform credit agenciescreate a variety of payment challenges unknown in India. Delivery of goods to consumer by couriers andpostal services is not very reliable in smaller cities, towns and rural areas. India has less credit card population, lack of fast postal services in rural India. Accessing the Internet iscurrentlyhindered down by slow transmission speeds, frequentdisconnects, cost of Wireless connection and wirelesscommunication standards over which data is transmitted.High-speed-bandwidth Internet connection is not available tomost citizens of the nation at an affordable rate. In India,mostly people are not aware about the English language or not so good in English language. So that for the transaction over internet through electronic devices, language becomes one of the major factors to purchases, hire and sell a particularproduct or services. Multiple issues of trust in e-commercetechnology and lack of widely accepted standards, lack ofpayment gateways, privacy of personal and business dataconnected over the Internet not assured security andconfidentiality of data not in place to deploy ubiquitous ITInfrastructure and its maintenance.However, many Indian Bankshave put the Internet banking facilities. The speed post and courier system has also improved tremendouslyin recent years. Modern computer technology like secured socket layer helps to protect againstpayment fraud, and to share information with suppliers and business partners. With further improvement inpayment and delivery system it is expected that India will soon become a major player in the e-commercemarket.While many companies, organizations, and communities in India are beginning to take advantage of the potential of e-commerce, critical challenges remain to be overcome before e-commerce would become an asset for common people.

Conclusion:-

E-commerce has undeniably become an important part of our society. The World Wide Web is and will have a large part in our daily lives. It is therefore critical that small businesses have their own to keep in competition with the larger websites. With the explosion of internet connectivity through mobile devices like Smartphone and tablets, millions of consumers are making decisions online and in this way enterprises can build the brand digitally and enhance productivity but government policies must ensure the cost effective methods/solutions.Advantages of e-commerce are cost savings, increased efficiency, and customization. In order to understand electronic commerce it is important to identify the different terms that are used, and to assess their origin and usage. These include information overload, reliability and security issues, and cost of access, social divisions and difficulties in policing the Internet. Successful e-commerce involves understanding the limitations and minimizing the negative impact. Ecommerce in India is destined to grow both in revenue and geographic reach. The challenge of establishing consumer trust in e-commerce poses problems and issues that need further research.

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COINCIDENCE AND FIXED POINT THEOREMS IN

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A B S T R A C T

The existence of fixed point theory for continuous mappings on Hausdorff topological spaces and regular compact space are proved. Our results are different from known, or are generalizations, extensions and improvements of the corresponding results due to Ciric, Jungck, Liu and Liu et al. Further, the Edelstein result for contractive mappings is extended to Hausdorff (not necessarily completely regular) topological spaces and generalized in many aspects. Examples and Theorems is presented to show that our results are genuine generalizations of the Edelstein result.

Keywords: Fixed point, Hausdorff topological spaces, Pseudo-compact space

INTRODUCTION

The purpose of this paper is to provide sufficient conditions for the existence and uniqueness of a fixed point and existence theorems of coincidence point of continuous mapping on Hausdorff topological spaces.

The fixed point theory is a combination of analysis, topology and geometry. The theory of existence of fixed points of maps has been depicted as very important tools in the study of non-linear phenomena. The fixed point theory is much extended when a topological space $(X, \mathfrak{I})$ is a metric space or a linear topological space. On the other side, if $(X, \mathfrak{I})$ possess a topological structure only, in such types of spaces the fixed point theory is very rigid. The results for Hausdorff topological spaces are different from known, or are generalization and improvement of the Edelstein [3] result for contractive mappings.

The material of this paper has been derived from the paper of Ciric [2] in which he improved and extended the result due to Jungck [4], Liu [5] and Liu et al. [6].

In 2014, Shah, Hussain and Ciric [7] generalized, extended and improved the results given by Ciric [2], Jungck [4], Liu [5] and Liu et al [6]. Ciric [2] worked on completely regular space for existence of a fixed point and Hausdorff topological space for the uniqueness of a fixed point. But Shah, Hussain and Ciric [7] extended the results and proved that unique fixed point exists on compact topological space.

The material of my paper has been extended from the paper of Ciric and research paper of Shah, Hussain and Ciric.

Thetheorem (12) is generalization and improvement of the results due to Ciric [2], Edelstein [3], Jungck [4] and Liu et al. [6].

Notation 1 : $\psi = \{F\}$ where $F : X \times X \to [0, \infty)$ is continuous, symmetric and such that $F(x, y) = 0$ if and only if $x = y$.

Theorem 1 : Let $(X, \Im)$ be a completely regular topological space, K be a non-empty pseudo-compact subset of X and $T : K \to K$ be a continuous self mapping on X. Suppose that for some $F \in \Psi$, a mapping T satisfies the following condition:

$$F(T^n(x), T^n(y)) < \max\{F(x, y), \min\{F(x, T(x)), F(y, T(y))\}\} + \lambda \min\{F(x, T(y)), F(y, T(x))\} \tag{1.1}$$

for all $x \neq y$; $x, y \in K$, where $n = n(x, y)$ is a positive integer and λ is an arbitrary positive real number. Then T has at least one fixed point.

Proof : As F and T are continuous functions, this implies that the function $F(x, T(x))$ is continuous on K. Since K is a pseudo-compact subset of X, there exists a point, say $w \in K$ such that

$$F(w, T(w)) = \inf\{F(x, T(x)) : x \in K\}. \tag{1.2}$$

Now, to prove that $T(w) = w$. Let us consider, to the contrary, that $w \neq T(w)$. Then from (1.1),

$$\begin{aligned} F(T^n(w), T^n(T(w))) &< \max\{F(w, T(w)), \min\{F(w, T(w)), F(T(w), T(T(w)))\}\} \\ &\quad +\lambda \min\{F(w, T(T(w))), 0\} \\ &= \max\{F(w, T(w)), \min\{F(w, T(w)), F(T(w), T(T(w)))\}\}. \end{aligned}$$

Thus, since $F(T^n(w), T^n(T(w))) = F(T^n(w), T(T^n(w)))$ and, by (1.2),

$$\min\{F(w, T(w)), F(T(w), T(T(w)))\}\} = F(w, T(w)),$$

we obtained

$$F(T^n(w), T(T^n(w))) < F(w, T(w)),$$

a contradiction by (1.2). Hence, our assumption $w \neq T(w)$ is wrong. So, w is the fixed point of T.

Theorem 2 : Let $(X, \Im)$ be a completely regular topological space, K be a non-empty pseudo-compact subset of X and $T : K \to K$ be a continuous self mapping on X. Suppose that for some $f \in \Psi$, a mapping T satisfies the following condition :

$$F(T^n(x), T^n(y)) < \max\{F(x, y), [\min\{F(x, T(x), F(y, T(y))\} + \min\{F(x, T(y)), F(y, T(x))\}]\} \quad (1.3)$$

for all $x \neq y$; $x, y \in K$, where $n = n(x, y)$ is a positive integer. Then T has a unique fixed point.

Proof : It is clear that (1.3) implies (1.1). Hence, by theorem 1, there exist some $w \in K$ such that $T(w) = w$. Consider that, there is $v \in K$ such that $T(v) = v$ and $v \neq w$. Then by (1.3),

$$\begin{aligned} F(v, w) &= F(T^n(v), T^n(w)) \\ &< \max\{F(v, w), [\min\{F(v, T(v)), F(w, T(w))\} + \min\{F(v, w), F(w, v)\}]\} \\ &= F(v, w), \end{aligned}$$

a contradiction. Thus, T has a unique fixed point.

Theorem 3: Let (X, d) be a metric space and T be a self mapping of X such that

$$d(T(x), T(y)) < d(x, y) \text{ for all } x \neq y;\ x, y \in X.$$

If there exists a point $x \in X$ whose sequence of iterates $\{T^n(x)\}$ contains a convergent subsequence $\{T^{n_i}(x)\}$, then $\xi = \lim_{i\to\infty} T^{n_i}(x) \in X$ is a unique fixed point of T.

Theorem 4: Let $(X, \Im)$ be a Hausdorff topological space and $T : X \to X$ be a continuous self mapping and for some $F \in \Psi$,

$$F(T(x), T(y)) < \max\{F(x, y), F(x, T(x)), F(y, T(y))\} + \lambda \min\{F(x, T(y)), F(y, T(x))\}, \quad (1.4)$$

for all $x \neq y$; $x, y \in X$, where $\lambda \geq 0$. If there exists a point $x \in X$ whose sequence of iterates $\{T^n(x)\}$ contains a convergent subsequence $\{T^{n_i}(x)\}$, then $\xi = \lim_{i\to\infty} T^{n_i}(x) \in X$ is a fixed point of T. If T satisfies (1.4) with $\lambda = 0$, then T has a unique fixed point.

Proof: Suppose that $x_0 \in X$ is such that a subsequence $\{T^{n_i}(x)\}$ is convergent. Let $x_n = T^n(x_0)$, $n \geq 1$. We may consider that $x_{n+1} \neq x_n$ for all n. For this, if we suppose on the contrary, that $x_{n_0+1} = T(x_{n_0}) = x_{n_0}$ for some n_0, then $x_n = x_{n_0}$ for each $n \geq n_0$. Thus $x_{n_0} = T(x_{n_0}) = \xi = T(\xi)$ and hence the proof.

For any $i \geq 1$, consider

$n_{i+1} = n_i + p_i$,

where $p_i = n_{i+1} - n_i \geq 1$. Then for $p_i > 1$, by (2.1.4),

$$F\left(x_{n_{i+1}}, T\left(x_{n_{i+1}}\right)\right) = F\left(x_{n_i+p_i}, T\left(x_{n_j+p_i}\right)\right) = F\left(T\left(x_{n_i+p_i-1}, T\left(T\left(x_{n_i+p_i-1}\right)\right)\right)\right)$$

$$< \max\left\{F\left(x_{n_i+p_i-1}, T\left(x_{n_i+p_i-1}\right)\right), F\left(x_{n_i+p_i-1}, T\left(x_{n_i+p_i-1}\right)\right), F\left(x_{n_i+p_i}, T\left(x_{n_i+p_i}\right)\right)\right\}$$

$$+\lambda \min\left\{F\left(x_{n_i+p_i-1}, T\left(x_{n_i+p_i}\right)\right), F\left(x_{n_i+p_i}, T\left(x_{n_i+p_i-1}\right)\right)\right\}$$

$$= \max\left\{F\left(x_{n_i+p_i-1}, T\left(x_{n_i+p_i-1}\right)\right), F\left(x_{n_i+p_i-1}, T\left(x_{n_i+p_i-1}\right)\right), F\left(x_{n_i+p_i}, T\left(x_{n_i+p_i}\right)\right)\right\}$$

$$= \max\left\{F\left(x_{n_i+p_i-1}, T\left(x_{n_i+p_i-1}\right)\right), F\left(x_{n_i+p_i}, T\left(x_{n_i+p_i}\right)\right)\right\}.$$

Thus, since $F\left(x_{n_i+p_i}, T\left(x_{n_i+p_i}\right)\right) < F\left(x_{n_i+p_i}, T\left(x_{n_i+p_i}\right)\right)$ is impossible,

hence, $F\left(x_{n_i+p_i}, T\left(x_{n_i+p_i}\right)\right) < F\left(x_{n_i+p_i-1}, T\left(x_{n_i+p_i-1}\right)\right)$. (1.5)

Continuing the process, we obtain

$$F\left(x_{n_i+p_i}, T\left(x_{n_i+p_i}\right)\right) < F\left(x_{n_i+p_i-1}, T\left(x_{n_i+p_i-1}\right)\right)$$

$$< F\left(x_{n_i+p_i-2}, T\left(x_{n_i+p_i-2}\right)\right)$$

$$\ldots$$

$$< F\left(x_{n_i+1}, T\left(x_{n_i+1}\right)\right)$$

$$< F\left(T\left(x_{n_i}\right), T^2\left(x_{n_i}\right)\right).$$

Hence, if $p_i > 1$, then

$$F\left(x_{n_{i+1}}, T\left(x_{n_i+1}\right)\right) < F\left(T\left(x_{n_i}\right), T^2\left(x_{n_i}\right)\right).$$

Clearly, if $p_i = 1$, then $x_{n_{i+1}} = x_{n_i} + 1 = T\left(x_{n_i}\right)$ and therefore

$$F\left(x_{n_{i+1}}, T\left(x_{n_{i+1}}\right)\right) = F\left(T\left(x_{n_i}\right), T^2\left(x_{n_i}\right)\right).$$

So, for all $p_i \geq 1$,

$$F\left(x_{n_{i+1}}, T\left(x_{n_{i+1}}\right)\right) \leq F\left(T\left(x_{n_i}\right), T^2\left(x_{n_i}\right)\right). \quad (1.6)$$

As T is continuous and $x_{n_i} \to \xi$ as $i \to \infty$, we obtain that $T\left(x_{n_i}\right) \to T(\xi)$, $T\left(x_{n_{i+1}}\right) \to T(\xi)$ and $T^2\left(x_{n_i}\right) \to T^2(\xi)$ as $i \to \infty$. Hence, taking the limit in (1.6) as $i \to \infty$, we obtain

$$F(\xi, T(\xi)) \leq F(T(\xi), T^2(\xi)). \quad (1.7)$$

Now we prove that $T(\xi) = \xi$. Assume, to the contrary, that $T(\xi) \neq \xi$. Then by (1.4),

$$F(T(\xi), T^2(\xi)) = F(T(\xi), T(T(\xi))$$

$$< \max\{F(\xi, T(\xi)), (\max\{F(\xi, T(\xi)), F(T(\xi), T^2(\xi))\} + \lambda \min\{F(\xi, T^2(\xi)), F(T(\xi), T(\xi))\})\}$$

$$= \max\{F(\xi, T(\xi)), \max\{F(\xi, T(\xi)), F(T(\xi), T^2(\xi))\}\}$$

$$= \max\{F(\xi, T(\xi)), F(T(\xi), T^2(\xi))\}.$$

Thus, since $F(T(\xi), T^2(\xi)) < F(T(\xi), T^2(\xi))$ is impossible, we get

$$F(T(\xi), T^2(\xi)) < F(\xi, T(\xi)),$$

which is a contradiction by (1.7).

Hence, our assumption is wrong.

Thus, $T(\xi) = \xi$.

Next, to prove that if T satisfies (1.4) with $\lambda = 0$, then T has a unique fixed point.

For **uniqueness**, let us suppose that x and y are two different fixed points of T with $\lambda = 0$.

Then, $T(x) = x$ and $T(y) = y$.

By (1.4), we obtain

$$\begin{aligned} F(T(x), T(y)) &< \max\{F(x, y), F(x, T(x)), F(y, T(y))\} \\ &= \max\{F(x, y), F(x, x), F(y, y)\} \\ &= F(x, y) \\ &= F(T(x), T(y)), \end{aligned}$$

which is a contradiction. Hence, our supposition is wrong.

Therefore, T has a unique fixed point with $\lambda = 0$.

This completes the proof.

Theorem 5: Let $(X, \Im)$ be a Hausdorff topological space and $T : X \to X$ be a continuous self mapping and for some $F \in \psi$,

$$F(T(x), T(y)) < \max\{F(x, y), [\max\{F(x, T(x)), F(y, T(y))\} + \min\{F(x, T(y)), F(y, T(x))\}]\}. \quad (1.8)$$

If there exists a point $x \in X$ whose sequence of iterates $\{T^n(x)\}$ contains a convergent subsequence $\{T^{n_i}(x)\}$, then $\xi = \lim_{i\to\infty} T^{n_i}(x) \in X$ is a unique fixed point of T.

Proof: The proof is similar to the proof of theorems 1 and 2.

Example which shows that Theorem 3 is a genuine generalization of the above result.

Theorem 6: Let $(X, \Im)$ be a completely regular topological space, K be a nonempty pseudo-compact subset of X and T, S, G and H be continuous self mappings on K such that T and S are surjective on K, T commutes with G and H, and S commutes with G and H. If, for some $F \in \psi$, mappings T, S, G and H satisfy the following conditions :

$$F(T(x), S(y)) > \inf\{F(t, P(t)), F(Q(t), P(t)), F(Q(t), P(Q(t))), F(Q(x), Q(y)): t \in \{x, y\}, P \in \{T, S\} \text{ and } Q \in \{G, H\}\}, \quad (1.9)$$

for any $x, y \in K$ with $T(x) \neq S(y)$, then at least one of the following assertions holds:

(1) T has a fixed point in K;

(2) S has a fixed point in K;

(3) T and G have a coincidence point in K;

(4) T and H have a coincidence point in K;

(5) S and G have a coincidence point in K;

(6) S and H have a coincidence point in K.

Proof: As K is a pseudo-compact and F, T, S, G and H are continuous functions, it obtains that the functions $F(x, P(x))$ and $F(Q(x), P(x))$ with $P \in \{T, S\}$ and $Q \in \{G, H\}$, are continuous on K, and that there exist a, b, p, q, r and $s \in K$ such that

$$F(a, T(a)) = \inf \{F(x, T(x)) : x \in K\};$$

$$F(b, S(b)) = \inf \{F(x, S(x)) : x \in K\};$$

$$F(G(p), T(p)) = \inf\{F(G(x), T(x)) : x \in K\};$$

$$F(H(q), T(q)) = \inf\{F(H(x), T(x)) : x \in K\};$$

$$F(G(r), S(r)) = \inf\{F(G(x), S(x)) : x \in K\};$$

$$F(H(s), S(s)) = \inf\{F(H(x), S(x)) : x \in K\}.$$

Consider the following cases:

Case 1 : Let

$$F(a, T(a)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\}. \quad (1.10)$$

As $S(K) = K$, there exist some $w \in K$ such that $S(w) = a$. Consider that $T(S(w)) \neq S(w)$, that is, $T(a) \neq a$.

As S commutes with G and H, from (1.9), we obtain

$$F(T(S(w)), S(w)) > \inf\{F(t, P(t)), F(Q(t), P(t)), F(Q(t), PQ(t)), F(Q(S(w)), Q(w)\}$$

$$: t \in \{S(w), w\}, P \in \{T, S\} \text{ and } Q \in \{G, H\}\}$$

$$\geq \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)),$$

$$F(H(q), T(q)), F(G(r), S(r), F(H(s), S(s))\}.$$

Thus, from (1.10), we obtain

$$F(a, T(a)) > F(a, T(a)), \text{ which is a contradiction.}$$

Hence, $T(a) = a$, that is, a is the fixed point of T.

Case 2 : Consider that

$$F(G(r), S(r)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)),$$

$$F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\}. \qquad (1.11)$$

As $T(K) = K$, there exist some $z \in K$ such that $T(z) = r$.

Consider that $S(T(z)) \neq G(T(z))$, that is, $S(r) \neq G(r)$.

By (1.9), we obtain

$$F(S(T(z)), T(G(z))) > \inf\{F(t, P(t)), F(Q(t), P(t)), F(Q(t), P(Q(t))),$$

$$F(Q(T(z)), Q(G(z))): t \in \{T(z), G(z)\}, P \in \{T, S\}$$

$$\text{and } Q \in \{G, H\}\}$$

$$\geq \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), F(H(q), T(q)),$$

$$F(G(r), S(r)), F(H(s), S(s))\}.$$

Thus, since $F(S(T(z)), T(G(z))) = F(S(T(z)), G(T(z))) = F(S(r), G(r))$, from (1.11), we obtain

$$F(G(r), S(r)) > F(G(r), S(r)), \text{ which is a contradiction.}$$

Hence, $G(r) = S(r)$ which means that r is the coincidence point of S and G.

Hence, proved assertions (1) and (5).

Case 3: Considering the remaining cases

$$F(b, S(b)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)),$$

$$F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\},$$

$$F(G(p), T(p)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)),$$

$$F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\},$$

$$F(H(q), T(q)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\},$$

or

$$F(H(s), S(s)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p), F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\}$$

Similarly, as in the proof of case 1, or case 2, the assertions (2), (3), (4) and (6) hold.

Lemma 1. Let X be a compact topological space and $f : X \to R$ is a function lsca. Then there exists $x_0 \in X$ such that $f(x_0) = \inf\{f(x) : x \in X\}$.

Proof : Suppose that f is lower semi-continuous from above on X. There exists a net $(x_t) \subset X$ such that $f(x_{t'}) \leq f(x_t)$ if $t' \geq t$ and $f(x_t) \to \inf_{x\in X} f(x)$. Since X is compact, without loss of generality, we may suppose that $x_t \to x_0$. By the lower semi-continuity from above of f(x), we obtain $f(x_0) \leq \lim_t f(x_t)$ and therefore

$$f(x_0) = \inf_{x\in X} f(x).$$

Lemma 2 : Let X be a topological space and $f : X \times X \to X$ be a continuous function. If $g : X \to R$ is a lsca function, then the composition function $F = g\circ f : X\times X \to R$ is also lsca.

Proof: Let $(x_0, y_0) \in X \times X$ and consider a net $\{(x_\lambda, y_\lambda)\}_{\lambda\in\Lambda}$ in $X \times X$ converging to (x_0, y_0) such that, for $\lambda_2 \leq \lambda_1$,

$$F(x_{\lambda_1}, y_{\lambda_1}) \leq F(x_{\lambda_2}, y_{\lambda_2})$$

Put $z_\lambda = f(x_\lambda, y_\lambda)$ and $z = f(x_0, y_0)$. Then, as f is continuous and g is lsca, $\lim_{\lambda\in\Lambda} f(x_\lambda, y_\lambda) = f(x_0, y_0) \in X$ and

$$g(z) = g(f(x_0, y_0)) \leq \lim_{\lambda\in\Lambda} g(f(x_\lambda, y_\lambda)) = \lim_{\lambda\in\Lambda} g(z_\lambda)$$

where $\quad g(z_{\lambda_1}) \leq g(z_{\lambda_2})$ for $\lambda_2 \leq \lambda_1$.

Remark: Let X be a topological space. Let $f : X \to X$ be a continuous function and $F : X \times X \to R$ be a lsca function. Then $g: X \to R$ defined by $g(x) = F(x, f(x))$ is also lsca. In fact, let $\{x_\lambda\}_{\lambda\in\Lambda}$ be a net in X converging to a point $x \in X$.

As f is continuous, so $\lim_{\lambda\in\Lambda} f(x_\lambda) = f(x)$

Let us consider that $g(x_{\lambda_1}) \leq g(x_{\lambda_2})$ for $\lambda_2 \leq \lambda_1$; then, since F is lsca, we obtain

$$g(x) = F(x, f(x)) \leq \lim_{\lambda\in\Lambda} F(x_\lambda, f(x_\lambda)) = \lim_{\lambda\in\Lambda} g(x_{\lambda_1})$$

and therefore g is lsca.

Theorem 7: Let X be a topological space, K be a nonempty compact subset of X and f : K $\to$ K be a continuous function. If $F \in \Phi$ and

$$F(fx, fy) < \max\{F(x, y), \min\{F(x, fx), F(y, fy)\}\} + \lambda \min\{|F(x, fy)|, F(fx, y)\} \quad (1.12)$$

for all $x, y \in K$ with $x \neq y$ and λ an arbitrary positive real number, then f has at least one fixed point.

Proof : Consider $\varphi : K \to (-\infty,\infty)$ which is defined by $\varphi(x) = F(x, fx))$ for $x \in K$.

Since f is continuous and F is lsca, therefore, from Remark, it obtains that the function φ is also lsca on K. By Lemma (1), there exists a point, say, $w \in K$ such that

$$\varphi(w) = F(w, f(w)) = \inf\{\varphi(x) : x \in K\}. \quad (1.13)$$

Now, to show that $f(w) = w$.

Let us consider, to the contrary, that $w \neq f(w)$.

Then by (1.12),

$$\begin{aligned} F(f(w), f(f(w))) &< \max\{F(w, f(w)), \min\{F(w, f(w)), F(f(w), f(f(w)))\}\} \\ &\quad + \lambda \min\{|F(w, f(f(w)))|, 0\} \\ &= \max\{F(w, f(w)), \min\{F(w, f(w)), F(f(w), f(f(w)))\}\}. \end{aligned}$$

From (1.13),

$$\min\{F(w, f(w)), F(f(w), f(f(w)))\} = F(w, f(w)).$$

Therefore, we obtain

$$F(f(w), f(f(w))) < F(w, f(w)),$$

which is a contradiction to (1.13).

Hence, $f(w) = w$, that is, w is a fixed point of f.

Corollary 1: Let X be a compact topological space, K be a closed subset of X and f: K $\to$ K be a continuous function. If $F \in \Phi$and

$$F(fx, fy) < \max\{F(x, y), \min\{F(x, fx), F(y, fy)\}\}+\lambda \min\{|F(x, fy)|, F(fx, y)\}$$

for all $x, y \in K$ with $x \neq y$ and λ is an arbitrary positive real number, then f has at least one fixed point.

Corollary 2: Let X be a topological space, K be a non-empty compact subset of X and f: K $\to$ K be a continuous function. If $F \in\Phi$ is symmetric and

$$F(fx, fy) < \max\{F(x, y), \min\{F(x, fx), F(y, fy)\}\}+\lambda \min\{|F(x, fy)|, F(fx, y)\}$$

for all x, y ∈ K with x ≠ y and λ is an arbitrary positive real number, then f has at least one fixed point.

Corollary 3: Let (X, d) be a metric space, K be a nonempty compact subset of X and f: K → K be a continuous function. If F ∈ Φ and

$$d(fx, fy) < \max\{d(x, y), \min\{d(x, fx), d(y, fy)\}\} +\lambda\min\{d(x, fy), d(fx, y)\}$$

for all x, y ∈ K with x ≠ y and λ is an arbitrary positive real number, then f has at least one fixed point.

Theorem 8: Let X be a topological space, K be a nonempty compact subset of X and f: K → K be a continuous mapping. Suppose that for F ∈ Φ, the mapping f satisfies the following condition :

$$F(f(x), f(y)) < \max\{F(x, y), [\min\{F(x, f(y)), F(y, f(y))\} + \min\{F(x, f(y)), F(f(x), y)\}]\} \quad (1.14)$$

for all x, y ∈ K with x ≠ y. Then f has a unique fixed point.

Proof: Since (1.12) is implied by (1.14), thus, by theorem (7), there exists some w ∈ K such that f(w) = w.

Next, to prove the **uniqueness** of fixed point, show that, for v, w ∈ K such that

$$f(v) = v \text{ and } f(w) = w \text{ then } v = w.$$

Let us consider that there is v ∈ K such that

$$f(v) = v,\ f(w)=w \text{ and } v \neq w.$$

Then by (1.14),

$$\begin{aligned} f(v, w) &= F(f(v), f(w)) \\ &< \max\{F(v, w), [\min\{F(v, f(v)), F(w, f(w))\} \\ &\qquad +\min\{F(v, f(w)), F(f(v), w)\}]\} \\ &= \max\{F(v, w), [\min\{F(v, v), F(w, w)\} \\ &\qquad +\min\{F(v, w), F(v, w)\}]\} \\ &= F(v, w), \end{aligned}$$

which is a contradiction. Thus v = w and hence, f has a unique fixed point.

Theorem 9: Let X be a topological space, K be a compact subset of X and f : K→K be a continuous function. If F ∈ Φand

$$F(f^n x, f^n y) < \max\{F(x, y), \min\{F(x, f^n x), F(y, f^n y)\}\}$$
$$+\lambda \min\{|F(x, f^n y)|, F(f^n x, y)\} \quad (1.15)$$

for all x, y $\in$ K with x $\neq$y; n = n(x, y) $\in$ N and λ is an arbitrary positive real number, then f has atleast one periodic point.

Proof : Let $\varphi : K \to (-\infty, \infty)$, defined by

$$\varphi(x) = F(x, f^n(x)), x \in K.$$

Since f is continuous and F is lsca, thus, from Remark, it obtains that the function φ is also lsca on K. By Lemma (1), there exists a point, say w $\in$ K such that

$$\varphi(w) = F(w, f^n(w)) = \inf\{\varphi(x) : x \in K\}. \quad (1.16)$$

To prove theorem, show that $f^n(w) = w$.

Let us consider, to the contrary, that $w \neq f^n(w)$.

Then by (1.15),

$$F(f^n(w), f^n(f^n(w))) < \max\{F(w, f^n(w)), \min\{F(w, f^n(w)), F(f^n(w), f^n(f^n(w)))\}\}$$
$$+\lambda\min\{|F(w, f^n(f^n(w)))|, 0\}$$
$$= \max\{F(w, f^n(w)), \min\{F(w, f^n(w)), F(f^n(w), f^n(f(w)))\}\}$$
$$+\lambda.0$$
$$= \max\{F(w, f^n(w)), \min\{F(w, f^n(w)), F(f^n(w), f^n(f^n(w)))\}\}.$$

Since from (1.16),

$$\min\{F(w, f^n(w)), F(f^n(w), f^n(f^n(w)))\}\} = F(w, f^n(w)),$$

we obtain

$$F(f^n(w), f^n(f^n(w))) < F(w, f^n(w)),$$

which is a contradiction to (1.16).

Hence, $f^n(w) = w$ for some n $\in$ N and thus w is a periodic point of f.

Theorem 10: Let X be a topological space, f: X $\to$ X be a continuous function. Let for F $\in \Phi$, the mapping f satisfying the contractive condition :

$$F(fx, fy) < \max\{F(x, y), \min\{F(x, fx), F(y, fy)\}\}$$
$$+\lambda\min\{|F(x, fy)|, F(fx, y)\}$$

for all x, y $\in$ X with x $\neq$ y and λ is an arbitrary positive real number. If, for $x_0 \in X$ and for some $K \subseteq X$

$$K = f(K) \cup \{x_0\} \Rightarrow K \text{ is relatively compact, then f has a fixed point.}$$

Proof : Let $x_1 = f(x_0)$ and consider the sequence $\{x_n\}$ in X as follows:

$$x_{n+1} = f(x_n), \text{ for } n \geq 1.$$

Suppose $A = \{x_n : n \geq 1\}$, then

$$A = \{x_{n+1} : n \geq 1\} \cup \{x_1\}$$
$$= f(A) \cup \{x_1\}$$

and so, by hypothesis, A is relatively compact.

Let $\varphi : \overline{A} \rightarrow R$, defined by

$$\varphi(x) = F(x, f(x)).$$

As f is continuous and F is lsca, so, from Remark, φ is lsca and thus, by Lemma (1), φ has a minimum, say, at $a \in \overline{A}$. Hence by theorem (7), a is a fixed point of f.

Result: Let X be a Hausdorff topological space and $f : X \rightarrow X$ be a continuous self mapping and such that for some $F \in \psi$,

$$F(f(x), f(y)) < \max\{F(x, y), F(x, f(x)), F(y, f(y))\}$$
$$+\lambda \min\{F(x, f(y)), F(y, f(x))\}, \qquad (1.17)$$

for all x, y $\in$ X with $x \neq y$ and $\lambda \geq 0$. If there exists a point $x \in X$ whose sequence of iterates $\{f^n(x)\}$ contains a convergent subsequence $\{f^{n_i}(x)\}$, then

$$\xi = \lim_{i \to \infty} f^{n_i}(x) \in X$$

is a fixed point of f. If f satisfies (1.17) with $\lambda = 0$, then f has the unique fixed point.

The following theorem is modification of the Result.

Theorem 11: Let X be a topological space, $f : X \rightarrow X$ be a continuous mapping and for some continuous

$$F : X \times X \rightarrow R \text{ (with } F(x, y) = 0 \text{ when } x = y),$$

$$F(f(x), f(y)) < \max\{F(x, y), [\max\{F(x, f(x)), F(x, f(y))\}$$
$$+\lambda \min\{F|x, f(y))|, F(f(x), y)\}]\}, \qquad (1.18)$$

for all x, y $\in$ X with $x \neq y$ and $\lambda \geq 0$. If there exists a point $x_0 \in X$ whose sequence of iterates $\{f^n(x_0)\}$ contains a convergent subsequence $\{f^{n_i}(x_0)\}$, then $a = \lim_{i \to \infty} f^{n_i}(x_0) \in X$ is a fixed point of f. If $\lambda = 0$, then f has a unique fixed point.

Proof : Suppose $x_0 \in X$ and consider the sequence $\{x_n\}$ in X such that $x_n = f^n(x_0)$, for $n \geq 1$.

Suppose $\{f^{n_i}(x_0)\}$ be a subsequence of $\{x_n\}$ such that $f^{n_i}(x_0) \to a \in X$, as $i \to\infty$. We may assume that $x_{n+1} \neq x_n$ for each n.

Indeed, if we suppose, to the contrary, that $x_{n_0+1} = f^{n_0+1}(x_0) = f(x_{n_0}) = x_{n_0}$ for some n_0, then $x_n == x_{n_0}$ for all $n \geq n_0$ and thus

$$x_{n_0} = f(x_{n_0}) = a = f(a) \text{ and hence the proof.}$$

For $i \geq 1$, set

$$p_i = n_{i+1} - n_i \geq 1, \text{ that is, } n_{i+1} = n_i + p_i.$$

Case 1 : Let $p_i > 1$. Then by (2.1.18), we obtain

$$\begin{aligned} F(x_{n_{i+1}}, f(x_{n_{i+1}})) &= F(x_{n_i+p_i}, f(x_{n_i+p_i})) \\ &= F(f(x_{n_i+p_i-1}), f(f(x_{n_i+p_i-1}))) \\ &< \max\{F(x_{n_i+p_i-1}, f(x_{n_i+p_i-1})), \\ &\qquad [\max\{F(x_{n_i+p_i-1}, f(x_{n_i+p_i-1})), F(x_{n_i+p_i}, f(x_{n_i+p_i}))\} \\ &\qquad + \lambda \min\{|F(x_{n_i+p_i-1}, f(x_{n_i+p_i-1}))|, F(f(x_{n_i+p_i-1}), x_{n_i+p_i})\}]\} \\ &= \max\{F(x_{n_i+p_i-1}, f(x_{n_i+p_i-1})), F(x_{n_i+p_i-1}, f(x_{n_i+p_i-1})), F(x_{n_i+p_i}, f(x_{n_i+p_i}))\} \\ &= \max\{F(x_{n_i+p_i-1}, f(x_{n_i+p_i-1})), F(x_{n_i+p_i}, f(x_{n_i+p_i}))\} \\ &= \max\{F(x_{n_i+p_i-1}, f(x_{n_i+p_i-1})), F(x_{n_i+1}, f(x_{n_i+1}))\}. \end{aligned}$$

Thus, since $F(x_{n_{i+1}}, f(x_{n_{i+1}})) = F(x_{n_i+p_i}, f(x_{n_i+p_i})) < F(x_{n_i+1}, f(x_{n_i+1}))$ is false, we obtain

$$F(x_{n_{i+1}}, f(x_{n_{i+1}})) = F(x_{n_i+p_i}, f(x_{n_i+p_i})) < F(x_{n_i+p_i-1}, f(x_{n_i+p_i-1})), \text{ where } p_i > 1. \qquad (1.19)$$

Continuing the above process, we obtain

$$\begin{aligned} F(x_{n_{i+1}}, f(x_{n_{i+1}})) &= F(x_{n_i+p_i}, f(x_{n_i+p_i})) \\ &< F(x_{n_i+p_i-1}, f(x_{n_i+p_i-1})) \\ &\;\;\cdots \\ &< F(x_{n_i+1}, f(x_{n_i+1})) \\ &= F(f(x_{n_i}), f^2(x_{n_i})). \end{aligned}$$

Hence, if $p_i > 1$, then

$$F(x_{n_{i+1}}, f(x_{n_{i+1}})) < F(f(x_{n_i}), f^2(x_{n_i})).$$

Case 2 : Suppose $p_i = 1$. Then

$$x_{n_{i+1}} = x_{n_i+1} = f^{n_i+1}(x_0) = f(f^{n_i}(x_0)) = f(x_{n_i}) \text{ and}$$

Therefore

$$F(x_{n_{i+1}}, f(x_{n_{i+1}})) = F(f(x_{n_i}), f^2(x_{n_i})).$$

Thus, for each $p_i \geq 1$, we obtain

$$F(x_{n_{i+1}}, f(x_{n_{i+1}})) \leq F(f(x_{n_i}), f^2(x_{n_i})). \qquad (1.20)$$

Since f is continuous and $x_{n_i} \to a$ as $i \to \infty$, it obtains that $f(x_{n_i}) \to f(a)$ and $f^2(x_{n_i}) \to f^2(a)$ as $i \to \infty$.

As $F : X \times X \to R$ is continuous, we obtain

$$F(a, f(a)) = \lim_{i \to \infty} F(x_{n_{i+1}}, f(x_{n_{i+1}})), .$$

and

$$F(f(a), f^2(a)) = \lim_{i \to \infty} F(f(x_{n_i}), f^2(x_{n_i})).$$

Combined with (1.20), these two equations imply that

$$F(a, f(a)) \leq F(f(a), f^2(a)). \qquad (1.21)$$

Now, to prove $f(a) = a$.

Assume, to the contrary, that $f(a) \neq a$.

Then by (1.18),

$$\begin{aligned} F(f(a), f^2(a)) &= F(f(a), f(f(a))) \\ &< \max\{F(a, f(a)), [\max\{F(a, f(a)), F(f(a), f^2(a))\} \\ &\qquad + \lambda \min\{|F(a, f^2(a))|, F(f(a), f(a))\}]\} \\ &= \max\{F(a, f(a)), \max\{F(a, f(a)), F(f(a), f^2(a))\}\} \\ &= F(f(a), f^2(a)), \end{aligned}$$

which is a contradiction.

Hence, $f(a) = a$ and thus f has a fixed point.

Further, we show that if $\lambda = 0$, then f has a unique fixed point.

Assume, to the contrary, that f has not a unique fixed point, that is, there is $b \in X$ such that $f(b) = b$ and $b \neq a$ for $\lambda = 0$.

Then by (1.18),

$$
\begin{aligned}
F(a, b) &= F(f(a), f(b)) \\
&< \max\{F(a, b), [\max\{F(a, f(a)), F(b, f(b))\}]\} \\
&= \max\{F(a, b), [\max\{F(a, a), F(b, b)\}]\} \\
&= \max\{F(a, b), \max\{0, 0\}\} \\
&= F(a, b),
\end{aligned}
$$

which is a contradiction.

Hence $a = b$ and thus, f has a unique fixed point for $\lambda = 0$.

Theorem 12: Let X be a topological space, $f : X \to X$ be a continuous mapping and for some $F \in \Phi$, we have

$$
\begin{aligned}
F(f(x), f(y)) < \max\{F(x, y), [&\max\{F(x, f(x)), F(y, f(y))\} \\
&+\min\{|F(x, f(y))|, F(f(x), y)\}]\}, \qquad (1.22)
\end{aligned}
$$

for all $x, y \in X$ with $x \neq y$. If there exists a point $x_0 \in X$ whose sequence of iterates $\{f^n(x_0)\}$ contains a convergent subsequence $\{f^{n_i}(x_0)\}$, then $a = \lim_{i\to\infty} f^{n_i}(x_0) \in X$ is a unique fixed point of f.

Proof : Since (1.18) is implied by (1.22), so by Theorem (11), there exists some $a \in K$such that $f(a)=a$. Assume, to the contrary, that there exists $b \in K$ such that $f(b) = b$ and $a \neq b$.

Then by (1.22), we obtain

$$
\begin{aligned}
F(a, b) &= F(f(a), f(b)) \\
&< \max\{F(a, b), [\max\{F(a, f(a)), F(b, f(b))\} \\
&\qquad + \min\{|F(a, f(b))|, F(f(a), b)\}]\} \\
&= \max\{F(a, b), [\max\{F(a, a), F(b, b)\} + \min\{|F(a, b)|, F(a, b)\}]\} \\
&= \max\{F(a, b), [0 + F(a, b)]\} \text{ as } |F(a, b)| \geq F(a, b) \\
&= F(a, b),
\end{aligned}
$$

which is a contradiction. Thus $a = b$ and hence, f has a unique fixed point.

Theorem 13: Let X be a topological space, K be a nonempty compact subset of X and T, S, G, H : K   K be continuous self mappings on K such that T, S are surjective on K, T commutes with {G, H} and S commutes with {G, H}. If for F   Φ, the four mappings T, S, G and H satisfy the following condition :

$$
\begin{aligned}
F(S(x), T(y)) > \inf\{&F(t, P(t)), F(Q(t), P(t)), F(Q(t), P(Q(t))), F(Q(x), Q(y)) \\
&: t \in \{x, y\}, P \in \{T, S\} \text{ and } Q \in \{G, H\}\} \qquad (1.23)
\end{aligned}
$$

for any x, y $\in$ K with $T(x) \neq S(y)$, then at least one of the following assertions hold:

(1) T has a fixed point in K;

(2) S has a fixed point in K;

(3) T and G have a coincidence point in K;

(4) T and H have a coincidence point in K;

(5) S and G have a coincidence point in K;

(6) S and H have a coincidence point in K

Proof : Since K is a compact and F, T, S, G and H are continuous functions, it follows from Remark, it obtained that the functions $x \to F(x, P(x))$ and $x \to F(Q(x), P(x))$ with $P \in \{T, S\}$ and $Q \in \{G, H\}$, are continuous on K, and by Lemma (1) there exist a, b, p, q, r and s $\in$ K such that

$$F(a, T(a)) = \inf\{F(x, T(x)) : x \in K\};$$

$$F(b, S(b)) = \inf\{F(x, S(x)) : x \in K\};$$

$$F(G(p), T(p) = \inf\{F(G(x), T(x)) : x \in K\};$$

$$F(H(q), T(q) = \inf\{F(H(x), T(x)) : x \in K\};$$

$$F(G(r), S(r) = \inf\{F(G(x), S(x)) : x \in K\};$$

$$F(H(s), S(s) = \inf\{F(H(x), S(x)) : x \in K\}.$$

To prove the results, require to consider the following cases:

Case (1) : Suppose that

$$F(a, T(a)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\}.$$

As $S(K) = K$, there exists some $w \in K$ such that $S(w) = a$. Assume that $T(S(w)) \neq S(w)$, that is, $T(a) \neq a$.

Since S commutes with G and H, by (1.23), we obtain

$$\begin{aligned} F(S(w), T(S(w))) &> \inf\{F(t, P(t), F(Q(t), P(t)), F(Q(t), PQ(t), F(Q(S(w)), Q(w)) \\ &\qquad : t \in \{S(w), w\}, P \in \{T, S\} \text{ and } Q \in \{G, H\}\} \\ &= \min\left\{ \inf_{t\in\{S(w), w\}}\{F(t, P(t))\}, \inf_{t\in\{S(w), w\}}\{F(Q(t), P(t))\} \right. \\ &= \inf_{t\in\{S(w), w\}}\{F(Q(t), PQ(t))\}, \inf_{t\in\{S(w), w\}}\{F(S(Q(w), Q(w))\} \\ &\qquad \left. : P \in \{T, S\} \text{ and } Q \in \{G, H\}\right\} \\ &\geq \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), \\ &\qquad F(H(q), T(q), F(G(r), S(r)), F(H(s), S(s))\}. \end{aligned}$$

As

$$F(t, P(t)) = F(t, T(t)), \text{ or } F(t, P(t)) = F(t, S(t)),$$

then, $\inf_{t\in\{S(w), w\}} \{F(t, P(t))\} \geq \inf_{t\in K} \{F(t, P(t))\} = \min\{F(a, T(a)), F(b, S(b))\}$

Similarly, since

$$F(Q(t), P(t)) = F(G(t), T(t)), \text{ or } F(Q(t), P(t)) = F(G(t), S(t)),$$

or $F(Q(t), P(t)) = F(H(t), T(t)), \text{ or } F(Q(t), P(t)) = F(H(t), S(t)),$

implies that,

$$\inf_{t\in\{S(w), w\}} F(Q(t), P(t)) \geq \min\{F(G(p), T(p)), F(G(r), S(r)), F(H(q), T(q)), F(H(s), S(s))\}$$

Further,

$$F(Q(t), PQ(t)) = F(Q(t), TQ(t)) \text{ or } F(Q(t), PQ(t)) = F(Q(t), SQ(t)),$$

then, we obtain

$$\inf_{G(t)\in K, H(t)\in K; t\in\{S(w), w\}} F(Q(t), TQ(t)) \geq \inf_{x\in K} F(x, T(x)) = F(a, T(a)),$$

$$\inf_{G(t)\in K, H(t)\in K; t\in\{S(w), w\}} F(Q(t), SQ(t)) \geq \inf_{x\in K} F(x, S(x)) = F(b, S(b)) .$$

Also, since

$F(Q(S(w), Q(w)) = F(S(Q(w)), Q(w)) = F(S(G(w)), G(w)), \text{ or}$

$F(Q(S(w)), Q(w)) = F(S(G(w)), H(w)), \text{ or } F(Q(S(w)), Q(w)) = F(S(H(w)), G(w)), \ldots..$

then, we obtain

$$\inf_{G(w)\in K, H(w)\in K} F(Q(S(w), Q(w)) \geq \inf_{x\in K} F(S(x), x) = F(b, S(b)) .$$

Therefore, we get

$$\begin{aligned} F(S(w), T(S(w))) &> \inf\{F(t), P(t)), F(Q(t), P(t)), F(Q(t), PQ(t), F(Q(S(w)), Q(w)) \\ &\geq \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), \\ &\qquad F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\}. \end{aligned}$$

Thus, from (1.24), we obtain

$$\begin{aligned} F(S(w), T(S(w))) &= F(a, T(a)) \\ &> \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), \\ &\qquad F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\} \\ &= F(a, T(a)), \end{aligned}$$

which is a contradiction.

Hence $T(a) = a$, that is, a is the fixed point of T.

Case (2) : Assume that

$$F(G(r), S(r)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\}. \tag{1.25}$$

Since $T(K) = K$, there exists some $z \in K$ such that $T(z) = r$.

Assume that $S(T(z)) \neq G(T(z))$, that is, $S(r) \neq G(r)$.

By (1.23), we obtain

$$\begin{aligned} F(T(G(z), S(T(z))) &> \inf\{F(t, P(t)), F(Q(t), P(t)), \\ &\qquad F(Q(t), PQ(t))), F(Q(T(z)), Q(G(z))) \\ &\qquad : t \in\{T(z), G(z)\}, P\in\{T, S\} \text{ and } Q \in\{G, H\}\} \\ &\geq \min\{F(a, T(a), F(b, S(b)), F(G(p), T(p)), \\ &\qquad F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\}. \end{aligned}$$

Thus, since $F(T(G(z)), S(T(z))) = F(G(T(z)), S(T(z))) = F(G(r), S(r))$,

from (1.25), we obtain

$$F(G(r), S(r)) > F(G(r), S(r)),$$

which is a contradiction.

Hence, $G(r) = S(r)$ which means that r is the coincidence of S and G. Thus, Assertion (1) and (5) are proved.

Next, we discuss about the remaining cases :

$$F(b, S(b)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\}.$$

$$F(G(p), T(p)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\}.$$

$$F(H(q), T(q)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)), F(H(q), T(q)), F(G(r), S(r)), F(H(s), S(s))\}.$$

or

$$F(H(s), S(s)) = \min\{F(a, T(a)), F(b, S(b)), F(G(p), T(p)). F(H(q), T(q), F(G(r), S(r)), F(H(s), S(s))\}.$$

and proceeding on lines as in the proof of Assertion (1) or Assertion (5), Assertions (2), (3), (4) and (6) also hold.

Corollary 4: Let X be a topological space, K be a non empty compact subset of X and T, S, G, H : $K \to K$ be continuous self mapping on K such that S, T are surjective on K and commutes with G, H. If, for $F \in \Phi$, the four mappings T, S, G and H satisfy the following condition :

$$F(S(x), T(y)) > \text{Sup}\{F(t, P(t)), F(Q(t), P(t)), F(Q(t), P(Q(t))), F(Q(x), Q(y)) \\ : t \in \{x, y\}, P \in \{T, S\} \text{ and } Q \in \{G, H\}\}$$

for any $x, y \in K$ with $T(x) \neq S(y)$, then at least one of the following assertions hold:

(1) T has a fixed point in K;

(2) S has a fixed point in K;

(3) T and G have a coincidence point in K;

(4) T and H have a coincidence point in K;

(5) S and G have a coincidence point in K;

(6) S and H have a coincidence point in K.

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Assessing the AI Maturity of Cloud Providers

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A B S T R A C T

This article examines the AI maturity levels of leading cloud providers to evaluate their capabilities in delivering innovative and efficient AI solutions within their respective platforms

Keywords: AI-Maturity, AWS, Azure, Google, MLOps

INTRODUCTION

The Cloud Services Industry reached the next level with Artificial Intelligence (AI) Technologies advancement which pushed market titans Amazon Web Services (AWS), Microsoft Azure and Google Cloud Platform to the top of the revolution and market leaders. These three cloud providers are all actively acquiring and building AI/ML. They each have pros and cons.

- AWS, with the strength of matured and most advanced platform, big ecosystem, large experience. But it can be complex and expensive.

- Azure (Microsoft Azure) Strong AI/ML focus, Support with Microsoft products and

rowing ecosystem with partners and tools. But they are not quite experienced.

- Google Cloud Platform (GCP) – Latest AI/ML technology, Affordable Price and Expertise in unique areas like MLOps, NPL.

AI Maturity and innovative AI solutions

AI maturity - it is the level of AI maturity of an organization (eg, ability to scale, track, use and automate AI models and workflows within the enterprise). Maturity of AI determinant cloud as the mature, powerful and configurable AI products offered by cloud service providers the higher their chances to gain market share. All AWS, Azure, Google have their AI maturity that they use to win an edge, AI maturity is a term for how proficiently they sell you machine learning, natural language processing, data analytics etc. The more enterprises demand AI driven decision making, the more AI-savvy the Cloud provider will be and the bigger its selection advantage in customer base and market share. Best in class AI Solutions – All three top cloud platforms provide thousands of AI services for all business needs. AWS all in, Amazon SageMaker - Data analysis, Algorithm building, Model development and delivery.

- Azure is already pretty familiar with cloud AI (i.e., Azure Cognitive Services, Azure Machine Learning).

- Google Cloud's ML-first nature is what makes it the one that brings you tools like Vertex AI, TensorFlow.

Each solution by the provider differs and varies but all are competing on scale, simplicity and rapid deployment. They are both providers' accessibility and usability approach which is also where they have been heading towards AI maturity.

Customer happiness in the cloud — and most importantly, in AI products — is just about stability, scale and innovation — and these things are done when AI is mature.

- AWS is the customer care king with the biggest and most complete cloud AI services available.
- Azure uses the fact that it's deeply integrated with Microsoft's enterprise services product suite, so customers will experience a unified, natural interface.
- Google Cloud is better however with its open-source philosophy and superpower of Google AI. This contrast shows that AI maturity is needed for not just customers today but future customers who will be measuring customer satisfaction in the future.

Improvements to AI maturity in the major cloud players - In order to climb the maturity chain, the big cloud players take on several projects of technology, organizational and skill enhancement. AWS develops AI infrastructure through research into AI, and the development of super-flexible AI systems. Azure can use its extensive network and existing enterprise contracts to centralize its AI infrastructure and bring AI into the cloud. Google Cloud is geared toward open source and cross-platform integration to reach more people and contribute to the latest AI studies. The trick to these giants is creating and training AI experts who can learn and grow, constantly, to stay on top of technology. Alliances and alliances are also the backbone of their AI systems that drive maturity and scale.

Security in AI Maturity

AI maturity and security & morality within cloud service providers - AI maturity isn't simply a technical development; it's also about having solid security & behavior. As we use AI more and more, the risks of data breaches, algorithmic distortions and tampering rise exponentially. AWS, Azure, Google all have security infrastructure — encryption, access control, surveillance — at the heart of their AI services. Morality is dealt with by explicit policy definitions, data privacy policies and AI equity standards. They are fundamental to customer trust and complying with national laws worldwide, and they are the balance between technology nimbleness and governance in AI maturity in the cloud.

It allows cloud providers to use AI with minimal risk and with empowered responsibility in building and running these disruptive technologies.

Future trends in AI maturity

AI maturity in the cloud will presumably be an extension of new technologies like quantum computing and blockchain. Future improvements envision AI models with unmatched precision and decision-making power as well as edge computing with real-time processing. This transition to autonomous AI platforms has disruptive effects for cloud platforms operations — from efficiency, sustainability and AI democratization more generally. A vision of AI maturity at asymptotic levels promises an age in which cloud computing will break out of existing limits and acclimate to hyper-individuated and self-governing service outputs. In the face of such utopian trends, we'll see new business models and operating ecosystems on the cloud computing platform that are based on advanced AI intelligence.

Conclusions

The current focus on AI maturity by cloud providers hints at a marketplace where innovation, customer satisfaction, and responsible governance are leadership. AWS, Azure, Google – all these show AI maturation through various strategies and products that will determine the next generation of cloud computing with AI paving the way for future innovations. Examining where they are today, and looking ahead, will enable industries and stakeholders to gain strategic insight, which will then help guide decisions and drive innovation. Furthermore, the deep moral and technological foundations of the ethically sound system ensure long-term sustainability and trust in a constantly evolving digital economy.

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