ISSN : 2394 - 7139

Aryabhatta Journal of Mathematics and Informatics

VOLUME NO. 17

ISSUE NO. 2

MAY - AUGUST 2025

ENRICHED PUBLICATIONS PVT. LTD

S-9, IInd FLOOR, MLU POCKET,
MANISH ABHINAV PLAZA-II, ABOVE FEDERAL BANK,
PLOT NO-5, SECTOR-5, DWARKA, NEW DELHI, INDIA-110075,
PHONE: - + (91)-(11)-47026006

ISSN : 2394 - 7139

Chief Editor

Dr. T.P. Singh
Professor (Maths & O.R.) YIET, Gadholi Ph.D. (Mathematics) - 1986 specialized in the area of Operation Research, Probability, Queuing Theory, Production Scheduling & Reliability theory, Fuzzy Logic. (Garhwal University, Srinagar Garhwal), PGDCA-1997 from Kurukshetra University, Kurukshetra., M.Sc. - (Mathematics) -1978 in 1st Division from Meerut University, Meerut., B.Sc. (PCM) -1970 from Meerut University, Meerut. More than 30 years of teaching Experience. Currently working in Yamuna Instt. of Engg. & Technology, Gadholi, YamunaNagar as a Professor of Mathematics & O.R. from 1-4-2010 till date.The post is approved by Kurukshetra University kurukshetra.

Editorial Board Members

Dr. S.M. Rizwan, Prof. and Head Deptt. of Mathematics & Statistics, Caledonian College of Engineering Sultanate of Oman	**Dr. Madhu Jain,** Professor in Mathematics, IIT Roorkee
Dr. Shakti Kumar , Prof. & Director , Computational Intelligence Lab. ISTK	Er. Abhishek Pratap Singh Manager Technology, Sapient Corporation Virginia (USA) 22201.
Dr. O.P. Vinocha. Principal- Director Ferozpur College of Engineering , Feroz Shah Punjab	**Dr. Sanjay Jain** Associate Prof., Deptt. Of Mathematical Sciences , Govt. PG. College Ajmer
Dr. RK Tuteja Pof. Stat. & O.R. and Principal MCA , N.C. College of Engineering , Israna (Panipat)	**Dr. Om Parkash** Proff , Guru Nanak Dev University Amritsar (Punjab)
Dr. Rajendar Kumar Principal S.S.V. (PG) College , Hapur (UP)	**Er. Dilip Aditya** , Senior Software Engineer/ Team Lead. , J.P. Morgans & Chase , Mumbai
Dr. Vikram Singh Prof Deptt of Computer Science & Engg. CDL University Sirsa (Haryana)	**Dr. S.K.Tomar ,** Proff & Chairman (Maths) Punjab University , Chandigarh
Dr. S. Lakshmi Associate Prof. , KN Govt. Arts College (w) Thanjavur (Tamil Nadu)	**Dr. Chanchal Kumar Sharma.** Associate Editor & Regional Coordinator SAJOSPS fellow, CMF,Indian Insti of Social Sciences New Delhi
Dr. Deepak Gupta Prof and Head Deprt. Of Maths MM University Mullana, Ambala	**Dr. M. Reni Sagayaraj** Head & Associate Professor, PG and Research Dept. of Mathematics, Sacred Heart College Tirupattur.- 635601 Vellore District. TamilNadu,
Prof.Dr. Sumit Kumar Banerjee Professor, Dhirajlal Gandhi College of Technology,Salem, Tamilnadu	**Dr. Srichandan Mishra,** Lecturer in Mathematics, Govt. Science College, Malkangiri, Odisha,
Dr. Pawan Kumar Mahajan Professor and Head; Department of Basic Sciences, Dr. Y. S. Parmar University of Horticulture & Forestry Nauni (Solan) - 173230; Himachal Pradesh	**Dr. M. Reni Sagayaraj,** Head & Associate Professor, PG and Research Dept. of Mathematics, Sacred Heart College Tirupattur.- 635601
Dr.G.S.Sao Asst.Prof & Head of Dept, Govt.ERR PG Sc.College Bilaspur	**Dr.Premlata Verma** Asst.Prof & Head of Dept., Govt.Bilas Girls College,Bilaspur

ISSN : 2394 - 7139

Editorial Board Members

Dr.Aradhana Sharma Asst.Prof.of Mathematics, Govt.Bilasa Girls College,Bilaspur	**Dr. Deepika Garg** Assistant Professor, Amity University Haryana
Vinod Kumar Bais Assitant Professor, Manav Bharti University, Solan, H.P.,India	**Dr. Nahid Fatima** Asst. Prof. College/Institute/University/Organisation : Amity University Haryana
Dr. Mavurapu Satyanarayana, Assistant Professor Department of Pharmaceutical Chemistry University College of Science, Telangana niversity Nizamabad,	**Dr Sebastian Vadakan** Vice Principal (Quality and Research) St. Xavier's College, Ahmedabad Gujarat.
Dr.K.Bageerathi Assistant Professor of Mathematics College/Institute/University : Aditanar College of Arts and Science, Tiruchendur, Tamil Nadu Learning (iFEEL), Lonavala residential campus,	**Dr. Dhananjaya Reddy** Assistant Prof. in Mathematics College/Institute/University : Govt.Degree College,PUTTUR, Chittoor(Dt), Andhra Pradesh, India
Avinash Pokhriyal Dean College/Institute/University : FMCA, RBS College, Agra	**Lalit Mohan Upadhyaya** Associate Professor. Department of Mathematics, Municipal Post Graduate College, Mussoorie, Dehradun, Uttarakhand -248179, India.

ISSN : 2394 - 7139

ISSN : 2394 - 7139

Aryabhatta Journal of Mathematics and Informatics

(Volume No. 17, Issue No. 2, May - August 2025)

Contents

ISSN : 2394 - 7139

Primitive central idempotents of certain finite semisimple group algebras

Shalini Gupta
Department of Mathematics,
Punjabi University, Patiala, India.

ABSTRACT

The objective of this paper is to give a complete algebraic structure of semisimple group algebras of some finite indecomposable groups, whose central quotient is the Klein's four group, over a finite field.

Keywords: semisimple group algebra, metabelian groups , indecomposable groups, primitive central idempotents, Wedderburn decomposition.

MSC2000: 16S34; 20C05; 16K20

1. INTRODUCTION

Let Fq be a finite field with q elements and G be a finite group of order coprime to q, so that the group algebra Fq[G] is semisimple. The most important problem in the area of group algebras is to find a complete set of primitive central idempotents of semisimple group algebra Fq[G]. The knowledge of primitive central idempotents is useful in finding Wederburn decomposition, unit group of integral group ring, various parameters in error correcting codes **[1,2,4,5,10,11,13,14,15,16,17,18].** In **[3]**, Bakshi et.al. obtained a complete algebraic structure of Fq[G], G metabelian, using Strong Shoda pairs. They further illustrated their result by providing a complete set of primitive central idempotents and the Wedderburn decomposition of certain finite group algebras of indecomposable groups whose central quotient is Klein's four group. Further, Neha et. al. **[12]** obtained a complete Wedderburn decomposition of group algebras of all such indecomposable groups using the method developed by Ferraz in **[6]**. In this paper, we give a complete algebraic structure of |Fq[G] or some indecomposable groups G, as classified by Milies **[7]**, using the method developed in **[3]**.

2. Metabelian groups

We recall the structure of metabelian group algebras over finite field as given in [3].
Let $H \trianglelefteq K \trianglelefteq G$ with K/H cyclic of order n. Let $Irr(K/H)$ be the set of irreducible characters of K/H over the algebraic closure $\overline{\mathbb{F}}_q$ of $\mathbb{F}_q$. Let $\mathcal{C}\,(K/H)$ be the set of q-cyclotomic cosets of $Irr(K/H)$ containing the generators of $Irr(K/H)$, i.e., if χ is a generator of $Irr(K/H)$, then an

element C of $\mathcal{C}(K/H)$ containing χ is the set $\{\chi, \chi^q, \ldots, \chi^{q^{o-1}}\}$, where $o = o_n(q)$, the order of q modulo n. Consider the action of $N_G(H)$, the normalizer of H in G, on $\mathcal{C}(K/H)$ by conjugation. Let $E_G(K/H)$ denote the stabilizer of $C \in \mathcal{C}(K/H)$ and $\mathcal{R}(K/H)$ denote the set of distinct orbits of $\mathcal{C}(K/H)$ under this action. Set

$$\varepsilon_C(K,H) = |K|^{-1} \sum_{g \in K} tr_{\mathbb{F}_q(\zeta)/\mathbb{F}_q}(\chi(\bar{g}))\, g^{-1},$$

where χ is a representative of the q-cyclotomic coset C and ζ is a primitive nth root of unity in $\overline{\mathbb{F}}_q$, $C \in \mathcal{C}(K/H)$. Let $e_C(\mathrm{G,K,H})$ be the sum of distinct G-conjugates of $e_C(K,H)$. For a ring R, let $R^{(n)}$ denote the n-copies of R.

For a normal subgroup N of G, let A_N/N be a maximal abelian subgroup of G/N containing its derived subgroup$(G/N)'$. Let T be the set of all subgroups D/N of G/N with $D/N \leq A_N/N$ and A_N/D cyclic. Consider $T_{G/N}$ to be a set of representatives of the distinct equivalence classes of T under the equivalence relation of conjugacy in G/N. Define

$$S_{G/N} \coloneqq \{(D/N, A_N/N) \mid D/N \in T_{G/N}, D/N \text{ core free in } G/N\} \text{ and}$$

$$S \coloneqq \{(N, D/N, A_N/N) \mid N \trianglelefteq \mathrm{G}, S_{G/N} \neq \emptyset, (D/N, A_N/N) \in S_{G/N}\}$$

Theorem 1 [**3**] Let $\mathbb{F}_q$ be a finite field with q elements and G a finite metabelian group. Suppose that $\gcd(q,|G|) = 1$. Then a complete set of primitive central idempotents of $\mathbb{F}_q[G]$ is given by the set

$$\{e_C(G, A_N, D) \mid (N, D/N, A_N/\mathrm{N}) \in S,\ C \in \mathcal{R}(A_N/\mathrm{D})\}.$$

Moreover, the corresponding simple component $F_q[G]e_c(G, A_N, D)$ is isomorphic to $M_{[G:A_N]}\left(\mathbb{F}_{q^{o(A_{N,D})}}\right)$, the algebra of $[G:A_N] \times [G:A_N]$ matrices over the field $F_{q^{o(A_{N,D})}}$, where $o(A_N, D) = \frac{o_{[A_N:D]}(q)}{[E_G(A_N/D):A_N]}$.

The groups G of the type $G/\mathbb{Z}(G) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$, where $\mathbb{Z}(G)$ denotes the centre of the group G, are studied by Goodaire [**8,9**]. It has been proved in **[7]**, that the finite indecomposable groups with $G/\mathbb{Z}(G) \cong \mathbb{Z}_2 \times \mathbb{Z}_2$ break into five classes as follows:

Group	Generators	Relations
D_1	x, y, t	$x^2, y^2, t^{2^m}, y^{-1}x^{-1}yxt^{2^{m-1}}, t$ central, $m \geq 1$
D_2	x, y, t	$x^2t^{-1}, y^2t^{-1}, t^{2^m}, y^{-1}x^{-1}yxt^{2^{m-1}}, t$ central, $m \geq 1$
D_3	a, b, x, y	$a^2, b^2y^{-1}, x^{2^{m_1}}, y^{2^{m_2}}, b^{-1}a^{-1}bax^{2^{m_1-1}}, x, y$ central, $m_1, m_2 \geq 1$
D_4	a, b, x, y	$a^2x^{-1}, b^2y^{-1}, x^{2^{m_1}}, y^{2^{m_2}}, b^{-1}a^{-1}bax^{2^{m_1-1}}, x, y$ central, $m_1, m_2 \geq 1$
D_5	a, b, x, y, z	$a^2y^{-1}, b^2z^{-1}, x^{2^{m_1}}, y^{2^{m_2}}, z^{2^{m_3}}, b^{-1}a^{-1}bax^{2^{m_1-1}}, x, y, z$ central, $m_1, m_2, m_3 \geq 1$

The complete algebraic structure of $\mathbb{F}_q[G]$, G of type D_1, D_2, is studied in [**3**]. In this paper, we will find the complete algebraic structure of $\mathbb{F}_q[G]$, G of type D_3.

3. Groups G of type D_3

$G \coloneqq D_3 =< a, b, x, y \mid a^2 = 1, b^2 = 1, x^{2^{m_1}} = y^{2^{m_2}} = 1, a^{-1}b^{-1}ab = x^{2^{m_1-1}}, x, y$ central in $G >$

Theorem 2 For $m_1 = 1$, $m_2 \geq 1$ the complete algebraic structure of semisimple group algebra, $\mathbb{F}_q[G]$, G of type D_3, is given as follows:

Primitive Central Idempotents

$e_C(G, G, < x, a >),\ C \in \mathcal{R}(G/< x, a >);$

$e_C(G, G, < x, b >),\ C \in \mathcal{R}(G/< x, b >);$

$e_C\left(G, G, < x, a, b^{2^i} >\right),\ C \in \mathcal{R}(G/< x, a, b^{2^i} >);$

$e_C\left(G, G, < x, ab^{2^i} >\right),\ C \in \mathcal{R}(G/< x, ab^{2^i} >);$

$e_C\left(G, G, < x^2, x^i a, xb^{2^j} >\right),\ C \in \mathcal{R}(G/< x^2, x^i a, xb^{2^j} >),\ i, j = 0,1;$

$e_C\left(G, G, < x^{2^v}, x^i a, b >\right),\ C \in \mathcal{R}(G/< x^{2^v}, x^i a, b >);$

$e_C\left(G, G, < x^{2^v}, x^i a, x^j b >\right),\ C \in \mathcal{R}(G/< x^{2^v}, x^i a, x^j b >),\ 1 \leq v \leq m_1 - 1,\ i = 0, 2^{v-1},$
$1 \leq j \leq v - 1,\ \gcd(j, 2^v) \geq 2^{v-2};$

$e_C(G, < b, x >, < b >),\ C \in \mathcal{R}(< b, x >/< b >);$

$e_C\left(G, < a, x, y >, < a, x^{2^{m_1-1}} y >\right),\ C \in \mathcal{R}(< a, x, y >/< a, x^{2^{m_1-1}} y >).$

Wedderburn Decomposition

$$\mathbb{F}_q[G] \cong \mathbb{F}_q^{(8)} \oplus (\mathbb{F}_{q^{f_2}})^{(\frac{8}{f_2})} \oplus_{v=2}^{m_1-1} (\mathbb{F}_{q^{f_v}})^{(\frac{2^{v+2}}{f_v})} \oplus M_2(\mathbb{F}_{q^{f_{m_1}}})^{(\frac{2^{m_1}}{f_{m_1}})}$$

where $f_i = o_{2^i}(q)$, the order of q modulo 2^i, $i \geq 1$.

In order to find a complete set of primitive central idempotents and Wedderburn decomposition of $\mathbb{F}_q[G]$, G of type D_3, we need to find all the normal subgroups N of G and the corresponding $S_{G/N}$.

Lemma 1 All the distinct normal subgroups of G are as follows:

$\{e\}, < y >, < x^{2^{m_1-1}}y >$;

$< x >, < x, a >, < x, b^{2^i} >, < x, ab^{2^i} >, < x, a, b^{2^i} >, \quad i = 0,1;$

$< x^{2^v} >, < x^{2^v}, x^j a >, < x^{2^v}, x^j b^{2^i} >, < x^{2^v}, x^j ab^{2^i} >, < x^{2^v}, x^j a, b^{2^i} >,$

$< x^{2^v}, x^j a, x^{2^{v-1}} b^{2^i} >, \ 1 \le v \le m_1 - 1, \ j = 0, 2^{v-1}, i = 0, 1;$

$< x^{2^v}, x^j b >, < x^{2^v}, x^j ab >, < x^{2^v}, x^j a, x^i b >, \ 2 \le v \le m_1 - 1, \ j = 0, 2^{v-1},$

$i = 2^{v-2}, 3.2^{v-2}$.

Proof: Let $N \trianglelefteq G$ be such that $N \cap < x > = \{e\}$, then $N = < y >$ or $< x^{2^{m_1-1}} y >$ or $\{e\}$. For $N \cap < x > = < x >$, it is easy to see that N is either $< x >$ or $< x, a >$ or $< x, b^{2^i} >$ or $< x, ab^{2^i} >$ or $< x, a, b^{2^i} >, i = 0, 1$.

Let us assume that $N \cap < x > = < x^{2^v} >, \ 1 \le v \le m_1 - 1$. Now, $N/< x^{2^v} >$ is isomorphic to one of the following: $< x >, < a < x >>, < b^{2^i} < x >>, < ab^{2^i} < x >>$, or $< a < x >, b^{2^i} < x >>, 0 \le i \le 1$. Let $N/< x^{2^v} >$ be isomorphic to $< x >$, then $N = < x^{2^v} >$, further if $N/< x^{2^v} >$ is isomorphic to $< a < x >>$ then N is either $< x^{2^v}, a >$ or $< x^{2^v}, x^j a >, \ 1 \le j \le 2^{v-1}$. But if $\gcd(j, 2^v) = 2^\alpha$, then $(x^{2^\alpha} a)^2 = x^{2^{\alpha+1}}$ which will lie in $< x^{2^v}, x^j a >$ if, and only if, $\alpha \ge v - 1$. Thus $j = 2^{v-1}$, hence in this case, $N = < x^{2^v}, a >$ or $< x^{2^v}, x^{2^{v-1}} a >$.

Now, if $N/< x^{2^v} > \cong < b < x >>$, then for $v = 1, N/< x^2 > \cong < b < x >>$, and $N = < x^2, b >$ or $< x^2, xb >$. Similarly, for $2 \le v \le m_1 - 1$, we have N is either $< x^{2^v}, b >$ or $< x^{2^v}, x^j b >, \ 1 \le j \le 2^{v-1}$. Let $\gcd(j, 2^v) = 2^\alpha$, then $(x^{2^\alpha} b)^4 = x^{2^{\alpha+2}}$, which will lie in $< x^{2^v}, x^j b >$ if, and only if, $\alpha \ge v - 2$. Thus, for $2 \le v \le m_1 - 1$, $N = < x^{2^v}, x^{2^{v-2}} b >$ or $< x^{2^v}, x^{3.2^{v-2}} b >$ or $< x^{2^v}, x^{2^{v-1}} b >$.

Similarly for $N/< x^{2^v} > \cong < b^2 < x >>$, either $N = < x^2, b^2 >$ or $< x^2, xb^2 >$ or $< x^{2^v}, b^2 >$ or $< x^{2^v}, x^{2^{v-1}} b^2 >$, $2 \le v \le m_1 - 1$. Next, for $N/< x^{2^v} > \cong < ab < x >>$, either $N = < x^2, ab >$ or $< x^2, xab >$ or $< x^{2^v}, ab >$ or $< x^{2^v}, x^{2^{v-1}} ab >$ or $< x^{2^v}, x^{2^{v-2}} ab >$ or $< x^{2^v}, x^{3.2^{v-2}} ab >, 2 \le v \le m_1 - 1$.

Further for $N/< x^{2^v} > \cong < ab^2 < x >>$, either $N = < x^{2^v}, ab^2 >$ or $< x^{2^v}, x^{2^{v-1}} ab^2 >, \ 1 \le v \le m_1 - 1$ Next, for $N/< x^{2^v} > \cong < a < x >, b < x >>, N = < x^2, a, xb >$, $< x^2, xa, xb >$, $< x^{2^v}, a, b >$, $< x^{2^v}, x^{2^{v-1}} a, b >, \ 1 \le v \le m_1 - 1$, $< x^{2^v}, x^i a, x^{2^{v-2}} b >$, $< x^{2^v}, x^i a, x^{3.2^{v-2}} b >$, $< x^{2^v}, x^i a, x^{2^{v-1}} b >$, $2 \le v \le m_1 - 1$, $i = 0, 2^{v-1}$. Finally, if $N/< x^{2^v} > \cong < a < x >, b^2 < x >>$, then $N = < x^{2^v}, a, b^2 >$, $< x^{2^v}, x^{2^{v-1}} a, b^2 >$, $< x^{2^v}, x^i a, x^{2^{v-1}} b^2 >, 1 \le v \le m_1 - 1, \ i = 0, 2^{v-1}$.

Observe that if $N \cap <x> = <x^{2^v}>$, $0 \le v \le m_1 - 1$ then $G' = <x^{2^{m_1-1}}> \subseteq N$ and hence G/N is abelian. Thus,

$$S_{G/N} = \begin{cases} (<1>, G/N), & if\ G/N\ is\ cyclic, \\ \emptyset, & otherwise. \end{cases}$$

Out of the normal subgroups N, listed above, the following have cyclic quotient with G:
$<x, a>, <x, b>, <x, a, b^{2^i}>, <x, ab^{2^i}>$, $i = 0,1$;
$<x^{2^v}, x^j a, x^{2^{v-2}} b>, <x^{2^v}, x^j a, x^{3.2^{v-2}} b^2>$, $2 \le v \le m_1 - 1$, $j = 0, 2^{v-1}$;
$<x^{2^v}, x^j a, x^{2^{v-1}} b>, <x^{2^v}, x^j a, b>$, $1 \le v \le m_1 - 1$, $j = 0, 2^{v-1}$;
$<x^2, a, xb^2>, <x^2, xa, xb^2>$.
Further, if $N = \{e\}$, then $S_{G/N} = \emptyset$, whereas for $N = <y>$,
$S_{G/N} = \{(<b>/N, <b, x>/N)\}$, and for $N = <x^{2^{m_1-1}} y>$,
$S_{G/N} = \{(<a, x^{2^{m_1-1}} y>/N, <a, x, y>/N)\}$.

Hence, the primitive central idempotents of Fq [G], as stated in Theorem 2, are obtained with the help of Theorem 1.

The Wedderburn decomposition of Fq [G] can now be easily obtained with the help of following table and Theorem 1.

N	(D, A_N)	$o(A_N, D)$	$\lvert \mathcal{R}(A_N, D) \rvert$
$<x, a>$	(N, G)	f_2	$\frac{2}{f_2}$
$<x, b>$	(N, G)	1	1
$<x, a, b^{2^i}>$, $0 \le i \le 1$	(N, G)	1	1
$<x, ab>$	(N, G)	1	1
$<x, ab^2>$	(N, G)	f_2	$\frac{2}{f_2}$
$<x^{2^v}, x^j a, x^{2^{v-1}} b>$, $j = 0, 2^{v-1}$,	(N, G)	f_v	$\frac{2^{v-1}}{f_v}$

$1 \leq v \leq m_1 - 1$			
$< x^{2^v}, x^j a, b >$, $j = 0, 2^{v-1}$, $1 \leq v \leq m_1 - 1$	(N, G)	f_v	$\frac{2^{v-1}}{f_v}$
$< x^2, x^i a, x b^2 >$, $i = 0, 1$	(N, G)	f_2	$\frac{2}{f_2}$
$< x^{2^v}, x^j a, x^{2^{v-2}} b >$, $j = 0, 2^{v-1}$, $2 \leq v \leq m_1 - 1$	(N, G)	f_v	$\frac{2^{v-1}}{f_v}$
$< x^{2^v}, x^j a, x^{3.2^{v-2}} b >$, $j = 0, 2^{v-1}$, $2 \leq v \leq m_1 - 1$	(N, G)	f_v	$\frac{2^{v-1}}{f_v}$
$< y >$	$(< b >, < b, x >)$	f_{m_1}	$\frac{2^{m_1-1}}{f_{m_1}}$
$< x^{2^{m_1-1}} y >$	$(< a, x^{2^{m_1-1}} y >, < a, x, y >)$	f_{m_1}	$\frac{2^{m_1-1}}{f_{m_1}}$

Theorem 3 For $m_1 = 1$, $m_2 \geq 1$ the complete algebraic structure of semisimple group algebra, $\mathbb{F}_q[G]$, G of type D_3, is given as follows:

Primitive Central Idempotents

$e_C(G, G, < x, b >),\ C \in \mathcal{R}(G/< x, b >);$

$e_C\left(G, G, < x, ab^{2^i} >\right),\ C \in \mathcal{R}(G/< x, ab^{2^i} >),\ 0 \leq i \leq m_2;$

$e_C\left(G, G, < x, a, b^{2^i} >\right),\ C \in \mathcal{R}(G/< x, a, b^{2^i} >),\ 0 \leq i \leq m_2 + 1;$

$e_C\left(G, < a, x, y >, < a, xy^{2^i} >\right),\ C \in \mathcal{R}(< a, x, y >/< a, xy^{2^i} >),\ 0 \leq i \leq m_2 - 1;$

$e_C(G, < b, x >, < b >),\ C \in \mathcal{R}(< b, x >/< b >).$

Wedderburn Decomposition

$$\mathbb{F}_q[G] \cong \mathbb{F}_q^{(4)} \oplus_{i=1}^{m_2} \left(\mathbb{F}_{q^{f_{i+1}}}\right)^{\left(\frac{2^i}{f_{i+1}}\right)} \oplus_{i=2}^{m_2+1} \left(\mathbb{F}_{q^{f_i}}\right)^{\left(\frac{2^{i-1}}{f_i}\right)} \oplus M_2(\mathbb{F}_q) \oplus_{i=0}^{m_2-1} M_2(\mathbb{F}_{q^{f_{i+1}}})^{\left(\frac{2^i}{f_{i+1}}\right)}$$

where $f_i = o_{2^i}(q)$.

Proof: In order to prove this, we need to find all the distinct normal subgroups of G. Let N be a normal subgroup of G such that $N \cap < x >= \{e\}$ then clearly N is $\{e\}$ or $< y^{2^i} >$ or $< xy^{2^i} >$

$,0 \le i \le m_2 - 1$. Now let us assume that $N \cap <x> = <x>$, then $N/N \cap <x>$ is isomorphic to $<x>$ or $<a<x>>$ or $<b^{2^i}<x>>$ or $<ab^{2^i}<x>>$ or $<a<x>, b^{2^i}<x>>$, $0 \le i \le m_2$. Let $N/<x> \cong <x>$, thus $N \cong <x>$. If $N/<x> \cong <a<x>>$, then $N \cong <x,a>$. Similarly in other cases N will be one of the following $<x, ab^{2^i}>, <x, b^{2^i}>, <x, a, b^{2^i}>, 0 \le i \le m_2$.

Observe that if $N \cap <x> = <x>$, then G/N is abelian and hence

$$S_{G/N} = \begin{cases} (<1>, G/N), & if\ G/N\ is\ cyclic, \\ \emptyset, & otherwise. \end{cases}$$

It can again be seen easily that for $N \trianglelefteq G, N \cap <x> = <x>$, the following have cyclic quotients with G:

$<x,a>, <x,b>, <x, ab^{2^i}>, <x, a, b^{2^i}>, \ 0 \le i \le m_2$.

Also observe that for $N = \{e\}, <y^{2^i}>, 1 \le i \le m_2 - 1$, $S_{G/N} = \emptyset$, whereas for $N = <y>$, $S_{G/N} = \{(<b>/N, <b,x>/N)\}$ and for $N = <xy^{2^i}>$, $0 \le i \le m_2 - 1$, $S_{G/N} = \{(<a, xy^{2^i}>/N, <a,x,y>/N)\}$.

The primitive central idempotents stated in Theorem 3 are thus obtained with the help of Theorem 1.

To find the Wedderburn decomposition of $\mathbb{F}_q[G]$, we compute the required parameters $o(A_N, D)$ and $|\mathcal{R}(A_N, D)|$ as follows:

N	$S_{G/N}$	$o(A_N, D)$	$\lvert\mathcal{R}(A_N, D)\rvert$
$<x,a,b>$	$\{(<1>,<1>)\}$	1	1
$<x,b>$	$\{(<1>, G/N)\}$	1	1
$<x,a,b^{2^i}>$, $1 \le i \le m_2 + 1$	$\{(<1>, G/N)\}$	f_i	$\frac{2^{i-1}}{f_i}$
$<x, ab^{2^i}>$, $1 \le i \le m_2$	$\{(<1>, G/N)\}$	f_{i+1}	$\frac{2^i}{f_{i+1}}$
$<y>$	$\{(<b>/N, <b,x>/N)\}$	1	1
$<xy^{2^i}>$, $1 \le i \le m_2 - 1$	$\{(<a, xy^{2^i}>/N, <a,x,y>/N)\}$	f_{i+1}	$\frac{2^i}{f_{i+1}}$

With the help of this table, the Wedderburn decomposition of Fq [G] , as stated in Theorem 3, is obtained.

The proof of the following theorem is similar to the previous one, so we omit the details here.

Theorem 4 Let $m_1, m_2 > 1$. Then (i) For $m_1 = m_2$, the complete algebraic structure of semisimple group algebra $\mathbb{F}_q[G]$ is given as:

Primitive Central Idempotents

$e_C(G, G, < x, a >),\ C \in \mathcal{R}(G/< x, a >);$
$e_C(G, G, < x, b >),\ C \in \mathcal{R}(G/< x, b >);$
$e_C\left(G, G, < x, a, b^{2^i} >\right),\ C \in \mathcal{R}\left(G/< x, a, b^{2^i} >\right),\ 1 \le i \le m_1;$
$e_C\left(G, G, < x, ab^{2^i} >\right),\ C \in \mathcal{R}\left(G/< x, ab^{2^i} >\right),\ 1 \le i \le m_1;$
$e_C\left(G, G, < x^{2^v}, x^j a, b >\right),\ C \in \mathcal{R}\left(G/< x^{2^v}, x^j a, b >\right),\ 1 \le v \le m_1 - 1,\ j = 0, 2^{v-1};$
$e_C\left(G, G, < x^{2^v}, x^k a, x^j b >\right),\ C \in \mathcal{R}(G/< x^{2^v}, x^k a, x^j b >),\ 1 \le v \le m_1 - 1,$
$\gcd(j, 2^v) \ge 1,\ k = 0, 2^{v-1};$
$e_C\left(G, G, < x^{2^v}, x^k a, x^j b^{2^\beta} >\right),\ C \in \mathcal{R}(G/< x^{2^v}, x^k a, x^j b^{2^\beta} >),\ 1 \le v \le m_1 - 1,$
$\gcd(j, 2^v) = 1,\ 1 \le \beta \le m_1 + 1 - v,\ k = 0, 2^{v-1};$
$e_C(G, < b, x >, < b >),\ C \in \mathcal{R}(< b, x >/< b >);$
$e_C\left(G, < a, x, y >, < a, x^j y >\right),\ C \in \mathcal{R}\left(< a, x, y >/< a, x^j y >\right),\ \gcd(j, 2^v) \ge 1.$

Wedderburn Decomposition

$$\mathbb{F}_q[G] \cong \mathbb{F}_q^{(6)} \oplus_{i=2}^{m_1+1} \left(\mathbb{F}_{q^{f_i}}\right)^{\left(\frac{2^i}{f_i}\right)} \oplus_{v=2}^{m_1-1} \left(\mathbb{F}_{q^{f_v}}\right)^{\left(\frac{2^v}{f_v}\right)} \oplus_{v=1}^{m_1-1} \oplus_{\beta=0}^{v-1} \left(\mathbb{F}_{q^{f_v}}\right)^{\left(\frac{2^{2v-\beta-1}}{f_v}\right)}$$
$$\oplus_{v=1}^{m_1-1} \oplus_{\beta=1}^{m_1+1-v} \left(\mathbb{F}_{q^{f_{\beta+v}}}\right)^{\left(\frac{2^{2v+\beta-1}}{f_{\beta+v}}\right)} \oplus M_2(\mathbb{F}_{q^{f_{m_1}}})^{\left(\frac{2^{m_1-1}}{f_{m_1}}\right)(2^{m_1})}.$$

(ii) For $m_1 > m_2$, the complete algebraic structure of semisimple group algebra Fq[G] is given as:

Primitive Central Idempotents

$e_C(G, G, < x, a >),\ C \in \mathcal{R}(G/< x, a >);$
$e_C(G, G, < x, b >),\ C \in \mathcal{R}(G/< x, b >);$
$e_C\left(G, G, < x, a, b^{2^i} >\right),\ C \in \mathcal{R}\left(G/< x, a, b^{2^i} >\right),\ 1 \le i \le m_2;$
$e_C\left(G, G, < x, ab^{2^i} >\right),\ C \in \mathcal{R}\left(G/< x, ab^{2^i} >\right),\ 1 \le i \le m_2;$
$e_C\left(G, G, < x^{2^v}, x^j a, b >\right),\ C \in \mathcal{R}\left(G/< x^{2^v}, x^j a, b >\right),\ 1 \le v \le m_1 - 1,\ j = 0, 2^{v-1};$
$e_C\left(G, G, < x^{2^v}, x^k a, x^j b >\right),\ C \in \mathcal{R}(G/< x^{2^v}, x^k a, x^j b >),\ 1 \le v \le m_1 - 1,$
$\gcd(j, 2^v) \ge \max\{1, 2^{v-m_2-1}\},\ k = 0, 2^{v-1};$
$e_C\left(G, G, < x^{2^v}, x^k a, x^j b^{2^\beta} >\right),\ C \in \mathcal{R}(G/< x^{2^v}, x^k a, x^j b^{2^\beta} >),\ 1 \le v \le m_1 - 1,$
$\gcd(j, 2^v) = 1,\ 1 \le \beta \le m_2 + 1 - v,\ k = 0, 2^{v-1};$
$e_C(G, < b, x >, < b >),\ C \in \mathcal{R}(< b, x >/< b >);$
$e_C\left(G, < a, x, y >, < a, x^j y >\right),\ C \in \mathcal{R}\left(< a, x, y >/< a, x^j y >\right),\ \gcd(j, 2^v) \ge$
$\max\{1, 2^{m_1-m_2}\}.$

Wedderburn Decomposition

$$\mathbb{F}_q[G] \cong \mathbb{F}_q{}^{(6)} \oplus_{\mathrm{i}=2}^{\mathrm{m}_2+1} \left(\mathbb{F}_{q^{f_i}}\right)^{\left(\frac{2^i}{f_i}\right)} \oplus_{v=2}^{m_1-1} \left(\mathbb{F}_{q^{f_v}}\right)^{\left(\frac{2^v}{f_v}\right)} \oplus_{v=1}^{m_2} \oplus_{\beta=0}^{v-1} \left(\mathbb{F}_{q^{f_v}}\right)^{\left(\frac{2^{2v-\beta-1}}{f_v}\right)}$$

$$\oplus_{v=m_2+1}^{m_1-1} \oplus_{\beta=v-m_2-1}^{v-1} \left(\mathbb{F}_{q^{f_2}}\right)^{\left(\frac{2^{2v-\beta-1}}{f_v}\right)} \oplus_{v=1}^{m_2} \oplus_{\beta=1}^{m_2+1-v} \left(\mathbb{F}_{q^{f_{\beta+v}}}\right)^{\left(\frac{2^{2v+\beta-1}}{f_{\beta+v}}\right)}$$

$$\oplus\, M_2(\mathbb{F}_{q^{f_{m_1}}})^{\left(\frac{2^{m_1-1}}{f_{m_1}}\right)(2^{m_2})}.$$

(ii) For $m_1 > m_2$, the complete algebraic structure of semisimple group algebra Fq [G] is given as:

Primitive Central Idempotents

$e_C(G, G, < x, a >),\ C \in \mathcal{R}(G/< x, a >);$

$e_C(G, G, < x, b >),\ C \in \mathcal{R}(G/< x, b >);$

$e_C\left(G, G, < x, a, b^{2^i} >\right),\ C \in \mathcal{R}\left(G/< x, a, b^{2^i} >\right),\ 1 \le i \le m_2;$

$e_C\left(G, G, < x, ab^{2^i} >\right),\ C \in \mathcal{R}\left(G/< x, ab^{2^i} >\right),\ 1 \le i \le m_2;$

$e_C\left(G, G, < x^{2^v}, x^j a, b >\right),\ C \in \mathcal{R}\left(G/< x^{2^v}, x^j a, b >\right),\quad j = 0, 2^{v-1};$

$e_C\left(G, G, < x^{2^v}, x^k a, x^j b >\right),\ C \in \mathcal{R}(G/< x^{2^v}, x^k a, x^j b >),\ 1 \le v \le m_1 - 1,$
$\gcd(j, 2^v) \ge \max\{1, 2^{v-m_2-1}\},\ k = 0, 2^{v-1};$

$e_C\left(G, G, < x^{2^v}, x^k a, x^j b^{2^\beta} >\right),\ C \in \mathcal{R}(G/< x^{2^v}, x^k a, x^j b^{2^\beta} >),\ 1 \le v \le m_1 - 1,$
$\gcd(j, 2^v) = 1,\ 1 \le \beta \le m_2 + 1 - v,\quad k = 0, 2^{v-1};$

$e_C(G, < b, x >, < b >),\ C \in \mathcal{R}(< b, x >/< b >);$

$e_C\left(G, < a, x, y >, < a, x^j y >\right),\ C \in \mathcal{R}\left(< a, x, y >/< a, x^j y >\right),\quad \gcd(j, 2^v) = 2^\beta,$
$0 \le \beta \le v - 1;$

$e_C\left(G, < a, x, y >, < a, x^j y^{2^\beta} >\right),\ C \in \mathcal{R}\left(< a, x, y >/< a, x^j y^{2^\beta} >\right),\quad \gcd(j, 2^v) = 1,$
$1 \le \beta \le m_2 - m_1.$

Wedderburn Decomposition

$$\mathbb{F}_q[G] \cong \mathbb{F}_q{}^{(6)} \oplus_{\mathrm{i}=2}^{\mathrm{m}_2+1} \left(\mathbb{F}_{q^{f_i}}\right)^{\left(\frac{2^i}{f_i}\right)} \oplus_{v=2}^{m_1-1} \left(\mathbb{F}_{q^{f_v}}\right)^{\left(\frac{2^v}{f_v}\right)} \oplus_{v=1}^{m_1-1} \oplus_{\beta=0}^{v-1} \left(\mathbb{F}_{q^{f_v}}\right)^{\left(\frac{2^{2v-\beta-1}}{f_v}\right)}$$

$$\oplus_{v=1}^{m_1-1} \oplus_{\beta=1}^{m_2+1-v} \left(\mathbb{F}_{q^{f_{\beta+v}}}\right)^{\left(\frac{2^{2v+\beta-1}}{f_{\beta+v}}\right)} \oplus M_2(\mathbb{F}_{q^{f_{m_1}}})^{\left(\frac{2^{m_1-1}}{f_{m_1}}\right)(2^{m_1})}$$

$$\oplus_{\beta=1}^{m_2-m_1} M_2(\mathbb{F}_{q^{f_{m_1+\beta}}})^{\left(\frac{2^{2m_1+\beta-2}}{f_{m_1+\beta}}\right)}.$$

References

[1] Bakshi, Gurmeet K.; Raka, Madhu (2003). Minimal cyclic codes of length pnq. Finite Fields Appl. 9(4), 432-448.

[2] Bakshi, Gurmeet K.; Raka, Madhu & Sharma, Anuradha (2008). Idempotent generators of irreducible cyclic codes. Number theory and discrete geometry, Ramanujan Math. Soc. Lect. Notes Ser., 6, 13-18.

[3] Bakshi, Gurmeet K.; Gupta, S. & Passi, I.B.S. (2015). The algebraic structure of finite metabelian group algebras, Comm. Algebra 43(6), 2240-2257.

[4] Berman, S.D. (1969). On the theory of group codes. Translated as cybernetics 3(1), 25-31.

[5] Broche, Osnel; del Rio, Angel (2007). Wedderburn decomposition of finite group algebras. Finite Fields Appl. 13(1), 71-79.

[6] Ferraz, Raul A. (2008). Simple components of the centre of FG/J(FG).Comm. Algebra, 36(9), 3191-3199.

[7] Ferraz, R. A., Goodaire, E. G. & Milies, C. P. (2010). Some classes of semisimple group (loop) algebra over finite fields. J. Algebra 324(12), 3457-3469.

[8] Goodaire, E. G. (1983). Alternative loop rings. Publ. Math. Debrecen 30(1-2), 31-38.

[9] Goodaire, E. G.; Jespers, E. & Miles, C.P. (1996). Alternative loop rings. North- Holland Mathematics Studies 184. Amsterdam: North-Holland Publishing Co.

[10] Khan, Manju (2009). Structure of the unit group of Fd10. Serdica Math. J. 35(1), 15-24.

[11] Khan, M,; Sharma, R.K. & Srivastav, J. B. (2008). The unit group of FS4. Acta Math. Hunger. 118(1-2), 105-113.

[12] Makhijani, N.; Sharma, R.K. & Srivastav, J. B. (2014). Structure of some classes of semisimple group algebras over finite fields, Bull. Korean Math. Soc. 51(6), 1605-1614.

[13] Makhijani, N.; Sharma, R.K. & Srivastav, J. B. (2016). The unit group of some special semi-simple group algebras, Questiones Mathematicae 39(1), 9-28.

[14] Pruthi, Manju & Arora S.K.(1997). Minimal codes of prime-power length. Finite Fields Appl. 3(2), 99-113.

[15] Sharma, Anuradha; Bakshi, Gurmeet K.; Dumir V.C. & Raka, Madhu (2004). Cyclotomic Numbers and primitive idempotents in the ring $GF(q)[x]/(x^{p^n}-1)$ *Finite Fields Appl. 10(4), 653-673.*

[16] Sharma, Anuradha; Bakshi, Gurmeet K. & Dumir V.C. ; Raka, Madhu (2008). Irreducible cyclic codes of length 2n. Ars Combin. 86, 133-146.

[17] Sharma, R.K.; Srivastav, J.B. & Khan, Manju (2007). The unit group of Fs3. Acta Math. Acad. Paedagog. Nyhzi. (N.S), 23(2), 129-142.

[18] Sharma, R.K.; Srivastav, J.B. & Khan, Manju (2007). The unit group of Fa4. Publ. Math. Debrecen 71(1-2), 21-26.

ISSN : 2394 - 7139

Degree Based Indices of Rhomtrees and Line Graph of Rhomtrees

1R.Anuradha, 2V.Kaladevi, 3A.Abinayaa
1Research Scholar, Research and Development centre, Bharathiar University, Coimbatore & Assistant Professor of Mathematics, Thanthai Hans Roever College, Perambalur-20
2 Professor Emeritus, PG and Research Department of Mathematics, Bishop Heber College, Trichy-17.
3 Research Scholar (FT), PG and Research Department of Mathematics, Bishop Heber College, Trichy-17.

ABSTRACT

Rhotrix theory deals with array of numbers in rhomboid mathematical form. The graphical representation of rhotrix of dimension n is known as rhomtree. In this paper the degree based indices of rhomtrees and line graph of rhomtrees are computed.

Keywords: first Zagreb index, forgotten index, hyper Zagreb index, irregularity index, Rhotrix, second Zagreb index.

1. INTRODUCTION

Let G (V, E)be a simple undirected graph. In the field of chemical graph theory and in mathematical chemistry, a topological index, also known as a connectivity index, is a type of a molecular descriptor that is calculated based on the distance between the atoms of molecular graph. Topological indices [3] are used for example in the development of quantitative structureactivity relationship (QSAR) and quantitative structure - property relationship (QSPR) in which the biological activity or other properties of molecules are correlated with their chemical structure. Among different topological indices, degree-based topological indices are most studied and have some important applications in chemical graph theory [8]. In [7] it was reported that the first and second Zagreb indices are useful in anti-inflammatory activities study of certain chemicals. In the same paper the F-index was introduced which is the sum of the cubes of the vertex degrees. In [4, 6], the authors reinvestigated the index and named it forgotten topological index or F-index. The F-index is defined as $F(G)=\sum_{uv\in E(G)}\left[d_G(u)^2+d_G(v)^2\right]$. In [4] this index is studied for different graph operations and in [5] the co-index version is introduced. Albetson in [2] defined another degree based topological index called irregularity of G as

Irr (G) = $\sum_{uv\in E(G)}|d_G(u)-d_G(v)|$. The first and second Zagreb indices of a graph are denoted by $M_1(G)$ and $M_2(G)$ and are, respectively, defined as $M_1(G)=\sum_{uv\in E(G)}[d_G(u)+d_G(v)]$ and $M_2(G)=\sum_{uv\in E(G)}[d_G(u)d_G(v)]$. These indices are one of the oldest and extensively studied topological indices in both mathematical and chemical literature; for details interested readers are referred to [10]. Shirdel et al. [13] introduced a new version of Zagreb index and named as hyper-Zagreb index, which is defined as HM (G)= $\sum_{uv\in E(G)}[d_G(u)+d_G(v)]^2$.

Construction of Rhomtree

The structure of n-dimensional rhotrix is as follows:

$$R_n=\left\langle \begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & a_{21} & c_{11} & a_{12} & & & \\ & & a_{31} & c_{21} & a_{22} & c_{12} & a_{13} & & \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ a_{t1} & c_{t-11} & & & h(R) & & & c_{1t-1} & a_{1t} \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \\ & & a_{tt-2} & c_{t-1t-2} & a_{t-1t-1} & c_{t-2t-1} & a_{t-2t} & & \\ & & & a_{tt-1} & c_{t-1t-1} & a_{t-1t} & & & \\ & & & & a_{tt} & & & & \end{array}\right\rangle$$

where a_{11}, a_{12}, … a_{tt} denote the major entries and c_{11}, c_{12}, … $c_{t-1}c_{t-1}$ in R_n denote the minor entries of the rhotrix by sani [11,12]. A Rhotrix would always have an odd dimension. Any n-dimensional Rhotrix R_n, will have $|R_n| = \frac{1}{2}(n^2+1)$ entries by Ajibade [1] and $n \in 2Z^+ + 1$. A heart of a Rhotrix denoted by h(R) is defined as the element at the perpendicular intersection of the two diagonals of a Rhotrix. Let $\hat{R}(n)$ be a set consisting all real rhotrices of dimension $n \in 2Z^+ + 1$ and let R(n) be any rhotrix in $\hat{R}(n)$. Then the graphical representation of rhotrix R(n) is a rhomtree T(m), with m = $\frac{1}{2}(n^2+1)$ number of vertices and $\frac{1}{2}(n^2-1)$ number of edges, having four components of binary branches and each component is bridged to the root vertex by one incident edge.

If n=7, then $\hat{R}(7)$ is a set consisting of all real rhotrices of dimension three and let R(7) be any element in $\hat{R}(7)$ given by

$$R_7 = \left\langle \begin{array}{ccccccc} & & & a_{11} & & & \\ & & a_{21} & c_{11} & a_{12} & & \\ & a_{31} & c_{21} & a_{22} & c_{12} & a_{13} & \\ a_{14} & c_{31} & a_{32} & c_{22} & a_{23} & c_{13} & a_{14} \\ & a_{42} & c_{32} & a_{33} & c_{23} & a_{24} & \\ & & a_{43} & c_{33} & a_{34} & & \\ & & & a_{44} & & & \end{array} \right\rangle$$

with r_{13} = h(R) is the heart of the rhotrix. If we take each entry in R(7) as a node point and connecting all of the entries as network of twenty five vertices using a particular pattern or style for the construction, in such a way that the heart vertex will serve as the root of the tree while the non heart vertices will serve as branches, then a rhomtree T(25) corresponding to the rhotrix R(7) is obtained and shown in Fig.1.

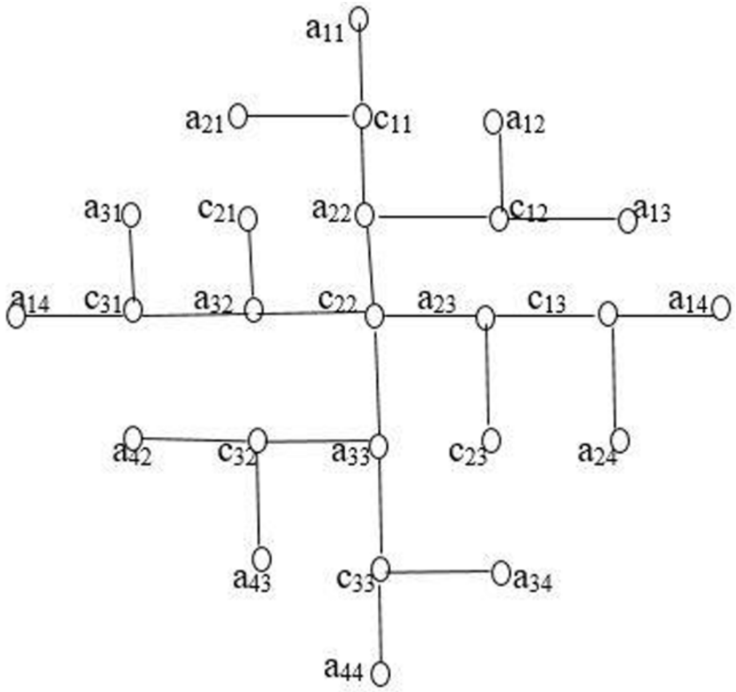

fig.1 Rhomtree T(25)

The line graph of [T (25)] is given in Fig.2

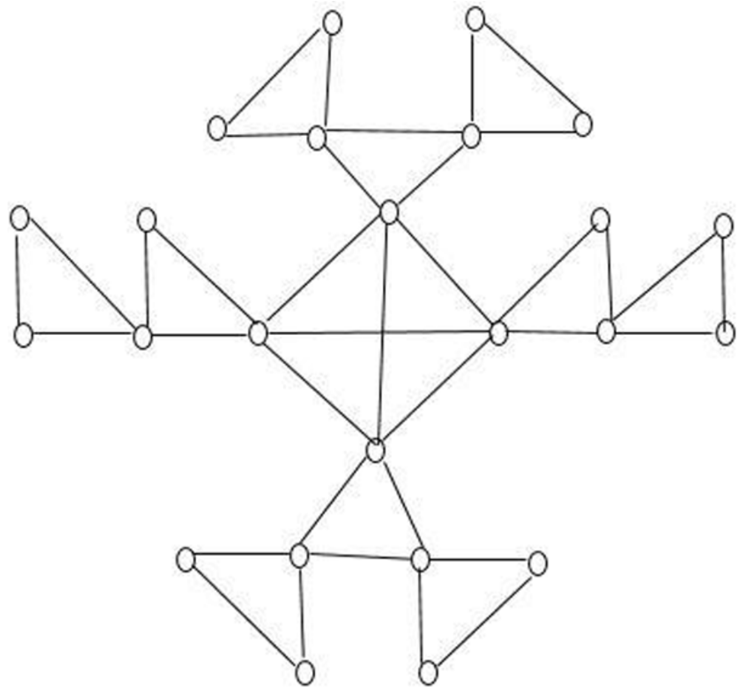

fig.2 L(T(25))

Let G=T(m) be the rhomtree of order m=$\frac{1}{2}(n^2+1)$. The partitions of the vertex set V (G) are denoted by V_i (G), where v∈V_i(G) if d(v)= i. Thus the following partitions of the vertex set are obtained.

V_1= {v∈V(G): d(v)=1},V_3= {v∈V(G):d(v)=3} and V_4= {v∈V(G):d(v)=4}

From the structure of rhomtree, the cardinality of V_1, V_3 and V_4 are given below:

$|V_1| = \frac{1}{4}(n^2+7)$, $|V_3| = \frac{1}{4}(n^2-9)$ and $|V_4| = 1$

The edge set of G can also be divided into three partitions based on the sum of degrees of the end vertices and it is denoted by E_j so that if $e = uv \in E_j$ then d (u) + d(v) = j for δ(G) ≤ j ≤ Δ(G). Thus the edge set of G is the union of E_4, E_6 and E_7. The edge sets E_4, E_6 and E_7, which are subsets of E (G) are as follows:

E_4={e=uv∈E(G):d(u)=1, d(v)=3}, E_6={e=uv∈E(G):d(u)=3, d(v)=3} and

E_7= {e=uv∈E(G): d(u)=3, d(v)=4}.

In this case from direct calculations, the cardinality of E_4, E_6 and E_7 are respectively $\frac{1}{4}(n^2+7)$, $\frac{1}{4}(n^2-25)$ and 4. The partitions of the vertex set V (G) and edge set E (G) are given in Table 1 and Table 2 respectively.

Vertex partition	V_1	V_3	V_4
Cardinality	$\frac{1}{4}(n^2+7)$	$\frac{1}{4}(n^2-9)$	1

Table1: The vertex partition of Rhomtree T (m)

Edge partition	E_4	E_6	E_7
Cardinality	$\frac{1}{4}(n^2+7)$	$\frac{1}{4}(n^2-25)$	4

Table 2: The edge partition of Rhomtree T (m)

Similarly the vertex set and edge set of line graph of rhomtree can be partitioned. The partitions of the vertex set of L (G) are given by
$V_2^* = \{v \in V(L(G)): d(v)=2\}, V_4^* = \{v \in V(L(G)):d(v)=4\}$ and $V_5^* = \{v \in V(L(G)):d(v)=5\}$.

Vertex Partition of L(G)	V_2^*	V_4^*	V_5^*
Cardinality	$\frac{1}{4}(n^2+7)$	$\frac{1}{4}(n^2-25)$	4

Table 3: The Vertex partition of L (T (m))

The partitions of the edge set of L(G) are given by
$E_4^* = \{e=uv \in E(L(G)): d(u)=2 , d(v)=2\}$, $E_6^* = \{e=uv \in E(L(G)) :d(u)=2, d(v)=4\}$ and
$E_7^* = \{e=uv \in E(G): d(u)=2, d(v)=5\}$, $E_8^* = \{e=uv \in E(G): d(u)=4, d(v)=4\}$
$E_9^* = \{e=uv \in E(G): d(u)=4, d(v)=5\}$, $E_{10}^* = \{e=uv \in E(G): d(u)=5, d(v)=5\}$

Edge partition of L(G)	E_4^*	E_6^*	E_7^*	E_8^*	E_9^*	E_{10}^*
Cardinality	n-1	$\frac{1}{2}(n^2-4n+7)$	2	$\frac{1}{4}(n^2+4n-69)$	6	6

Table 4: The Edge partition of L (T (m))

2. F-index, irregularity index of T(m) and L(T(m))

Theorem 2.1 The F- index of Rhomtree T (m) is given by F (G)= $\frac{1}{2}\left(7n^2+5\right)$

Proof F index of rhomtree T (m) is

$$F(G) = \sum_{uv \in E(G)} \left[d_G(u)^2 + d_G(v)^2\right]$$

$$= \sum_{uv \in E_4} \left[d_G(u)^2 + d_G(v)^2\right] + \sum_{uv \in E_6} \left[d_G(u)^2 + d_G(v)^2\right] + \sum_{uv \in E_7} \left[d_G(u)^2 + d_G(v)^2\right]$$

$$=\left|E_4^*\right|(10)+\left|E_6^*\right|(18)+\left|E_7^*\right|(25) = \frac{1}{4}(n^2+7)(10)+\frac{1}{4}(n^2-25)(18)+4(25) = \frac{1}{2}(7n^2+5)$$

Theorem 2.2 The F index of Line graph of Rhomtree L (T (m)) is given by

F (L (T(m))) = $18n^2+114$

Proof The F-index of Line graph of T (m) is

$$F(L(G)) = \sum_{uv\in E(G)} [d_G(u)^2 + d_G(v)^2]$$

$$= \sum_{uv\in E_4^*} [d_G(u)^2 + d_G(v)^2] + \sum_{uv\in E_6^*} [d_G(u)^2 + d_G(v)^2] + \sum_{uv\in E_7^*} [d_G(u)^2 + d_G(v)^2] +$$

$$\sum_{uv\in E_8^*} [d_G(u)^2 + d_G(v)^2] + \sum_{uv\in E_9^*} [d_G(u)^2 + d_G(v)^2] + \sum_{uv\in E_{10}^*} [d_G(u)^2 + d_G(v)^2]$$

$$=\left|E_4^*\right|(8)+\left|E_6^*\right|(20)+\left|E_7^*\right|(29)+\left|E_8^*\right|(32)+\left|E_9^*\right|(41)+\left|E_{10}^*\right|(50)$$

$$= (n-1)(8)+\frac{1}{2}(n^2-4n+7)(20)+2(29)+\frac{1}{4}(n^2+4n-69)(32)+6(41)+6(50) = 18n^2+114.$$

Theorem 2.3 The third Zagreb index or irregularity index of Rhomtree T (m) is given by

iir (T(m))= $\frac{1}{2}(n^2+15)$

Proof The irr-index of T (m) is

$$irr(G) = \sum_{uv\in E(G)} \left|d_G(u) - d_G(v)\right|$$

$$= \sum_{uv\in E_4^*} \left|d_G(u) - d_G(v)\right| + \sum_{uv\in E_6^*} \left|d_G(u) - d_G(v)\right| + \sum_{uv\in E_7^*} \left|d_G(u) - d_G(v)\right|$$

$$=\left|E_4^*\right|(2)+\left|E_6^*\right|(0)+\left|E_7^*\right|(1) = \frac{1}{4}(n^2+7)(2)+\frac{1}{4}(n^2-25)(0)+4(1) = \frac{1}{2}(n^2+15)$$

Theorem 2.4 The third Zagreb index or irregularity index of Line graph of Rhomtree L (T (m)) is given by iir (L (T(m)))= n^2-4n+19

Proof The irr-index of Line graph of T (m) is

$$irr(L(G)) = \sum_{uv\in E_4^*} \left|d_G(u) - d_G(v)\right| + \sum_{uv\in E_6^*} \left|d_G(u) - d_G(v)\right| + \sum_{uv\in E_7^*} \left|d_G(u) - d_G(v)\right| + \sum_{uv\in E_8^*} \left|d_G(u) - d_G(v)\right| +$$

$$\sum_{uv\in E_9^*} \left|d_G(u) - d_G(v)\right| + \sum_{uv\in E_{10}^*} \left|d_G(u) - d_G(v)\right|$$

$$= \left|E_4^*\right|(0)+\left|E_6^*\right|(2)+\left|E_7^*\right|(3)++\left|E_8^*\right|(0)+\left|E_9^*\right|(1)+\left|E_{10}^*\right|(0)$$

$$= (n-1)(0)+\frac{1}{2}(n^2-4n+7)(2)+2(3)+\frac{1}{4}(n^2+4n-69)(0)+6(1)+6(0) = n^2-4n+19$$

3. First, Second Zagreb index and hyper Zagreb index of Rhomtree and Line graph of Rhomtree

Theorem 3.1 The First Zagreb index of Rhomtree T (m) is given by $M_1(T(m)) = \frac{5}{2}\left(n^2-1\right)$

Proof The M_1-index of T (m) is

$$M_1(G) = \sum_{uv\in E(G)}\left[d_G(u)+d_G(v)\right]$$

$$= \sum_{uv\in E_4^*}\left[d_G(u)+d_G(v)\right]+\sum_{uv\in E_6^*}\left[d_G(u)+d_G(v)\right]+\sum_{uv\in E_7^*}\left[d_G(u)+d_G(v)\right]$$

$$=\left|E_4^*\right|(4)+\left|E_6^*\right|(6)+\left|E_7^*\right|(7) = \frac{1}{4}\left(n^2+7\right)(4)+\frac{1}{4}\left(n^2-25\right)(6)+4(7) = \frac{5}{2}\left(n^2-1\right)$$

Theorem 3.2 The First Zagreb index of Line graph of Rhomtree L (T (m)) is given by $M_1(L(T(m))) = 5n^2+7$

Proof The M_1-index of Line graph of T (m) is

$$M_1(G) = \sum_{uv\in E(G)}\left[d_G(u)+d_G(v)\right]$$

$$= \sum_{uv\in E_4^*}\left[d_G(u)+d_G(v)\right]+\sum_{uv\in E_6^*}\left[d_G(u)+d_G(v)\right]+\sum_{uv\in E_7^*}\left[d_G(u)+d_G(v)\right]$$

$$+\sum_{uv\in E_8^*}\left[d_G(u)+d_G(v)\right]+\sum_{uv\in E_9^*}\left[d_G(u)+d_G(v)\right]+\sum_{uv\in E_{10}^*}\left[d_G(u)+d_G(v)\right]$$

$$=\left|E_4^*\right|(4)+\left|E_6^*\right|(6)+\left|E_7^*\right|(7)+\left|E_8^*\right|(8)+\left|E_9^*\right|(9)+\left|E_{10}^*\right|(10)$$

$$= (n-1)(4)+\frac{1}{2}\left(n^2-4n+7\right)(6)+2(7)+\frac{1}{4}\left(n^2+4n-69\right)(8)+6(9)+6(10) = 5n^2+7$$

Theorem 3.3 The Second Zagreb index of Rhomtree T (m) is given by $M_2(T(m)) = 3\left(n^2-1\right)$

Proof The M_2-index of T (m) is

$$M_2(G) = \sum_{uv\in E(G)}\left[d_G(u)d_G(v)\right].$$

$$= \sum_{uv\in E_4^*} d_G(u)d_G(v)+\sum_{uv\in E_6^*} d_G(u)d_G(v)+\sum_{uv\in E_7^*} d_G(u)d_G(v)$$

$$=\left|E_4^*\right|(3)+\left|E_6^*\right|(9)+\left|E_7^*\right|(12)$$

$$= \frac{1}{4}\left(n^2+7\right)(3)+\frac{1}{4}\left(n^2-25\right)(9)+4(12)$$

$$= 3\left(n^2-1\right)$$

Theorem 3.4 The Second Zagreb index of Line graph of Rhomtree L(T(m)) is given by

M_2 (L(T(m)))= $8n^2+4n+38$

Proof The M_2-index of line graph of T(m) is

$$M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

$$= \sum_{uv\in E_4^*} d_G(u)d_G(v) + \sum_{uv\in E_6^*} d_G(u)d_G(v) + \sum_{uv\in E_7^*} d_G(u)d_G(v) + \sum_{uv\in E_8^*} d_G(u)d_G(v) + \sum_{uv\in E_9^*} d_G(u)d_G(v) + \sum_{uv\in E_{10}^*} d_G(u)d_G(v)$$

$$= |E_4^*|(4)+|E_6^*|(8)+|E_7^*|(10)+|E_8^*|(16)+|E_9^*|(20)+|E_{10}^*|(25)$$

$$= (n-1)(4)+\frac{1}{2}(n^2-4n+7)(8)+2(10)+\frac{1}{4}(n^2+4n-69)(16)+6(20)+6(25) = 8n^2+4n+38$$

Theorem 3.5 The HM index of Rhomtree T(m) is given by HM(T(m))= $13n^2-1$

Proof The HM-index of T(m) is

$$HM(G) = \sum_{uv\in E(G)} [d_G(u)+d_G(u)]^2$$

$$= \sum_{uv\in E_4^*} [d_G(u)+d_G(v)]^2 + \sum_{uv\in E_6^*} [d_G(u)+d_G(v)]^2 + \sum_{uv\in E_7^*} [d_G(u)+d_G(v)]^2$$

$$= |E_4^*|(16)+|E_6^*|(36)+|E_7^*|(49) = \frac{1}{4}(n^2+7)(16)+\frac{1}{4}(n^2-25)(36)+4(49) = 13n^2-1$$

Theorem 3.6 The HM index of Line graph of Rhomtree L (T(m)) is given by

HM (L(T(m)))= $34n^2+8n+190$

Proof The HM-index of line graph of T (m) is

$$HM(L(G)) = \sum_{uv\in E(G)} [d_G(u)+d_G(u)]^2$$

$$= \sum_{uv\in E_4^*} [d_G(u)+d_G(v)]^2 + \sum_{uv\in E_6^*} [d_G(u)+d_G(v)]^2 + \sum_{uv\in E_7^*} [d_G(u)+d_G(v)]^2 + \sum_{uv\in E_8^*} [d_G(u)+d_G(v)]^2 + \sum_{uv\in E_9^*} [d_G(u)+d_G(v)]^2 + \sum_{uv\in E_{10}^*} [d_G(u)+d_G(v)]^2$$

$$= |E_4^*|(16)+|E_6^*|(36)+|E_7^*|(49)+|E_8^*|(64)+|E_9^*|(81)+|E_{10}^*|(100)$$

$$= (n-1)(16)+\frac{1}{2}(n^2-4n+7)(36)+2(49)+\frac{1}{4}(n^2+4n-69)(64)+6(81)+6(100) = 34n^2+8n+190$$

Conclusion

The molecular name for T(25) is 4,4- Bis-(1-isopropyl-2-methyl-propyl)-2,3,5,6-tetramethylheptane and that of T(41) is 4-(1-Isopropyl-2-methyl-propyl)-5-[1-(1-isopropyl-2-methylpropyl)-2,3-dimethyl-butyl]-2,3,6,7,8-pentamethyl-5-(1,2,3-trimethyl-butyl)-nonane. In chemical graph theory, topological indices provide an important tool to quantify the molecular structure and it is found that there is a strong correlation between the properties of chemical compounds and their molecular structure. Among different topological indices, degree-based topological indices are most studied and have some important applications. In this study, degreebased topological indices are calculated for rhomtrees and line graph of rhomtrees.

References

[1] Ajibade, A.O, "The Concept of Rhotrix in Mathematical Enrichment", International journal of Mathematical Education in Science and Technology, 34, 175-179, 2003.

[2] Albertson. M.O, "The irregularity of a graph," Ars Combinatoria, vol.46, pp. 219–225, 1997.

[3] Basker Babujee.J and Ramakrishnan.J, "Topological indices for Graphs and Chemical Reactions", proceedings of ICMCS, pp: 81-88, 2011.

[4] De.N, Nayeem.S.M.A, and Pal.A, "F-index of some graph operations," Discrete Mathematics, Algorithms and Applications, vol. 8, no. 2, Article ID 1650025, 17 pages, 2016.

[5] De.N, Nayeem.S.M.A, and Pal.A, "The F-coindex of somegraph operations," SpringerPlus, vol. 5, article 221, 2016.

[6]Furtula.B and Gutman.I, "A forgotten topological index", Journal of Mathematical Chemistry, vol. 53, no. 4, pp. 1184–1190, 2015.

[7]Gutman.I and Trinajsti c.N, "Graph theory and molecular orbitals. Total φ-electron energy of alternant hydrocarbons" Chemical Physics Letters, vol. 17, no. 4, pp. 535–538, 1972.

[8] Hall.L.H, Kier.L.B, "Molecular connectivity in chemistry and drug research", ityplaceBoston: Academic Press (1976).

[9] Miliˇcevi´c.A, Nikoli´c.S, and Trinajsti´c.N, " On reformulatedZagreb indices," Molecular Diversity, vol. 8, no. 4, pp. 393–399, 2004.

[10] Nilangan De, "Research Article On Molecular Topological Properties of TiO2 Nanotubes" Hindawi Publishing Corporation Journal of Nanoscience, Volume 2016, Article ID1028031, http://dx.doi.org/10.1155/2016/1028031 , pp: 1-5.

[11] Sani, B, "An Alternative Method for Multiplication of Rhotrices", International journal of Mathematical Education in Science and Technology, 35, No.5, 777- 781, 2004.

[12] Sani, B, "The Row- Column Multiplication of High Dimensional Rhotrices", International journal of Mathematical Education in Science and Technology, 38, No.5, 657- 662, 2007.

[13] Shirdel G.H, Rezapour.H, and Sayadi A.M , "The hyper Zagreb index of graph operations," Iranian Journal of Mathematical Chemistry, vol. 4, no. 2, pp: 213–220, 2013.

ISSN : 2394 - 7139

ANALYSING AMINO ACIDS IN HUMAN GALANIN AND ITS RECEPTORS - GRAPH THEORETICAL APPROACH

Suresh Singh G.a and Akhil C. K.b
aDepartment of Mathematics, University of Kerala,
Kariavattom, Thiruvananthapuram – 695581, Kerala, India,

bDepartment of Mathematics, University of Kerala,
Kariavattom, Thiruvananthapuram – 695581, Kerala, India,

Acknoldgements

We wish to thank Dr Achuthsankar S. Nair, Head of the Department, Department of Computational Biology and Bioinformatics, University of Kerala, Thiruvananthapuram along with the faculty members, Vijayalakshmi B. and Biji C. L. for their fruitful suggestions and constant encouragements.

A B S T R A C T

Graph theoretical analysis is an important area of research in biological networks. In this work we define some new graphs called bipartite Pt-graphs and their physicochemical subgraphs for peptides/proteins and their receptors based on the physicochemical properties of amino acids. Here we analyze bipartite Ptgraphs and their physicochemical subgraphs of human galanin and its three receptors graph theoretically. From the graph theoretical analysis of bipartite Ptgraphs and the physicochemical subgraphs we get some observations about the relations among the amino acids, physicochemical properties, galanin and its receptors. By a graph theoretical parameter of physicochemical subgraphs we get all the collections of maximum independent pairs of amino acids which connect the galanin and receptors by sharing exactly n (n = 1,2,3,…) common physicochemical properties. These analyses can be used to study all the relationships between peptide/protein ligands and their receptors and this may help in the field of drug designing.

Keywords: Amino acid, galanin, galanin receptor, bipartite Pt-graph, physicochemical subgraph.

1. INTRODUCTION

Proteins are polymers of amino acids, with each amino acid residue joined to its neighbour by a specific type of covalent bond [3]. Twenty different types of amino acids are commonly found in peptide/protein. The sequence of amino acids in a protein is characteristic of that protein and is called its primary structure [3]. Peptides/proteins are the compounds of amino acids in which a carboxyl group of one is united with an amino group of another. Neuropeptides are peptides formed and released by neurons. They are involved in a wide range of brain functions.

Galanin is a neuropeptide of 30 amino acids in humans and 29 amino acids in other species [4]. It is expressed in a wide range of tissues including the brain, spinal cord and gut. Its signaling occurs through three G protein-coupled receptors. It is linked to a number of diseases including Alzheimer's disease, epilepsy, depression, eating disorders, cancer, etc.

In [7], we can see so many graph theoretical applications in various fields. Amino acid network with in protein was studied by S. Kundu [5]. By using some physicochemical properties (Hydrophobicity, Hydrophilicity, Polarity, Non-polarity, Aliphaticity, Aromaticity and Charge (Positive and Negative)) of amino acids, the amino acid network was studied by Adil Akthar and Nisha Gohan graph theoretically [1]. The centralities in amino acid networks were used by Adil Akhtar and Tazid Ali [2]. By using the concept of amino acid network we defined and analysed the peptide/protein graph (Pt-graph) and species peptide/protein graph (SPt-graph) of galanin present in fourteen species of animals graph theoretically [6]. In this work we define and analyse new graphs - bipartite Pt-graphs and physicochemical subgraphs (physicochemical propertywise) - of human galanin and its three receptors on the basis of the physicochemical properties of amino acids. The maximum matching of physicochemical subgraphs is done to get all the collections of maximum independent pairs of amino acids which connect the galanin and its receptors by sharing exactly n(n=1,2,3,…) common physicochemical properties of amino acids. This method can be applied for all relationships between peptides/proteins and their receptors and this may help in the field of drug designing.

2. Basic Concepts of Graph Theory

Definition 2.1: A Graph [7] $\mathcal{G}$ is a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consisting of a finite set $\mathcal{V}$ and a set $\mathcal{E}$ of 2-element subsets of $\mathcal{V}$. The elements of $\mathcal{V}$ are called vertices and elements of $\mathcal{E}$ are called edges. The set $\mathcal{V}$ is known as the vertex set of $\mathcal{G}$ and $\mathcal{E}$ as the edge set of $\mathcal{G}$. Two vertices u and v of $\mathcal{G}$ are said to be adjacent, if an edge join u and v, and two edges are adjacent if they have common vertex. The number of vertices in a graph $\mathcal{G}$ is called its order and the number of edges is its size. A graph with p vertices and q edges is said to be a (p, q) graph.

Definition 2.2: Centrality measures in Graphs [2] are the vertex representation which gives the relative importance within the graph. A centrality is a real-valued function f which assigns every vertex $v \in \mathcal{V}$ of a given graph $\mathcal{G}$ a value $f(v) \in \mathbb{R}$.

Definition 2.3: Let $\mathcal{G}$ be an arbitrary (p, q) graph. $\mathcal{M} \subset \mathcal{E}(\mathcal{G})$ is said to be a matching in $\mathcal{G}$ if its elements are links in $\mathcal{G}$ and no two elements of $\mathcal{M}$ are adjacent in $\mathcal{G}$. $\mathcal{M}$ is said to be a maximal matching if there exists no matching $\mathcal{M}'$ of $\mathcal{G}$ with $|\mathcal{M}'| > |\mathcal{M}|$. An edge $e \in \mathcal{E}(v)$ is said to be matched under $\mathcal{M}$ (resp. unmatched under $\mathcal{M}$) if $e \in \mathcal{M}$ (resp. if $e \notin \mathcal{M}$). A vertex v is said to be saturated by a matching $\mathcal{M}$ ($\mathcal{M}$- saturated) or matched vertex with respect to $\mathcal{M}$ if v is incident with an edge of $\mathcal{M}$. Otherwise, the vertex is said to be unsaturated by $\mathcal{M}$ ($\mathcal{M}$-

unsaturated) or a single vertex with respect to $\mathcal{M}$. A matching $\mathcal{M}$ of a graph $\mathcal{G}$ is said to be a perfect matching if all the vertices of $\mathcal{G}$ are saturated by $\mathcal{M}$.

Definition 2.4: A Pt-graph is defined as a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of a peptide/protein in which the vertex set, $\mathcal{V}$ is the collection of all different amino acids presented in the peptide/protein and weight of a vertex in $\mathcal{G}$ is the number of times it appears in the sequence of the peptide/protein. Two vertices are said to be adjacent in $\mathcal{G}$ if they are consecutive elements in the sequence and also have at least one common physicochemical property.

For all terminologies and notations not mentioned in this work, we follow [7] (related to graph theory) and [3] (related to biology).

Remark: Weight of a vertex implies the frequency of occurrence of a specific amino acid in a sequence. Obviously greater the weight of a vertex of a Pt-graph implies greater the characteristics of those particular amino acid can be attributed to the peptide/protein. Also the centrality measures of a Pt-graph help us to determine the number of amino acids possess inter-relationships with each other.

3. Bipartite Pt-graphs and physicochemical subgraphs of human galanin and its receptors

In this section we define some new graphs called bipartite Pt-graphs and physicochemical subgraphs for peptides/proteins and its receptors. Also we construct and analyze bipartite Ptgraphs and their physicochemical subgraphs of human galanin and its receptors using $\mathcal{C}$ -sets of corresponding Pt-graphs.

efinition 3.1: A bipartite Pt-graph is defined as a simple bipartite graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of a peptide/protein and its receptors with x and y as the partitions of the vertex sets of the corresponding Pt-graphs of the peptide/protein and its receptors respectively. Two vertices $x \in X$ and $y \in y$ are said to be adjacent if they have atleast one common physicochemical property.

Definition 3.2: A $\mathcal{C}$-set of a Pt-graph of a peptide/protein is defined as the subset of the vertex set whose elements are the amino acids which recieve the highest centrality measures for each physicochemical properties of amino acids.

Pt-graphs of human galanin and its receptors

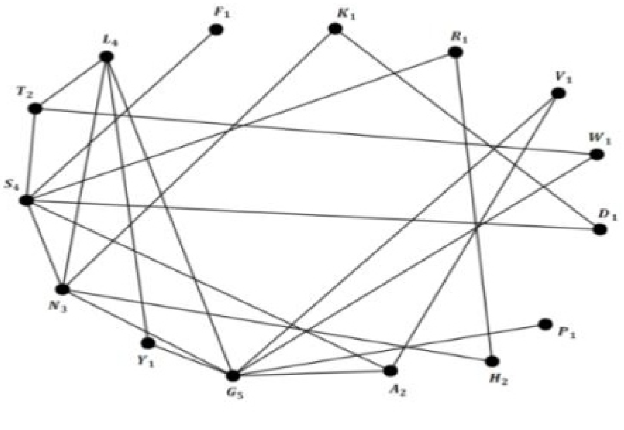

Figure 1: Pt-graph of human galanin

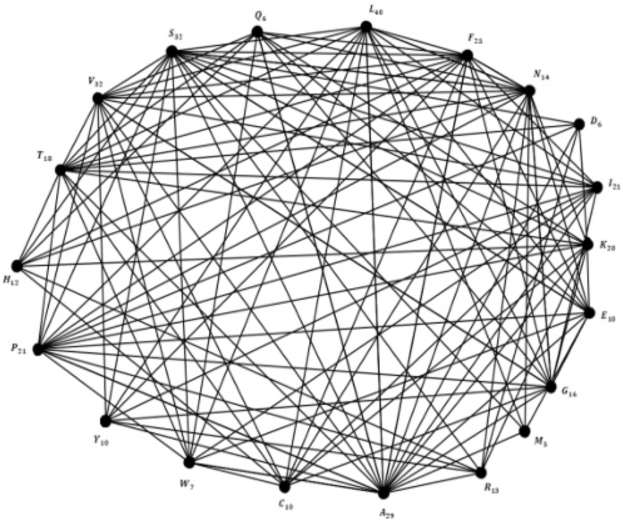

Figure 2: Pt-graph of receptor-1

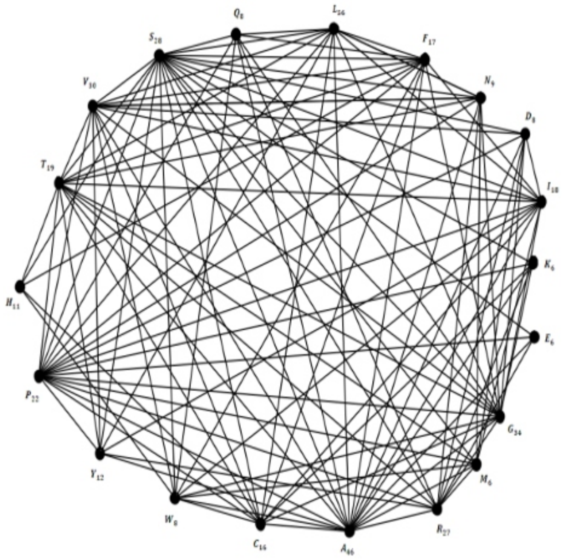

Figure 3: Pt-graph of receptor-2

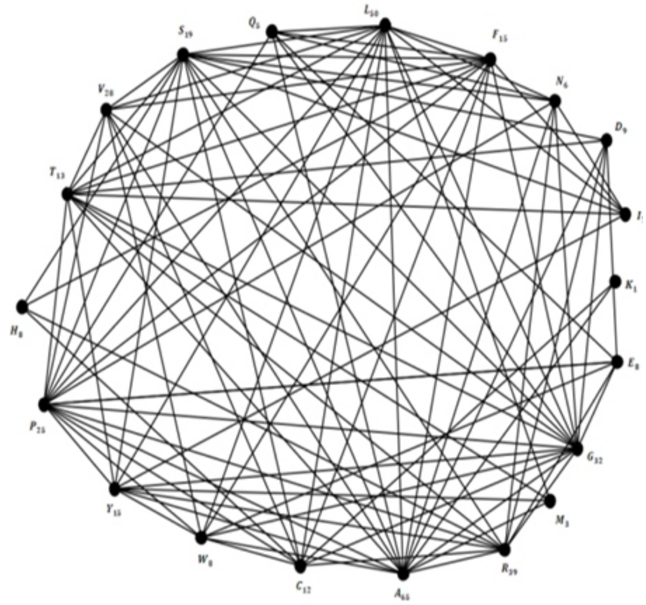

Figure 4: Pt-graph of receptor-3

From Pt-graphs we get the $\mathcal{C}$-set from the highest centralities of amino acids for each physicochemical property. For human galanin, G_5 (Hydrophoic and Non-polar), S_4 and N_3 (Hydrophilic and polar), L_4 (Aliphatic), Y_1 and W_1 (Aromatic), H_2, K_1 and R_1 (Positive), D_1 (Negative) are the amino acids which receive the highest centralities. Hence the $\mathcal{C}$-set for the galanin is $\{G_5, S_4, N_3, L_4, Y_1, W_1, H_2, K_1, R_1, D_1\}$. Similarly we get the $\mathcal{C}$-sets for the Pt-graphs of receptor-1, receptor-2 and receptor-3. Then the $\mathcal{C}$-set for receptor-1 is $\{A_{29}, N_{14}, S_{32}, L_{40}, F_{25}, K_{20}, E_{10}\}$, $\mathcal{C}$-set for receptor-2 is $\{A_{46}, S_{28}, V_{30}, F_{17}, R_{27}, D_8, I_{18}, W_8\}$ and $\mathcal{C}$-set for receptor-3 is $\{A_{65}, G_{32}, P_{25}, S_{19}, L_{50}, F_8, R_{39}, E_8, D_9, V_{28}\}$.

Next we analyse the bipartite Pt-graphs of human galanin and its receptors.

Bipartite Pt-graphs of human galanin and its receptors

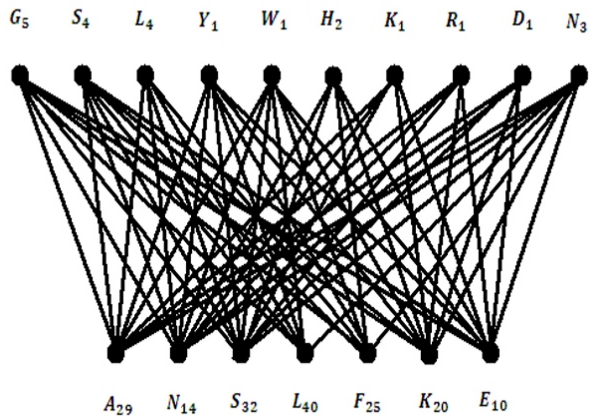

Figure 5: Bipartite Pt-graph of galanin and receptor-1

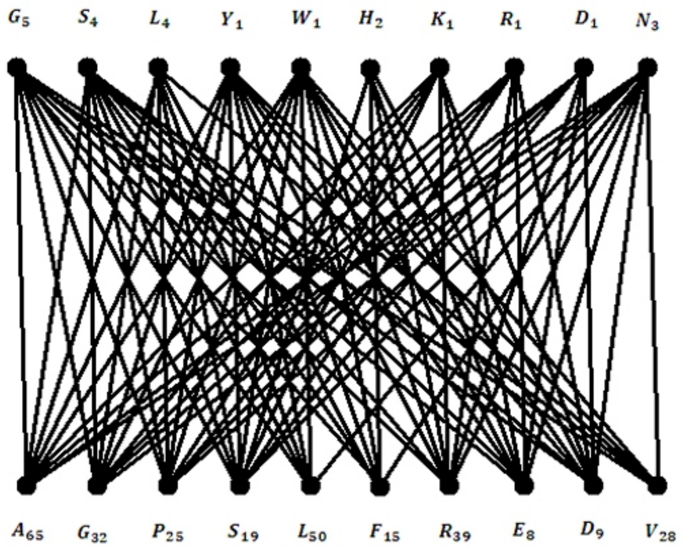

Figure7: Bipartite Pt-graph of galanin and receptor-3

Definition 3.3: Physicochemical subgraphs ${\mathcal{H}_k}^i$(for $i = 1,2,3, \ldots$) of a bipartite Pt-graph $\mathcal{G}$ of a peptide/protein and k receptors is defined as a subgraph whose vertex sets are same as that of $\mathcal{G}$ and two vertices in the different partitions are adjacent if they have exactly i common physicochemical properties.

Remark: Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a bipartite Pt-graph of a peptide/protein and its k receptors with $\mathcal{X}$and $\mathcal{Y}$ as the partitions of the vertex sets and let ${\mathcal{H}_k}^i(\mathcal{X}, \mathcal{Y}_i)$, where $\mathcal{Y}_i \subseteq \mathcal{Y}$ (for $i = 1,2, \ldots, n$) be n physicochemical subgraphs ,
Then,

$$\bigcap_{i=1,2,3,\ldots n} \mathcal{Y}_i = \emptyset \quad \text{and} \quad \bigcup_{i=1,2,3,\ldots n} \mathcal{Y}_i = \mathcal{Y}$$

Next we analyse physicochemical subgraphs of bipartite Pt-graphs of human galanin and its receptors. The dark edges indicate the maximum matching for physicochemical subgraphs as given below.

Physicochemical subgraphs of galanin and receptor-1

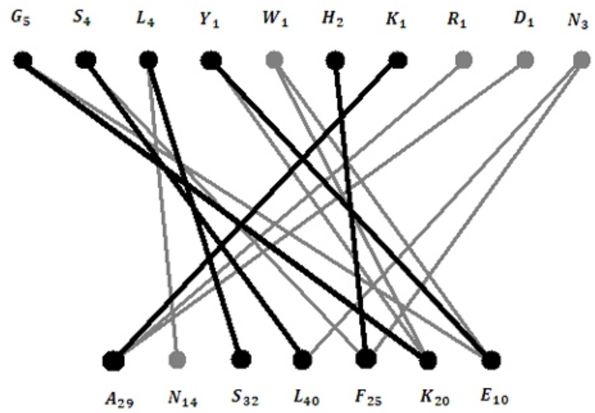

Figure 8: ${H_1}^1$ subgraph

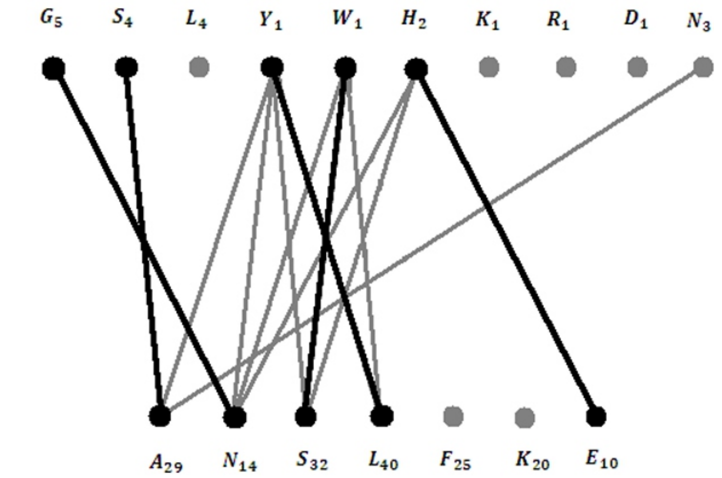

Figure 9: $H_1^{\ 2}$ **subgraph**

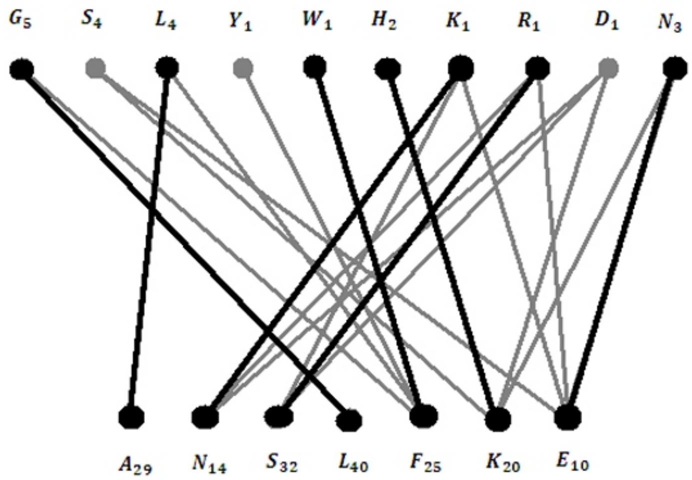

Figure 10: $H_1^{\ 3}$ **subgraph**

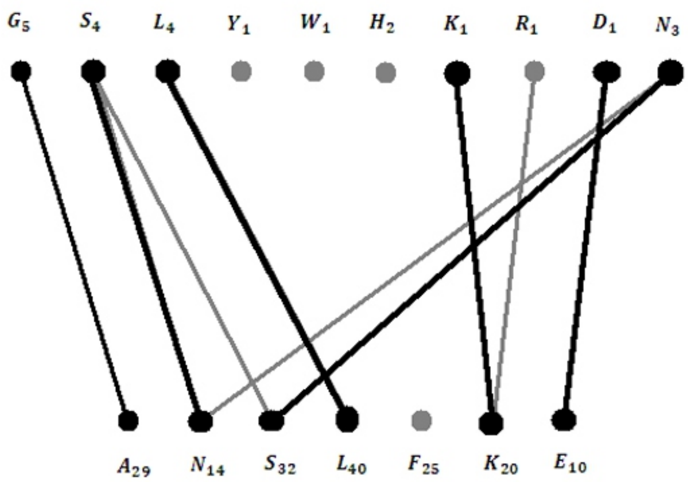

Figure 11: $H_1^{\ 4}$ **subgraph**

Physicochemical subgraphs of galanin and receptor-2

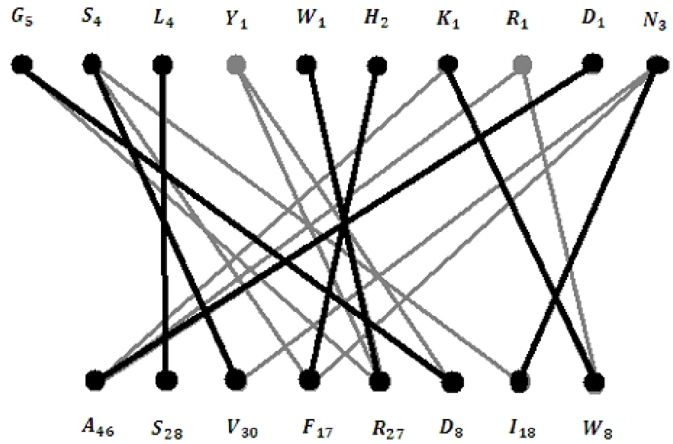

Figure 12: $H_2^{\ 1}$ **subgraph**

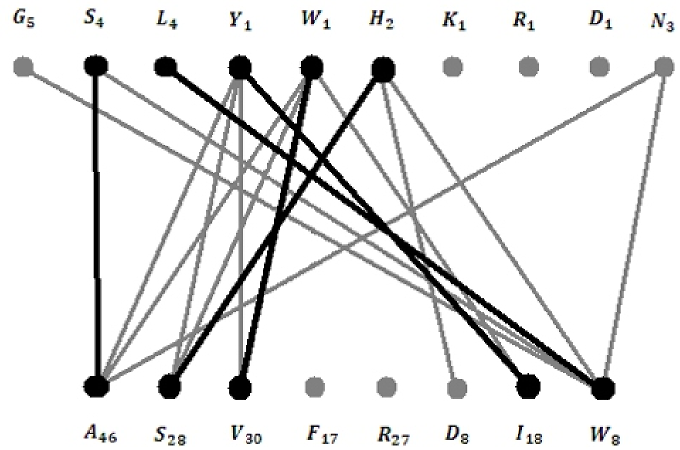

Figure 13: $H_2^{\ 2}$ **subgraph**

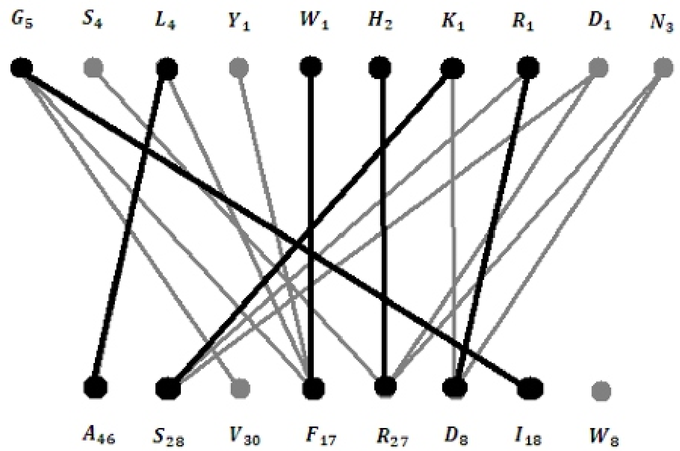

Figure 14: $H_2^{\ 3}$ **subgraph**

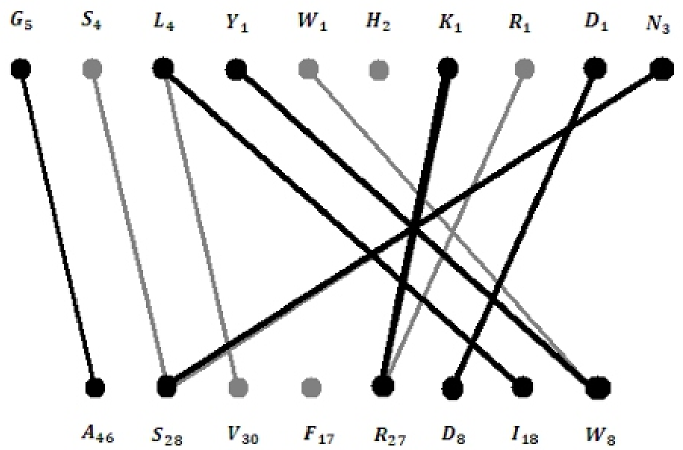

Figure 15: $H_2^{\ 4}$ **subgraph**

Physicochemical subgraphs of galanin and receptor-3

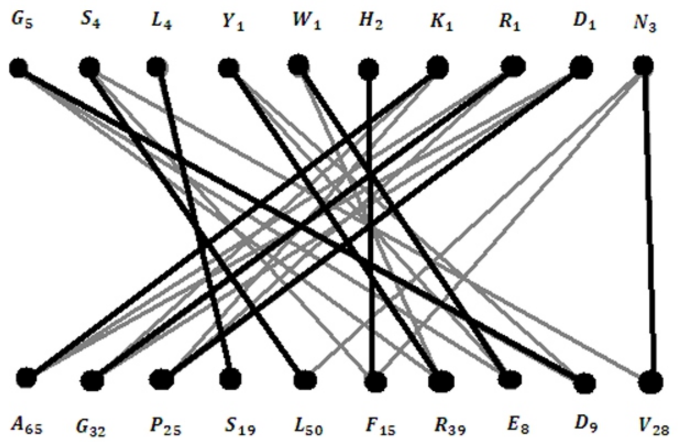

Figure 16: $H_3^{\ 1}$ **subgraph**

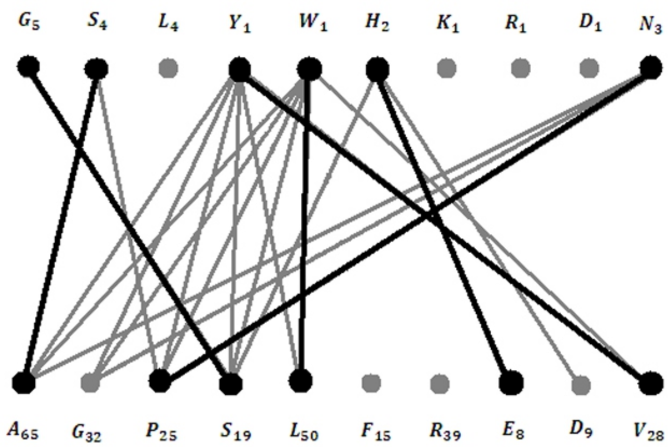

Figure 17: $H_3^{\ 2}$ **subgraph**

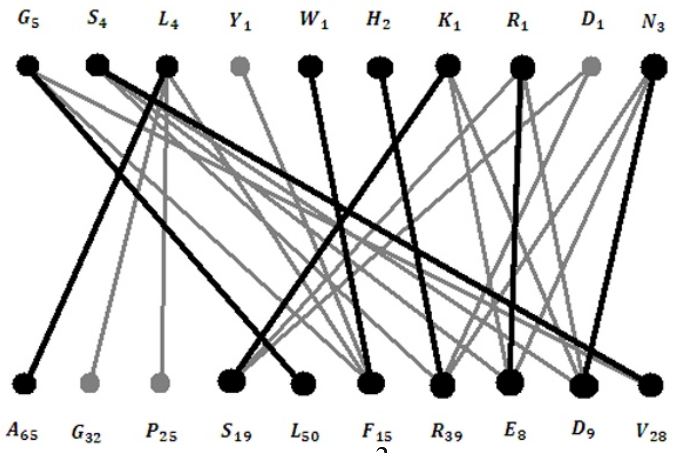

Figure 18: H_3^3 **subgraph**

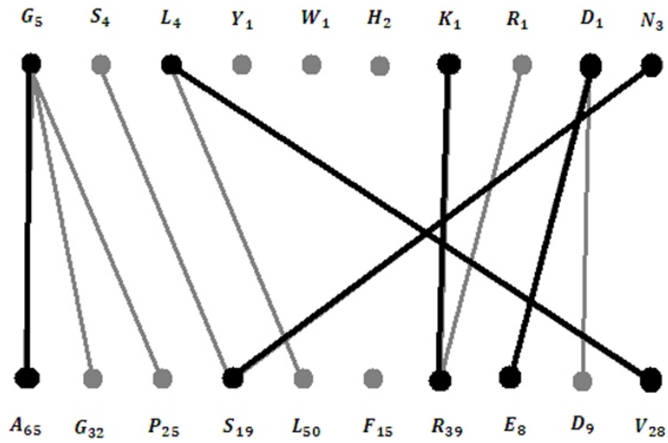

Figure 19: H_3^4 **subgraph**

Next we obtain the maximum matching of H_k^i subgraphs of bipartite Pt-graphs of human galanin and its receptors. Table 1 represents all the collections of maximum independent pairs of amino acids which connecting the human galanin and its receptors by sharing exactly $i(i=1,2,3,4)$ common physicochemical properties.

Receptors k	Maximum matching of $H_k{}^i$ subgraphs			
	$\mathcal{H}_k{}^1$ subgraphs	$\mathcal{H}_k{}^2$ subgraphs	$\mathcal{H}_k{}^3$ subgraphs	$\mathcal{H}_k{}^4$ subgraphs
$k = 1$	$(G_5, K_{20}), (S_4, L_{40}), (L_4, S_{32}), (Y_1, E_{10}), (H_2, F_{25}), (K_1, A_{29})$	$(G_5, N_{14}), (S_4, A_{29}), (Y_1, L_{40}), (W_1, S_{32}), (H_2, E_{10})$	$(G_5, L_{40}), (L_4, A_{29}), (W_1, F_{25}), (H_2, K_{20}), (K_1, N_{14}), (R_1, S_{32}), (N_3, E_{10})$	$(G_5, A_{29}), (S_4, N_{14}), (L_4, L_{40}), (K_1, K_{20}), (D_1, E_{10}), (N_3, S_{32})$
$k = 2$	$(G_5, D_8), (S_4, V_{30}), (L_4, S_{28}), (W_1, R_{27}), (H_2, F_{17}), (K_1, W_8), (D_1, A_{46}), (N_3, I_{18})$	$(S_4, A_{46}), (L_4, W_8), (Y_1, I_{18}), (W_1, V_{30}), (H_2, S_{28})$	$(G_5, I_{18}), (L_4, A_{46}), (W_1, F_{17}), (H_2, R_{27}), (K_1, S_{28}), (R_1, D_8)$	$(G_5, A_{46}), (L_4, I_{18}), (Y_1, W_8), (K_1, R_{27}), (D_1, D_8), (N_3, S_{28})$
$k = 3$	$(G_5, D_9), (S_4, L_{50}), (L_4, S_{19}), (Y_1, R_{39}), (W_1, E_8), (H_2, F_{15}), (K_1, A_{65}), (R_1, G_{32}), (D_1, P_{25}), (N_3, V_{28})$	$(G_5, S_{19}), (S_4, A_{65}), (Y_1, V_{28}), (W_1, L_{50}), (H_2, E_8), (N_3, P_{25})$	$(G_5, L_{50}), (S_4, V_{28}), (L_4, A_{65}), (W_1, F_{15}), (H_2, R_{39}), (K_1, S_{19}), (R_1, E_8), (N_3, D_9)$	$(G_5, A_{65}), (L_4, V_{28}), (K_1, R_{39}), (D_1, E_8), (N_3, S_{19})$

Table 1: Maximum matching of H_k^i subgraphs of human galanin and its receptors

Let $\mathcal{G}_k(\mathcal{X}, \mathcal{Y}_k)$ (for receptors $k = 1,2,3$) be three bipartite Pt-graphs of human galanin and its receptors, where $\mathcal{X}$ is the vertex set of Pt-graph of human galanin and $\mathcal{Y}_1, \mathcal{Y}_2$ and $\mathcal{Y}_3$ are the vertex sets of Pt-graphs of receptor-1, receptor-2 and receptor-3 respectively. Also let $\mathcal{H}_k{}^i(\mathcal{X}, \mathcal{Y}_k)$ be the physicochemical subgraphs of $\mathcal{G}_k(\mathcal{X}, \mathcal{Y}_k)$, where i indicates the number of common physicochemical properties of amino acids. Also let we denote the amino acids with physicochemical properties as

$P_1{}^+$ = Hydrophobic, $P_1{}^-$ = Hydrophilic,
$P_2{}^+$ =Polar, $P_2{}^-$ = Non-polar,
$P_3{}^+$ = Aliphatic, $P_3{}^-$ = Aromatic, $P_3{}^0$ = Neutral (aliphatic nor aromatic),
$P_4{}^+$ = Positive charge, $P_4{}^-$ = Negative charge, $P_4{}^0$ = Neutral in charge.

By analysing H_k^i subgraphs, we get some observations about the physicochmical property-wise connections of amino acids of galanin and its receptors.

Observation 3.1: In the analysis of the bipartite Pt-graphs of human galanin and receptors, we obtain N_{14}, S_{32} (in receptor 1), S_{28}, W_8 (in receptor 2) and S_{19} (in receptor 3) are the amino acids which receiving all highest centralities. Also we obtain G_5, S_4, Y_1, W_1 and N_3 are the common amino acids of human galanin which receive the highest centralities in all the bipartite Pt-graphs.

Observation 3.2: In the physicochemical subgraphs with amino acids sharing exactly one common property, the connections of amino acids of galanin to receptor-1, receptor-2 and receptor-3 are

(1) P_1^+ is not connected with P_1^+ and P_1^- is not connected with P_1^-
(2) P_2^- is not connected with P_2^-.
(3) P_3^+ is not connected with both P_3^+ and P_3^- but P_3^- is not connected with P_3^+.
(4) P_4^+ and P_4^- are not connected with both P_4^+ and P_4^-.

Observation 3.3: In the physicochemical subgraphs with amino acids sharing exactly two common properties, the connections of amino acids of galanin to receptor-1 and receptor-3 are

(1) P_1^+ and P_1^- are connected with both P_1^+ and P_1^- .
(2) P_2^- is not connected with P_2^-.
(3) P_3^+ is not connected with, P_3^+, P_3^- and P_3^0
P_3^- is not connected with P_3^-
P_3^0 is not connected with P_3^+ and P_3^-.
(4) P_4^+ is not connected with P_4^-
P_4^- is not connected with P_4^+, P_4^- and P_4^0
P_4^0 is not connected with P_4^+ and P_4^-.

Observation 3.4: In the physicochemical subgraphs with amino acids sharing exactly two common properties, the connections of amino acids of galanin to receptor-2 are

(1) P_1^+ and P_1^- are conected with both P_1^+ and P_1^-.
(2) P_2^- is not connected with P_2^-.
(3) P_3^+ is not connected with P_3^+ and P_3^0
P_3^- is not connected with P_3^-
P_3^0 is not connected with P_3^+.
(4) P_4^+ is not connected with P_4^-
P_4^- is not connected with P_4^+, P_4^- and P_4^0
P_4^0 is not connected with P_4^+ and P_4^-.

Remark: In the physicochemical subgraphs with amino acids sharing exactly two common properties, the amino acids P_3^+ and P_3^0 of galanin are conncted with the amino acids P_3^- of receptor-2 and not with receptor-1 and receptor-3. The only aromatic amino acid Tryptophan (W) of receptor-2 is connceted to the aliphatic and neutral (neither aliphatic nor aromatic) amino acids of galanin.

Observation 3.5: In the physicochemical subgraphs with amino acids sharing exactly three common properties, the connections of amino acids of galanin to receptor-1, receptor-2 and receptor-3 are

(1) P_1^+ is not connected with P_1^- and P_1^- is not connected with P_1^+.
(2) P_2^- is not connected with P_2^+.
(3) P_3^+ and P_3^- are not connected with P_3^+.
(4) P_4^+ is not connected with P_4^+.

Observation 3.6: In the physicochemical subgraphs with amino acids sharing exactly four common properties, the connections of amino acids of galanin to receptor-1, receptor-2 and receptor-3 are

(1) P_1^- have more neighbours than P_1^+ of X.
(2) P_2^- have more neighbours than P_2^+ of X.
(3) P_4^- have more neighbours than P_4^+ of X.

Observation 3.7: There is no neighbours for P_3^- in receptor-2 of the physicochemical subgraph with amino acids sharing exactly four common properties.

Observation 3.8: The physicochemical subgraph of galanin and receptor-3 with amino acids sharing exactly one common property (figure 16) is the subgraph having a maximum matching which is the only perfect matching among all the physicochemical subgraphs of galanin and its receptors.

Conclusion

Here we analysed the amino acids and some of their physichochemical properties which involved in the human galanin neuropeptide and its receptors graph theoretically. We have constructed and analysed some newly defined graphs - bipartite Pt-graphs and their physicochemical subgraphs - of galanin and its receptors using $\dot{\mathcal{C}}$-sets of the corresponding peptide/protein graphs (Pt-graphs). We observed from the analysis of the bipartite Pt-graphs of galanin and its receptors that G_5 ,S_4, $Y_{1,}$ W_1 and N_3 (in galanin), N_{14} S_{32} (in receptor 1), S28 W8 (in receptor 2) and S_{19} (in receptor 3) are the amino acids which receive all the highest centralities. The analysis of physicochemical subgraphs shows that, (i) if the amino acids of galanin and its receptors share exactly two common properties, the aliphatic and neutral (neither aliphatic nor aromatic) amino acids of galanin are connected with an aromatic amino acids (ie., Tryptophan (W)) only in receptor 2 (figure 13). (ii) if the amino acids of galanin and its receptors share exactly three common properties, (a) hydrophilic amino acids of galanin are more connected than hydrophobic amino acids, (b) non-polar amino acids of galanin are more connected than polar amino acids and (c) negatively charged amino acids of galanin are more connected than positively chareged amino acids. The maximum matching of physicochemical subgraphs shows that Leucine (L), Glutamate (E) and Alanine (A) (in receptor 1), Alanine (A) and Isoleucine (I) (in receptor 2), Serine (S), Alanine (A) and Valine (V) (in receptor 3) and Glycine (G), Leucine (L) and Aspargine (N) (in galanin) are the most repeated amino acids in the independent pairs. The physicochemical subgraph of galanin and receptor 3 with amino acids sharing exactly one common property (figure 16) is the subgraph having a maximum matching which which is the only perfect matching among all the physicochemical subgraphs of galanin and its receptors. These analyses can be used to study all the relationships between peptide/protein ligands and their receptors and this may help in the field of drug designing.

References

[1] Adil Akhtar and Nisha Gohain, Graph theoretic approach to analyze amino acid network, Int. J. Adv. Appl. Math. And Mech. 2(3) (2015) 31-37.

[2] Adil Akhtar and Tazid Ali, Analysis of Unweighted Amino Acids Network, Hindawi Publishing Corporation, Vol. 2014, Article ID 350276 (2014) 6 pages.

[3] David L. Nelson and Michael M. Cox, Lehninger Principles of Biochemistry, W. H Freeman and Company (2008).

[4] Mitsukawa K., Lu X., Bartfai T., Galanin, Galanin receptors and drug targets. Cell. Mol. Life. Sci. 65, 1976-180510.1007/s00018-008-8153-8 (2008)

[5] S. Kundu, Amino acid network with in protein, Physica A, 346 (2005) 104-109.

[6] Suresh Singh G., Akhil C. K., Analysing Amino Acids in Galanin – Graph Theoretical Appraoch, International Journal on Recent and Innovation Trends in Computing and Communication (IJRITCC), June 17 Volume 5 Issue 6, ISSN: 2321-8169, PP: 342-346 (2017).

[7] Suresh Singh G., Graph Theory, PHI Learning Private Limited (2010).

[8] Yang Wang, Mingnia Wang, Sanwen Yin, Richard Jang, Jian Wang, Zhidong Xue and Tao Xu, Neoropep: a comprehensive Resource of Neuropeptides, Database (Oxford) 2015 Apr 29. doi:10.1093/ database/ bar038.

ISSN : 2394 - 7139

GRILL ON GENERALIZED TOPOLOGICAL SPACES

Shyamapada Modak* and Sukalyan Mistry**
*Department of Mathematics, University of Gour Banga , Malda 732103, India
*Sahidgarh High School,P.O: Maynaguri, Jalpaiguri-735224, India

ABSTRACT

The aim of this paper is to introduce grill generalized topological spaces and to investigate the relationships between generalized topological spaces and grill generalized topological spaces. For establishment of their relationships, we define some closed sets in these spaces. Basic properties and characterization related to these sets are also discussed.

Keywords and phrases: generalized topological space, grill generalized topological space, g_μ-closed set, μ^Φ-closed set, μ-Gg-closed set.

1. INTRODUCTION

The study of grill topological spaces[16] as like ideal topological spaces[9] has been started from 2007 although the study of grill [3,1,2,18] in topological spaces was started from 1947 at different point of view. Generalized closed sets[10] in topological space as well as in grill topological space[11] has been discussed at various research papers.We have introduced the generalized closed sets in grill generalized topological space (generalized topological space(GTS) [5,6] with grill), and characterized the same at different aspect. We also obtain the relations with earlier generalized closed sets in topological space, generalized topological space and grill generalized topological space etc.

2 PRELIMINARIES

Definition 2.1[3]. A nonempty collection $\mathcal{G}$ of nonempty subsets of a topological space (X, τ) is called grill if

i) $A \in \mathcal{G}$ and $\subseteq B \subseteq X \Longrightarrow B \in X$, and
ii) $A, B \subseteq X$ and $A \cup B \in \mathcal{G} \Longrightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$

If $\mathcal{G}$ is grill on X,then $(X, \tau, \mathcal{G})$ is called a grill topological space[16]

Definition 2.2[16]. Let $(X, \tau, \mathcal{G})$ be a grill topological space.An operator Φ: $\exp(X) \rightarrow \exp(X)$ is called a local function with respect to τ and $\mathcal{G}$ is defined as follows: for $A \subseteq X$,$\Phi(A)(\mathcal{G}, \tau) = \Phi(A) = \{x \in X: U \cap A \in \mathcal{G}$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau: x \in U\}$[16].
It is well known from [16], $A \cap \Phi(A) = \psi(A)$ is a Kuratowsi closure operator [9].
Definition 2.3[16]. Corresponding to a grill on a topological space (X, τ), there exists a unique topology $\tau_\mathcal{G}$ on X

given by
$\tau_{\mathcal{G}} = \{U \subseteq X: \psi(X\backslash A) = (X\backslash A)\}$, where for any $A \subseteq X, \psi(A) = A \cup \Phi(A) = \tau_{\mathcal{G}}\text{-}cl(A)$.

Definition 2.4. Let $(X, \tau, \mathcal{G})$ be a grill topological space. A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is $\tau_{\mathcal{G}}$-closed[16] (resp. $\tau_{\mathcal{G}}$-dense in itself[11], $\tau_{\mathcal{G}}$-perfect) , if $\psi(A) = A$ or equivalently if $\Phi(A) \subseteq A$(resp. $A \subseteq \Phi(A), A = \Phi(A)$) .

Definition 2.5. Let $(X, \tau, \mathcal{G})$ be a grill topological space. A subset A of a grill topological space $(X, \tau, \mathcal{G})$ is g-closed with respect to the grill $\mathcal{G}$(briefly, $\mathcal{G}$-g-closed)[11] if $\Phi(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
A subset A of X is said to be $\mathcal{G}$-g-open if $X\backslash A$ is $\mathcal{G}$-g-closed.

Definition 2.6. Let (X, τ) be a topological space. A subset A of a space (X, τ) is said to be g-closed set[10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Remark 2.1[11]. Every g-closed set is a $\mathcal{G}$-g-closed but not vice versa.

Remark 2.2[19]. Every closed set is g-closed.

Very interesting notion in literature has been introduced by Csaszar[4] on 1997. Using this notion topology has been reconstructed. The concept is:
A map $\tau: \exp(X) \to \exp(X)$ is possessing the property monotony(i.e. such that $A \subseteq B$ implies $\tau(A) \subseteq \tau(B)$). We denote by $\Gamma(X)$ the collections of all mapping having this property.
One of the consequence of the above notion is generalized topological space(GTS) [5,6], its formal definition is:

Definition 2.7. Let X be a non-empty set , and $\mu \subseteq \exp(X)$, μ is called a generalized topology (GTS) on X if $\emptyset \in \mu$ and the union of elements of μ belongs to μ.
The member of μ is called μ-open set and the complement of μ-open set is called μ-closed set. Again c_μ is the notation of μ-closure[5,6,13,14].

Definition 2.8[15]. Let (X, μ) be a generalized topological space.Then the generalized kernel of $A \subseteq X$ is denoted by g-ker(A) and defined as g-ker$(A) = \bigcap\{G \in \mu: A \subseteq G\}$.

Lemma 2.1[15].]. Let (X, μ) be a generalized topological space and $A \subseteq X$. Then g-ker$(A) = \{x \in X: c_\mu(\{x\}) \cap A \neq \emptyset\}$.
If $\mathcal{G}$ is a grill on X, then $(X, \mu, \mathcal{G})$ is called a grill generalized topological space (GGTS).

3 GGTS

Definition 3.1. Let $(X, \mu, \mathcal{G})$ be a GGTS. A mapping $()^{\Phi\mu}$: $\exp(X) \to \exp(X)$ is defined as follows :
$(A)^{\Phi\mu} = (A)^{\Phi\mu}(\mathcal{G}, \mu) = \{x \in X: A \cap U \in \mathcal{G}\}$, where $U \in \psi(x)$[5].
The mapping is called the local function associated with the grill $\mathcal{G}$ and generalized topology μ.

Properties:

Theorem 3.1. Let $(X, \mu, \mathcal{G})$ be a GGTS.Then
(1) $(\emptyset)^{\Phi\mu} = \emptyset$.
(2) for $A, B \subseteq X$ and $A \subseteq B$, $(A)^{\Phi\mu} \subseteq (B)^{\Phi\mu}$.
(3) $(A)^{\Phi\mu} \subseteq c_\mu(A)$.
(4) $((A)^{\Phi\mu})^{\Phi\mu} \subseteq c_\mu(A)$.
(5) $(A)^{\Phi\mu}$ is a μ-closed set.
(6) $((A)^{\Phi\mu})^{\Phi\mu} \subseteq (A)^{\Phi\mu}$.
(7) for $\mathcal{G} \subseteq \mathcal{G}_1$ implies $(A)^{\Phi\mu}(\mathcal{G}_1) \supseteq (A)^{\Phi\mu}(\mathcal{G})$.
(8) for $\in \mu$, $U \cap (U \cap A)^{\Phi\mu} \subseteq U \cap (\mathrm{A})^{\Phi\mu}$.
(9) for $G \notin \mathcal{G}$, $(\mathrm{A}\backslash\mathrm{G})^{\Phi\mu} = (A)^{\Phi\mu} = (A \cup G)^{\Phi\mu}$.

Proof. (1). It is obvious from definition.
(2). It is done by the fact, $A \cap G \in \mathcal{G}$ implies $B \cap G \in \mathcal{G}$.

(3). Obvious from [5,13].

(4). $((A)^{\Phi\mu})^{\Phi\mu} \subseteq c_\mu\left(c_\mu(A)\right) = c_\mu(A)$ [5,13].

(5). From [5], for $G \in \mu$ and $x \in G$, there exists $V \in \psi(x)$ such that $V \subseteq G$.Now if $A \cap G \notin \mathcal{G}$ then for $A \cap V \subseteq A \cap G$, $A \cap V \notin \mathcal{G}$.It follows that $X\backslash(A)^{\Phi\mu}$ is the union of μ-open sets.We know that the arbitrary union of μ-open sets is a μ-open set. So $X\backslash(A)^{\Phi\mu}$ is a μ-open set and hence $(A)^{\Phi\mu}$ is a μ-closed set.

(6). From above, $((A)^{\Phi\mu})^{\Phi\mu} \subseteq c_\mu((A)^{\Phi\mu}) = (A)^{\Phi\mu}$, since $(A)^{\Phi\mu}$ is a μ-closed set.

(7). Obvious from $A \cap V \in \mathcal{G}$ implies $\cap V \in \mathcal{G}_1$.

(8). Since $U \cap A \subseteq A$,then $(U \cap A)^{\Phi\mu} \subseteq (A)^{\Phi\mu}$ so $U \cap (U \cap A)^{\Phi\mu} \subseteq U \cap (A)^{\Phi\mu}$.

(9). Let $x \in (A)^{\Phi\mu}$. If possible suppose that $x \notin (A\backslash G)^{\Phi\mu}$. Then there is a $V \in \psi(x)$,$V \cap (A\backslash G) \notin \mathcal{G}$.Therefore $(V \cap (A\backslash G)) \cup G \notin \mathcal{G}$, i.e., $G \cup (A \cap V) \notin \mathcal{G}$. Then $\cap A \notin \mathcal{G}$, a contradiction to the fact that $x \in (A)^{\Phi\mu}$. Hence , $(A\backslash G)^{\Phi\mu} = (A)^{\Phi\mu}$.

Proof of 2nd part is similar.

It is obvious from (2), $()^{\Phi\mu} \in \Gamma(X)$ [4].

Definition 3.2. Let (X, μ) be a GTS with a grill $\mathcal{G}$ on X.

The set operator $c^{\Phi\mu}$ is called a generalized $\Phi\mu$-closure and is defined as $(c)^{\Phi\mu}(A) = A \cup (A)^{\Phi\mu}$, for $A \subseteq X$. We will denote by $\mu^{\Phi}(\mu; \mathcal{G})$ the generalized structure , generated by $c^{\Phi\mu}$, that is, $\mu^{\Phi}(\mu; \mathcal{G}) = \{U \subseteq X: c^{\Phi\mu}(X\backslash U) = (X\backslash U)\}$. $\mu^{\Phi}(\mu; \mathcal{G})$ is called $\Phi\mu$-generalized structure with respect to μ and $\mathcal{G}$ (in short $\Phi\mu$-generalized structure) which is finear than μ.

The element of $\mu^{\Phi}(\mu; \mathcal{G})$ are called μ^{Φ}-open and the complement of μ^{Φ}-open is called μ^{Φ}-closed.

Theorem 3.2. The set operator $c^{\Phi\mu}$ satisfy following conditions:

(a) $A \subseteq c^{\Phi\mu}(A)$, for $A \subseteq X$.

(b) $c^{\Phi\mu}(\emptyset) = \emptyset$ and $c^{\Phi\mu}(X) = X$.

(c) $c^{\Phi\mu}(A) \subseteq c^{\Phi\mu}(B)$ if $A \subseteq B \subseteq X$.

(d) $c^{\Phi\mu}(A) \cup c^{\Phi\mu}(B) \subseteq c^{\Phi\mu}(A \cup B)$.

(e) $c^{\Phi\mu} \in \Gamma(X)$.

Proof: Proof is obvious from Theorem 3.1.

Definition 3.3. Let (X, μ) be a GTS. A subset A of X is said to be g_μ-closed set[12] if $c_\mu(A) \subseteq M$ whenever $A \subseteq M$ and $M \in \mu$.

Definition 3.4. A subset A of a GGTS $(X, \mu, \mathcal{G})$ is μ^{Φ}-dense in itself (resp. μ^{Φ}-perfect) if $A \subseteq (A)^{\Phi\mu}$ (resp. $(A)^{\Phi\mu} = A$).

Definition 3.5. A subset A of a GGTS $(X, \mu, \mathcal{G})$ is called μ- G-generalized closed (briefly, μ-G_g-closed) if $(A)^{\Phi\mu} \subseteq U$ whenever U is μ-open and $A \subseteq U$. A subset A of a GGTS $(X, \mu, \mathcal{G})$ is called μ- G-generalized open (briefly, μ-G_g-open) if $X\backslash A$ is μ-G_g-closed.

Theorem 3.3. Let $(X, \mu, \mathcal{G})$ be a GGTS. Every g_μ-closed set is μ-G_g-closed.

Proof: Let U any μ-open set containing A. Since A is g_μ-closed, then $c_\mu(A) \subseteq U$.By Theorem 3.1(3), we have $(A)^{\Phi\mu} \subseteq U$.

Remark 3.1. Let (X, τ) be a topological space. If we take $= \tau$, then g_μ-closed set coincide with g-closed sets[7,8].

Proposition 3.1. . Let $(X, \mu, \mathcal{G})$ be a GGTS.

(a) Every μ^{Φ}-perfect set is μ^{Φ}-dense in itself.

(b) Every μ^{Φ}-perfect set is μ^{Φ}-closed.

Proof: The proof can be easily done.

Remark 3.2. Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X. If we take $\mu = \tau$,then μ-G_g-closed (resp. μ^{Φ}-closed , μ^{Φ}-dense in itself) sets coincide with $\mathcal{G}$-g-closed [11] (resp.$\tau_{\mathcal{G}}$-closed[16], $\tau_{\mathcal{G}}$-dense in itself [11].

Theorem 3.4. If $(X, \mu, \mathcal{G})$ is a GGTS and $\subseteq X$, then A is μ-G_g-closed if and only if $c^{\Phi\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-open in X.

Proof: Since A is μ-G_g-closed, we have $(A)^{\Phi\mu} \subseteq U$ whenever $A \subseteq U$ and U is μ-open in X. $c^{\Phi\mu}(\mathrm{A}) = A \cup (A)^{\Phi\mu} \subseteq U$ whenever $A \subseteq U$ and U is μ-open in X.

Converse part: Let $A \subseteq U$ and U be μ-open in X. By hypothesis $c^{\Phi\mu}(\mathrm{A}) \subseteq U$. Since $c^{\Phi\mu}(\mathrm{A}) = A \cup (A)^{\Phi\mu}$, we have $(A)^{\Phi\mu} \subseteq U$.

Theorem 3.5. Let $(X, \mu, \mathcal{G})$ is a GGTS and $A \subseteq X$. Then the following are equivalent :

(a) A is μ-G_g-closed.

(b) $c^{\Phi\mu}(\mathrm{A}) \subseteq U$ whenever $A \subseteq U$ and U is μ-open in X.

(c) $c^{\Phi\mu}(\mathrm{A}) \subseteq g$-ker (A).

(d) $c^{\Phi\mu}(\mathrm{A}) \setminus A$ contains no nonempty μ-closed set.

(e) $(A)^{\Phi\mu} \setminus A$ contains no nonempty μ-closed set.

Proof: (a)$\Leftrightarrow$(b). It follows from Theorem 3.4.

(b)$\Longrightarrow$ (c). Suppose $x \in c^{\Phi\mu}(\mathrm{A})$ and $x \notin g$-ker (A).Then $c_\mu(\{x\}) \cap A = \emptyset$. Implies that $A \subseteq X \setminus (c_\mu(\{x\}))$. Now from (b), $c^{\Phi\mu}(\mathrm{A}) \subseteq X \setminus c_\mu(\{x\})$. This implies $c^{\Phi\mu}(\mathrm{A}) \cap \{x\} = \emptyset$, a contradiction. Hence the result.

(c) $\Longrightarrow$ (d). Suppose $\subseteq \left(c^{\Phi\mu}(\mathrm{A})\right) \setminus A$, F is μ-closed and $x \in F$. Since $F \subseteq \left(c^{\Phi\mu}(\mathrm{A})\right) \setminus A$, $F \cap A = \emptyset$. We have $c_\mu(\{x\}) \cap A = \emptyset$ because F is μ-closed and $x \in F$ .From (c), this is a contradiction.

(d) $\Longrightarrow$ (e). This is obvious from the definition of $c^{\Phi\mu}(\mathrm{A})$.

(e) $\Longrightarrow$ (a). Let U be a μ-open subset containing A. Since $(A)^{\Phi\mu}$ is μ-closed by means of Theorem 3.1(5). Now $(A)^{\Phi\mu} \cap (X \setminus U) \subseteq (A)^{\Phi\mu} \setminus A$. Since intersection of two μ-closed sets is a μ-closed set, then $(A)^{\Phi\mu} \cap (X \setminus U)$ is an μ-closed set contained in $(A)^{\Phi\mu} \setminus A$. By assumption, $(A)^{\Phi\mu} \cap (X \setminus U) = \phi$. Hence, we have $(A)^{\Phi\mu} \subseteq U$.

Remark 3.3. . Let $(X, \tau, \mathcal{G})$ be a GGTS. If $\mu = \tau$ then the above theorem coincides with Theorem 2.7 in [11].

Proposition 3.2. Let $(X, \mu, \mathcal{G})$ be a GGTS. Every μ^Φ-closed set is μ-G_g-closed.

Proof: Let A be a subset of X and A be μ^Φ-closed. Assume that $A \subseteq U$ and U is μ-open. Since A is μ^Φ-closed, we have $(A)^{\Phi\mu} \subseteq A$ and so A is μ-G_g-closed.

For the relationship related to several sets defined in the paper, we have the following diagram :

μ^Φ-dense in itself $\Leftarrow$ μ^Φ-perfect $\Rightarrow$ μ^Φ-closed $\Rightarrow$ μ-G_g-closed $\Leftarrow$ g_μ-closed $\Leftarrow$ μ-closed

The following examples show that the converse implications of the diagram are not satisfied.

Example 3.1.(a). Let $X = \{a, b, c\}$, $\mu = \{X, \emptyset, \{b\}, \{b.c\}\}$, $\mathcal{G} = \{\{a\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}, X\}$ and $A = \{b\}$. Here $(A)^{\Phi\mu} = \emptyset$ and $c_\mu(A) = X$. Thus, A is μ-G_g-closed . But A is not g_μ-closed.

(b) In (a), let $= \{a, b\}$. Note that the only μ-open set containing B is X. $c_\mu(\mathrm{B}) = X$ is also contained in X.Therefore B is g_μ-closed but not μ-closed.

(c) In (a), A is μ^Φ-closed but not μ^Φ-perfect .

(d) Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{b\}, \{b.c\}\}$, $\mathcal{G} = \{\{a\}, \{c\}, \{a, c\}, \{a, b\}, \{b, c\}, X\}$ and $A = \{b\}$. Then $(A)^{\Phi\mu} = \emptyset$ which is also a subset of $\{b\}$ and $\{b, c\}$. So, A is μ-G_g-closed but not μ^Φ-closed.

(e) In (a), let $B = \{c\}$. Then $(B)^{\Phi\mu} = \{a, c\}$, so B is μ^Φ-dense in itself but not μ^Φ-perfect.

Definition 3.6[17]. A space (X, μ) is called μ-T_1if any pair of distinct points x and y of X, there exists a μ-open set U of X containing x but not y and a μ-open set V of X containing y but not x.

It is obvious from definition that every singleton set is μ-closed if and only if the space is μ-T_1.

Remark 3.4. Let $(X, \mu, \mathcal{G})$ be a GGTS and $A \subseteq X$ .If (X, μ) is a μ-T_1 space, then A is μ^Φ-closed if and only if A is μ-G_g-closed.

Theorem 3.6. Let $(X, \mu, \mathcal{G})$ be a GGTS and $A \subseteq X$. If A is an μ-G_g-closed set, then the following are equivalent :

(a) A is an μ^Φ-closed set.

(b) $c^{\Phi\mu}(\mathrm{A}) \setminus A$ is an μ-closed set.

(c) $(A)^{\Phi\mu} \setminus A$ is an μ-closed set.

Proof: (a)⇒(b). If A is μ^{Φ}-closed, then $c^{\Phi\mu}(\mathrm{A})\backslash A=\emptyset$. $c^{\Phi\mu}(\mathrm{A})\backslash A$ is μ-closed.

(b)⇒(c). Since $c^{\Phi\mu}(\mathrm{A})\backslash A=(A)^{\Phi\mu}\backslash A$,it is clear.

(c) ⇒(a). If $(A)^{\Phi\mu}\backslash A$ is μ-closed and A is μ-G_g-closed ,from Theorem 3.5(e), $(A)^{\Phi\mu}\backslash A=\emptyset$ and so A is μ^{Φ}-closed .

Lemma 3.1. Let $(X,\mu,\mathcal{G})$ be a GGTS and $A\subseteq X$. If A is μ^{Φ}-dense in itself, then $(A)^{\Phi\mu}=c_{\mu}((A)^{\Phi\mu})=c_{\mu}(A)=c^{\Phi\mu}(\mathrm{A})$.

Proof: Let A be μ^{Φ}-dense in itself. Then we have $A\subseteq(A)^{\Phi\mu}$ and hence $c_{\mu}(A)\subseteq c_{\mu}((A)^{\Phi\mu})$. We know that $(A)^{\Phi\mu}=c_{\mu}((A)^{\Phi\mu})\subseteq c_{\mu}(A)$ from Theorem 3.1(5). In this case $c_{\mu}(A)=c_{\mu}((A)^{\Phi\mu})=(A)^{\Phi\mu}$. Since $(A)^{\Phi\mu}=c_{\mu}(A)$, we have $c^{\Phi\mu}(\mathrm{A})=c_{\mu}(A)$.

We obtained that every g_{μ}-closed set is μ-G_g-closed in Theorem 3.3 but not vice versa. For μ^{Φ}-dense in itself sets, g_{μ}-closedness and μ-G_g-closedness are equivalent.

Theorem 3.7. Let $(X,\mu,\mathcal{G})$ be a GGTS and $A\subseteq X$. If A is μ^{Φ}-dense in itself and μ-G_g-closed, then A is g_{μ}-closed.

Proof. Assume A is μ^{Φ}-dense in itself and μ-G_g-closed on X . If U is an μ-open set containing A, then we have $(A)^{\Phi\mu}\subseteq U$. Since A is μ^{Φ}-dense in itself, Lemma 3.1 implies $c_{\mu}(A)\subseteq U$ and so A is g_{μ}-closed.

Theorem 3.8. . Let $(X,\mu,\mathcal{G})$ be a GGTS and $A\subseteq X$. If A is μ-G_g-closed and μ-open then A is μ^{Φ}-closed.

Proof: Let A be an μ-open.Since A is μ-G_g-closed, we have $(A)^{\Phi\mu}\subseteq A$. Hence A is μ^{Φ}-closed.

REFERENCES

[1] K.C. Chattopadhya, O. Njastad and W.J. Thrown, Metrotopic and extensions of closure spaces, Can.J. Math.,4(1983), 613-629.

[2] K.C. Chattopadhya and W.J. Thrown, Extensions of closure spaces, Can.J. Math.,6(1977),1277-1286.

[3] G. Choquet, Sur les notions de filter et grille, Comptes Rendus Acda. Sci. Paris, 224(1947), 171-173.

[4] A. Csaszar, Generalized open sets, Acta Math. Hungar., 75(1-2) (1997), 65-87.

[5] A. Csaszar, Generalized topology, generalized continuity, Acta Math. Hungar., 94(4) (2002), 351-357.

[6] A. Csaszar, Generalized open sets in generalized topologies, Acta Math. Hungar., 106(1-2)(2005),57-66.

[7] J. Dontchev, M. Ganster and T. Noiri, Unified operation apporoach of generalized closed sets via topological ideals, Math. Japonica, 49 (1999), 395 - 401.

[8] E. Hayashi, Topologies defined by local properties, Math. Ann., 156, (1964), 205 -215.

[9] K. Kuratowski, Topology, Vol. I, Academic Press (New York).

[10] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo (2), 19(1970),89 -96.

[11] D. Mandal and M.N. Mukherjee, On a type of generalized closed sets, Bol. Soc. Paran. mat., 30(2012), 67-70.

[12] S.Maragathavalli, M.Sheik John and D.Sivaraj, On g-closed sets in generalized topological spaces, J. Adv. Res. Pure maths., 2(1)(2010), 57-64.

[13] T. Noiri and V. Popa, Between closed sets and g-closed sets, Rend. Circ. Mat. Palermo (2), 55(2006), 175184.

[14] T. Noiri and B. Roy, Unification of generalized open sets on topological spaces, Acta Math. Hungar., 130(4) (2011), 349 -357.

[15] B. Roy, On generalized R0 and R1 spaces, Acta Math. Hungar., 127(3) (2010), 291-300.

[16] B. Roy and M.N. Mukherjee, On typical topologies induced by a grill, Soochow J.Math., 33(4) (2007), 771786.

[17] M.S. Sarsak, Weak separation axioms in generalized toplogical spaces, Acta Math. Hun-Gar., 131(1-2) (2011), 110-121.

[18] W.J. Thron, Proximities structures and grill, Math. Ann., 206(1973), 35 - 62.

[19] R.Vaidyanathaswamy, The localization theory in set topology , Proc. Indian Acad. Sci. Sect A, 20 (1944), 5161.

[20] Shyamapada Modak & Sukalyan Mistry "Continuities on Ideal Minimal Spaces" Aryabhatta J. of Maths & informatics vol. 5 [1] 2013 pp. 101-106.

N-Generated Fuzzy Groups and Its Level Subgroups

Dr. M. Mary JansiRani1, B. Bakkiyalakshmi2, P. Sudhalakshmi3
1 .Head & Assistant professor, PG& Research Department of Mathematics,
Thanthai Hans Roever college of Arts & Science, Perambalur.
2&3. Research scholar, PG& Research Department of Mathematics,
Thanthai Hans Roever college of Arts & Science, Perambalur.

ABSTRACT

In this paper, we define the algebraic structures of n–generated fuzzy subgroups and some related properties are investigated. The purpose of this study is to implement the fuzzy set theory and group theory in n–generated fuzzy subgroups. Characterizations of n–generated level subsets of a n– generated fuzzy subgroups of a group are given

Keywords- Fuzzy set, multi-fuzzy set, fuzzy subgroup, multi-fuzzy subgroup, anti-fuzzy subgroup, multi-anti fuzzy subgroup,n–generated fuzzy subset, n–generated fuzzy subgroups, n–generated fuzzy level subsets,n-generated fuzzy level subgroups

1. INTRODUCTION

After the introduction of the concept of fuzzy sets by L.A.Zadeh [1], researchers were conducted the generalizations of the notion of fuzzy sets A. Rosenfeld [2] Introduced the concept of fuzzy group and the idea of "Intuitionistic Fuzzy set" was first published by K.T. Atanassov [3]. W.D.Blizard [4] Introduced the concept of fuzzy multi-set theory. Also Shinoj .T.K and Sunil Jacob [6] produced some results in Intuitionistic Fuzzy Multi-sets. In this chapter we define n–generated fuzzy sets and n–generated fuzzy subgroups and some of their properties.

2. PRELIMINARIES

2.1 Definition

Let X be a non-empty set. A fuzzy set A on X is a mapping A:X→[0,1] and is defined as
$A=\left\{x\in X/\left(x,\mu(x)\right)\right\}$

2.2. Definition

Let X and Y be any two sets. Let f: $X\rightarrow Y$ be a function. If μ is a fuzzy set on X then the image μ under f is a fuzzy set on Y and is defined by

$$f(\mu)(y) = v(y) = \underset{x \in f^{-1}(y)}{Sup} \mu(x), \ \forall \ y \in Y \text{ is called image of } \mu \text{ under } f$$

2.3. Definition

Let X and Y be any two sets. Let f: $X \rightarrow Y$ be a function. If S is a fuzzy set on Y then the preimage of S under f is a fuzzy set on X and is defined by

$$\left(f^{-1}(S) \right)(x) = S\left(f(x) \right)$$

2.4. **Definition**

Let A be a fuzzy subset of a set X. For $t \in [0, 1]$, $A_t = \{ x \in X / A(x) \geq t \}$ is called a level fuzzy subset of A

2.5. **Definition**

Let X be a non empty set. An Intuitionistic Fuzzy set A on X is an object having the form $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle / x \in X \}$, where $\mu_A : X \rightarrow [0,1]$ & $\gamma_A : X \rightarrow [0,1]$ are the degree of membership and non- membership functions respectively with $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$

2.6. Definition

Let X be a non-empty set. A Fuzzy Multi set (FMS) A drawn from X is characterized by a function ' Count membership' of A denoted by CM_A such that $CM_A : X \rightarrow Q$ where Q is the set of all crisp finite set drawn from the unit interval $[0,1]$. Then for any $x \in X$, the value $CM_A(x)$ is a crisp multi set drawn from $[0,1]$. For each $x \in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $\left(\mu_{A_1}(x), \mu_{A_2}(x), \ldots\ldots\ldots \mu_{A_k}(x) \right)$ where $\mu_{A_1}(x) \geq \mu_{A_2}(x) \geq \ldots\ldots\ldots \geq \mu_{A_k}(x)$

$$A = \left\{ \left\langle x : \left(\mu_{A_1}(x), \mu_{A_2}(x) \ldots\ldots \mu_{A_k}(x) \right) \right\rangle : x \in X \right\}$$

Example 2.7

Let $X = \{ x, y, z, w \}$ be a universal non empty set.For each $x \in X$, we can write a Fuzzy Multi set as follows

$A = \{ \langle x, (0.8, 0.7, 0.7, 0.6) \rangle, \langle y, (0.8, 0.5, 0.2) \rangle, \langle z, (1, 0.5, 0.5) \rangle \}$ Where

$CM_A(x) = (0.8, 0.7, 0.7, 0.6)$ with $0.8 \geq 0.7 \geq 0.7 \geq 0.6$

2.8. **Definition**

Let X be a non- empty universal set and let A be an Fuzzy Multi set on X. The n-generated Fuzzy set on X is constructed from the Fuzzy Multi set and is defined as $\lambda = \left\{ \left\langle x, \frac{1}{k}\left(\mu_{A_1}^{n}(x) + \mu_{A_2}^{n}(x) + ...\mu_{A_k}^{n}(x) \right) \right\rangle : x \in X \right\}$

where $\mu_{A_1}^{n}(x) \geq \mu_{A_2}^{n}(x) \geq .... \geq \mu_{A_k}^{n}(x)$ and n is the dimension of the Fuzzy Multi set A

2.9.**Definition Multi-level subset**

Let A be a multi-fuzzy subset of X. For $t_i \in [0,1]$, $i = 1, 2, ..., k$, $A_{t_i} = \{ x \in X / A(x) \geq t_i \}$ is called multi-level subset of A

2.10. Definition Multi-Fuzzy Mapping

Let $\mu = (\mu_1, \mu_2, ..., \mu_k)$ and $\nu = (\nu_1, \nu_2, ..., \nu_k)$ be two multi-fuzzy sets in X of dimenstion k and n respectively. A multi-fuzzy mapping is a mapping $F : M^k FS(X) \to M^n FS(X)$ which maps each $\mu \in M^k FS(X)$ into a unique multi-fuzzy set $\nu \in M^n FS(X)$

2.11. Definition Atanassov Intuitionistic Fuzzy Sets Generating Maps (AIFSGM)

A mapping $F : M^k FS(X) \to M^2 FS(X)$ is said to be an Atanassov Intuitionistic Fuzzy Sets Generating Maps(AIFSGM) if $F(\mu)$ is an Intuitionistic fuzzy set in $M^2 FS(X)$

2.12. Definition Multi-Fuzzy extensions of functions

Let $f : X \to Y$ and $h : \prod M_i \to \prod L_j$ be functions . The Multi-fuzzy extension and the inverse of the extension are $f : \prod M_i^X \to \prod L_j^Y$, $f^{-1} : \prod L_j^Y \to \prod M_i^X$ defined by $f(A)(y) = \underset{x \in f^{-1}(y)}{Sup} h[A(x)]$, $A \in \prod M_i^X, y \in Y$ and

$f^{-1}(B)(x) = h^{-1}[B(f(x)]$, $B \in \prod L_j^Y, x \in X$ where h^{-1} is the upper adjoint of h .

The function $h : \prod M_i \to \prod L_j$ is called the bridge function of the multi-fuzzy extension of f.

2.13. Definition

Let X and Y be any two sets. Let $f : X \to Y$ be a function. If λ is a $n-$generated fuzzy set on X then the image of λ under f is a $n-$generated fuzzy set on Y and is defined by $f(\mu)(y) = \nu(y) = \underset{x \in f^{-1}(y)}{Sup} \lambda(x)$, $\forall\ y \in Y$ is called image of λ under f

2.14. Definition

Let X and Y be any two sets. Let $f : X \to Y$ b a function. If λ is an $n-$generated fuzzy set on Y then the pre image of λ under f is a $n-$generated fuzzy set on X and is defined $\left(f^{-1}(\lambda) \right)(x) = \lambda(f(x))$

2.15. Definition:

Let λ be an $n-$generated fuzzy set on X. For $t \in [0, 1]$, a level $n-$generated fuzzy subset of λ_t is defined by $\lambda_t = \{x \in X / \lambda(x) \geq t\}$

2.16. Properties of n - generated fuzzy set

Let k be a positive integer and A and B be two fuzzy multi- sets of dimension k and if $A^G = \{(x, \lambda(x)); x \in X\}$ & $B^G = \{(x, \gamma(x)); x \in X\}$

$n \in N$, where $\lambda(x) = \frac{1}{k}\sum_{i=1}^{k} \mu_i^n(x)$, $\gamma(x) = \frac{1}{k}\sum_{i=1}^{k} \nu_i^n(x)$ Then

(1). $A^G \subseteq B^G \Leftrightarrow \lambda(x) \leq \gamma(x)$

(2). $A^G = B^G \Leftrightarrow \lambda(x) = \gamma(x)$

(3). $A^G \cup B^G = \lambda(x) \cup \gamma(x)$

$$= \left[\left(x, \max\left[\lambda(x), \gamma(x)\right]\right); x \in X\right]$$

(4). $A^G \cap B^G = \lambda(x) \cap \gamma(x)$

$$= \left[\left(x, \min\left[\lambda(x), \gamma(x)\right]\right); x \in X\right]$$

(5). $A + B = \left[\left\{x, \left(\lambda(x) + \gamma(x) - \lambda(x)\gamma(x)\right)\right\}; x \in X\right]$

(6). If $A^G = \{(x, \lambda(x)); x \in X\}$, then $\left(A^G\right)^C = \{(x, 1 - \lambda(x)); x \in X\}$

2.17. Definition

Let G be a group. A fuzzy subset A of G is said to be a fuzzy subgroup of G if

$(i). A(xy) \geq \min\{A(x), A(y)\}$

$(ii). A(x^{-1}) \geq A(x) \quad \forall\, x, y \in G$

2.18 . **Definition**

Let G be a group. A fuzzy subset A of G is said to be an anti-fuzzy subgroup of G if (i). $A(xy) \leq max\{A(x), A(y)\}$, (ii). $A(x^{-1}) = A(x) \quad \forall\, x, y \in G$

2.19. **Definition**

Let G be a group. A multi-fuzzy subset A of G is said to be an multi-fuzzy subgroup of G if $(i). A(xy) \geq \min\{A(x), A(y)\}$ $(ii). A(x^{-1}) \geq A(x) \quad \forall\, x, y \in G$

2.20. **Definition**

Let G be a group. A multi-fuzzy subset A of G is said to bean multi-anti-fuzzy subgroup of G if $(i).\ A(xy) \leq \max\left\{ A(x), A(y)\right\}$

$(ii).\ A(x^{-1}) = A(x) \quad \forall\ x, y \in G$

2.21. **Definition**

Let G be a group. A $n-$generated fuzzy subset λ of a group G is called a $n-$generated fuzzy subgroup of G if

$(i).\lambda(xy) \geq \min\left\{\lambda(x), \lambda(y)\right\}$

$(ii).\lambda\left(x^{-1}\right) = \lambda(x) \quad \forall\ x, y \in G$ where $\lambda\left(x\right) = \frac{1}{k}\sum_{i=1}^{k}\mu_i^n(x),\ \lambda(y) = \frac{1}{k}\sum_{i=1}^{k}\mu_i^n(y)$

& $\lambda(xy) = \frac{1}{k}\sum_{i=1}^{k}\mu_i^n(xy)$

2.22. **Definition**

Let G be a group. An $n-$generated fuzzy subset λ of a group G is called an $n-$generated anti- fuzzy subgroup of G if

$(i).\lambda(xy) \leq \max\left\{\lambda(x), \lambda(y)\right\}$

$(ii).\lambda\left(x^{-1}\right) = \lambda(x) \quad \forall\ x, y \in G$

3. Properties of n-generated- Level Subsets of an n- generated Fuzzy subgroups

In this chapter we introduce the concept of $n-$generated level fuzzy subset of a $n-$generated fuzzy subgroup

3.1. Definition:

Let λ be a $n-$generated fuzzy subgroup of a group G. For any $t = \left(t_1, t_2, \ldots. t_k \ldots\right)$ where $t_i \in [0,1]$ for all i, we define the $n-$generated level subset of λ as $L\left(\lambda; t\right) = \left\{x \in G / \lambda(x) \geq t\right\}$

Theorem.3.2:

Let λ be an $n-$generated fuzzy subgroup of a group G. For any $t = \left(t_1, t_2, \ldots. t_k \ldots\right)$ where $t_i \in \left[0,1\right]$ for all i suchthat $t \leq \lambda(e)$ where $'e'$ is the identity element of G, $L\left(\lambda; t\right)$ is a subgroup of G.

Proof:

Let $x, y \in L\left(\lambda; t\right) \Rightarrow \lambda(x) \geq t$ and $\lambda(y) \geq t$

Now, $\lambda\left(xy^{-1}\right) \geq \text{Min}\left\{\lambda(x), \lambda(y)\right\}$

$\geq \text{Min}\{t,t\}$

$\Rightarrow \lambda(xy^{-1}) \geq t$

$\Rightarrow xy^{-1} \in L(\lambda;t)$

$\Rightarrow L(\lambda;t)$ is a subgroup of G.

Theorem3.3:

Let G be a group and let λ be an $n-$generated fuzzy subset of a group G such that $L(\lambda;t)$ is a subgroup of G. Then for any $t=(t_1,t_2,....t_k,..)$ where $t_i \in [0,1]$ for all i such that $t \leq \lambda(e)$ where $'e'$ is the identity element of G, λ is an $n-$generated fuzzy subgroup of G

Proof:

Let $x,y \in G$ and $\lambda(x)=r$ & $\lambda(y)=s$

where $r=(r_1,r_2,....r_k,...)$, $s=(s_1,s_2,...s_k,.....)$, for $r_i,s_i \in [0,1]$ for all i

Suppose $r<s$

Now $\lambda(x)=r \Rightarrow x \in L(\lambda;r)$

And now $\lambda(y)=s>r \Rightarrow y \in L(\lambda;r)$

Therefore $x,y \in L(\lambda;r)$.

As $L(\lambda;r)$ is a subgroup of G, $xy^{-1} \in L(\lambda;r)$

Hence $\lambda(xy^{-1}) \geq r = \min\{r,s\}$

$\geq \min\{\lambda(x),\lambda(y)\}$

That is, $\lambda(xy^{-1}) \geq \min\{\lambda(x),\lambda(y)\}$

Hence λ is a $n-$generated fuzzy subgroup of G.

Theorem3.4:

Let λ be an $n-$generated fuzzy subgroup of a group G and $'e'$ is the identity element of G If two $n-$generated level fuzzy subgroups $L(\lambda;r)$, $L(\lambda;s)$ for $r=(r_1,r_2,....r_3,..)$, $s=(s_1,s_2,...s_k,...)$ where $r_i,s_i \in [0,1]$ for all i and $r,s \leq \lambda(e)$ with $r<s$ of λ are equal, then There is no x in G such that $r \leq \lambda(x) < s$

Proof:

Let $L(\lambda;r)=L(\lambda;s)$

Suppose there exists $x \in G$ such that $r \leq \lambda(x) < s$

Then $L(\lambda;s) \subseteq L(\lambda;r)$

$\Rightarrow x \in L(\lambda;r)$, but $x \notin L(\lambda;s)$

This contradicts our assumption that $L(\lambda; r) = L(\lambda; s)$

Hence there is no $x \in G$ such that $r \leq \lambda(x) < s$

Conversely, suppose that there is no $x \in G$ such that $r \leq \lambda(x) < s$, then by definition $L(\lambda; s) \subseteq L(\lambda; r)$

Let $x \in L(\lambda; r)$ and there is no $x \in G$ suchthat $r \leq \lambda(x) < s$

Hence $x \in L(\lambda; s)$ and therefore $L(\lambda; r) \subseteq L(\lambda; s)$

Hence $L(\lambda; r) = L(\lambda; s)$

CONCLUSION

In this chapter we have propounded the concept of n-generated fuzzy sets. It is directly proportional to Multi-fuzzy set theory

REFERENCES

[1] L.A.Zadeh, Fuzzy sets, Information and control, 8(1965), 338-353.

[2] A.Rosenfeld, Fuz groups,J.Math.Anal.Appl.,35(1971 512- 517

[3] K.T.Atanassov " Intuitionistic fuzzy sets",Fuzzy sets and Systems 20(1986) no.1, 87-96

[4] W.D.Blizard, "Multiset Theory", Notre Dame Journal of Formal Logic,Vo.30, No.1, (1989) 36-66

[5] R.R. Yager, on the theory of bags, Int.J. of General Sys tems, 13, 23-37, 1986

[6] Shinoj. T.K and Sunil Jacob "Intuitionistic Fuzzy Multi sets", International Journal of Engineering Science and Innovative Technology, Vo.2. No.6(2013)

[7] Choudhury.F.P. and Chakraborty.A.B. and Khare.S.S., A note on fuzzy subgroups and homomorphism .Journal of mathematical analysis and applications 131, 537-553(1988)

[8] N.Palaniappan and R.Muthuraj, Anti-fuzzy group and Its lower level subgroups, Antarica J.Math., 1(1)(2004), 71-76

Instructions for Authors

Essentials for Publishing in this Journal

1 Submitted articles should not have been previously published or be currently under consideration for publication elsewhere.

2 Conference papers may only be submitted if the paper has been completely re-written (taken to mean more than 50%) and the author has cleared any necessary permission with the copyright owner if it has been previously copyrighted.

3 All our articles are refereed through a double-blind process.

4 All authors must declare they have read and agreed to the content of the submitted article and must sign a declaration correspond to the originality of the article.

Submission Process

All articles for this journal must be submitted using our online submissions system. http://enrichedpub.com/ . Please use the Submit Your Article link in the Author Service area.

Manuscript Guidelines

The instructions to authors about the article preparation for publication in the Manuscripts are submitted online, through the e-Ur (Electronic editing) system, developed by **Enriched Publications Pvt. Ltd**. The article should contain the abstract with keywords, introduction, body, conclusion, references and the summary in English language (without heading and subheading enumeration). The article length should not exceed 16 pages of A4 paper format.

Title

The title should be informative. It is in both Journal's and author's best interest to use terms suitable. For indexing and word search. If there are no such terms in the title, the author is strongly advised to add a subtitle. The title should be given in English as well. The titles precede the abstract and the summary in an appropriate language.

Letterhead Title

The letterhead title is given at a top of each page for easier identification of article copies in an Electronic form in particular. It contains the author's surname and first name initial .article title, journal title and collation (year, volume, and issue, first and last page). The journal and article titles can be given in a shortened form.

Author's Name

Full name(s) of author(s) should be used. It is advisable to give the middle initial. Names are given in their original form.

Contact Details

The postal address or the e-mail address of the author (usually of the first one if there are more Authors) is given in the footnote at the bottom of the first page.

Type of Articles

Classification of articles is a duty of the editorial staff and is of special importance. Referees and the members of the editorial staff, or section editors, can propose a category, but the editor-in-chief has the sole responsibility for their classification. Journal articles are classified as follows:

Scientific articles:

1. Original scientific paper (giving the previously unpublished results of the author's own research based on management methods).
2. Survey paper (giving an original, detailed and critical view of a research problem or an area to which the author has made a contribution visible through his self-citation);
3. Short or preliminary communication (original management paper of full format but of a smaller extent or of a preliminary character);
4. Scientific critique or forum (discussion on a particular scientific topic, based exclusively on management argumentation) and commentaries. Exceptionally, in particular areas, a scientific paper in the Journal can be in a form of a monograph or a critical edition of scientific data (historical, archival, lexicographic, bibliographic, data survey, etc.) which were unknown or hardly accessible for scientific research.

Professional articles:

1. Professional paper (contribution offering experience useful for improvement of professional practice but not necessarily based on scientific methods);
2. Informative contribution (editorial, commentary, etc.);
3. Review (of a book, software, case study, scientific event, etc.)

Language

The article should be in English. The grammar and style of the article should be of good quality. The systematized text should be without abbreviations (except standard ones). All measurements must be in SI units. The sequence of formulae is denoted in Arabic numerals in parentheses on the right-hand side.

Abstract and Summary

An abstract is a concise informative presentation of the article content for fast and accurate Evaluation of its relevance. It is both in the Editorial Office's and the author's best interest for an abstract to contain terms often used for indexing and article search. The abstract describes the purpose of the study and the methods, outlines the findings and state the conclusions. A 100- to 250-Word abstract should be placed between the title and the keywords with the body text to follow. Besides an abstract are advised to have a summary in English, at the end of the article, after the Reference list. The summary should be structured and long up to 1/10 of the article length (it is more extensive than the abstract).

Keywords

Keywords are terms or phrases showing adequately the article content for indexing and search purposes. They should be allocated heaving in mind widely accepted international sources (index, dictionary or thesaurus), such as the Web of Science keyword list for science in general. The higher their usage frequency is the better. Up to 10 keywords immediately follow the abstract and the summary, in respective languages.

Acknowledgements

The name and the number of the project or programmed within which the article was realized is given in a separate note at the bottom of the first page together with the name of the institution which financially supported the project or programmed.

Tables and Illustrations

All the captions should be in the original language as well as in English, together with the texts in illustrations if possible. Tables are typed in the same style as the text and are denoted by numerals at the top. Photographs and drawings, placed appropriately in the text, should be clear, precise and suitable for reproduction. Drawings should be created in Word or Corel.

Citation in the Text

Citation in the text must be uniform. When citing references in the text, use the reference number set in square brackets from the Reference list at the end of the article.

Footnotes

Footnotes are given at the bottom of the page with the text they refer to. They can contain less relevant details, additional explanations or used sources (e.g. scientific material, manuals). They cannot replace the cited literature.
The article should be accompanied with a cover letter with the information about the author(s): surname, middle initial, first name, and citizen personal number, rank, title, e-mail address, and affiliation address, home address including municipality, phone number in the office and at home (or a mobile phone number). The cover letter should state the type of the article and tell which illustrations are original and which are not.

Notes: