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EP Journal on Digital Signal Processing

Aims and Scope

EP Journal on Digital Signal Processings has become very important with the ever-increasing demands of the software development to serve the millions of applications across various disciplines. For large software projects, innovative software development approaches are of vital importance. In order to gain higher software standard and efficiency, software process adaptation must be derived from social behavior, planning, strategy, intelligent computing, etc.,

based on various factors. International Journal of Software Engineering address the state of the art of all aspects of software engineering, highlighting the all tools and techniques for the software development process.

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Contents

Wireless Network Coding on Image Processing and its Major Applications: A Study

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A B S T R A C T

This research gives a general depiction of how researchers utilized the essential idea of network coding in a few potential approaches to improve the performance of wireless networks. The parameters of performance, delay, bundle conveyance report, jitter, and so forth. Wireless networks are framed when gadgets associate with different gadgets through electromagnetic energy noticeable all around and start their communication utilizing radio waves. These networks can be grouped into networks dependent on infrastructures and specially appointed networks. Infrastructure-put together networks depend with respect to an access point for every one of their communications, while specially appointed networks are self-sorted out networks.A wireless sensor network (WSN) incorporates base stations and some wireless sensors (nodes). The WSNs are extraordinarily assigned networks (wireless nodes that self-sort out in an infrastructure without a network). Wireless sensor networks are commonly relevant to numerous cutting-edge military or mechanical applications, including normal checking, perception, challenge observing or wellbeing checking.

1. OVERVIEW

In the execution of the utilizations of the wireless sensor network of the factory, the utilization of energy is the most key factor, since the nodes of the sensors have an amazingly constrained hold of energy and are important to operate independently. His primary idea relies upon the dispersion of the remaining task at hand of processing wavelet changes between various nodes. Two strategies for data exchanging have been proposed. Along these lines, broad research has concentrated on the most able technique to constrain energy use and broaden the helpful existence of the network. In this procedure, we consider the data dissemination plan proposed utilizing LS 9/7 DWT[1].

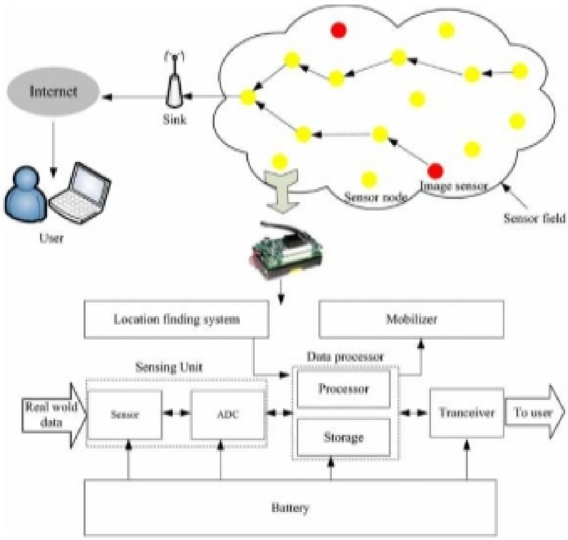

Figure 1: Sensor Network Architecture

It is believed that a routing algorithm is configured and that the nodes organize themselves in a two-level design. The research shows that SS is the only algorithm that presents the energy funds in the absence of pressure, which allows a decrease in power of about 29% of the proposed mechanism [2]. This father-young relationship is made to reinforce SPIHT fragilities in case of transmission of bit errors.

The pressure of the circulated image is taken into consideration for the images captured by the sensor nodes that have fields of vision. The approach uses a strategy such as the pressure of the stereo image to distinguish coverage in the images of adjacent sensor nodes. The image is the most important vector between the intercommunication of information in people's lives and the most important media that contain information.

2. IMAGE PROCESSING

Image processing is a technique for changing an image into a computerized structure and playing out certain tasks on it, considering a definitive objective of acquiring an improved image or expelling some significant data from it. It is a kind of banner harmony where the info is an image, like the edge of the video or photograph and the performance can be the image or characteristics identified with that image. As a rule, the image processing system fuses images as two-dimensional signs by applying authoritatively settled banner processing techniques. Today it is one of the most quickly advancing technologies, with its applications in various pieces of an organization. The image processing systems center the research region inside the development and data trains.

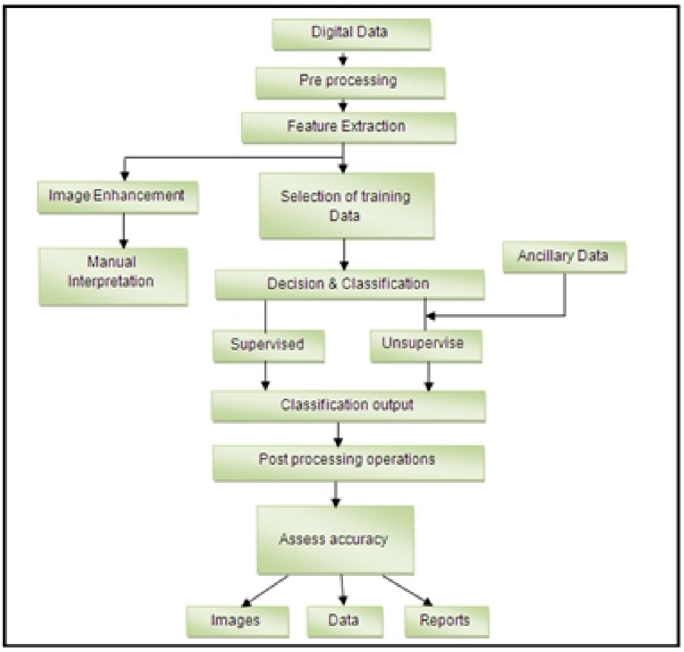

Figure 2: Image processing

The two kinds of strategies utilized for image processing are simple and digital image processing. Straightforward or visual image processing systems can be used for printed adaptations, for example, prints and photos. Image specialists utilize a few fundamental elements of explanation while utilizing these visual strategies. Image processing isn't just constrained to a district that should be inspected, in any case, in the analyst's information. The alliance is another essential device in image processing through visual systems. Hence, researchers apply a mix of individual information and assurance the data for image processing. The digital processing techniques help in the control of digital images by means of

C. Since the crude data from the satellite arena image sensors contain insufficiencies. To beat these deformities and get data innovativeness, it is important to try different things with various processing periods. The three general advances that must be confirmed by a wide scope of data when utilizing a digital system are the pre-processing, refreshing and data extraction.

3. IMAGE PROCESSING IN WIRELESS SENSOR NETWORKS

Due to the limited lifetime of the battery in each sensor, it is obvious that the reduction of the transmitted data will increase the energy efficiency and the useful life of the network. However, the most obvious solution is image compression. The purpose of image compression is to reduce the number of bits needed to represent the image, eliminating as much as possible spatial and spectral redundancies. In this research, the transmission scheme of the proposed image is based on the transformation of wavelet images. The structure of a transformation encoder is illustrated in.

Requirements: small size, high number, tether-less and low cost. The small size involves a small battery. Low cost and energy involve low power CPUs, radios with minimum bandwidth and range. The ad hoc implementation does not imply maintenance or replacement of the battery. To increase the useful life of the network, no raw data is transmitted.

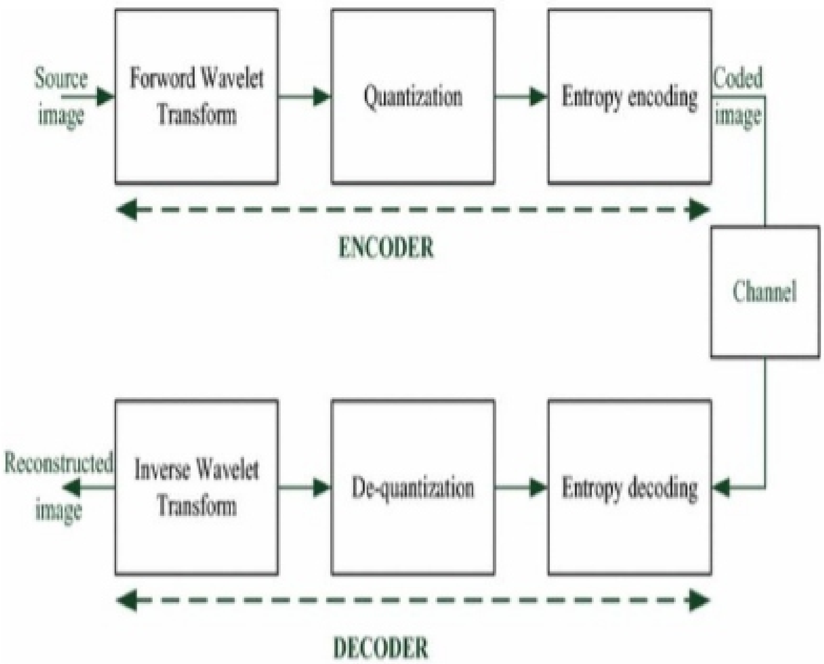

Figure 3: Functional Block Diagram of Jpeg 2000 Encoder

- **AD HOC Wireless Networks**

A large number of static or mobile self-classified nodes that are arbitrarily transmitted Communication with the nearest neighbour Wireless connections the connections are delicate and potentially unbalanced. The network is based on power and fades levels. The impedance is high for omnidirectional antennas. Sensor networks and sensor and actuator networks are a visible case.

- **System Model**

Consider a network of wireless multi-jump sensors that wirelessly interconnect sensor nodes prepared to recover and deal with a still image. The records, wherein the odd ones were discovered, including instances of information data, were at first spared toward the start of the calculation. In this research, no extra memory is required at any stage. For every model pixel, the low pass rot requires 8 turns (S) and 8 stage instructions (A), while the high advance requires 2 moves and 4 augmentations. The energy required for low pass/high pass rots can be described by the number of activities. The total calculation energy for this system can be prepared as the arrangement of the calculation load and the access to the data. Two degrees of wavelet rot are utilized.

4. NETWORK CODING FOR WIRELESS APPLICATIONS

Network Coding (NC) is a generally ongoing subset of network information theory that has prompted incredible advancement in enhancing network performance. It includes performing activities other than simply sending and replication in the nodes that make up a network. In this research, we attempt to inspect the developments in this field and look at the effect this has had on wireless networks, as far as the improvements it has made and the resulting application to different classes inside the wireless network.

Consider the theory behind NC, the different NC schemes that have been proposed and utilized throughout the years, the development of the NC application in the physical layer of networks and a few chose NC applications in wireless networks. This section endeavors to investigate the idea of network coding (NC), a moderately new subset of information theory, explicitly inside the space of wireless networks. Before the fundamental Section that depicts this field, the transmission of data through a network was seen basically as a progression of products, which is a trade of products without the capacity to process the products themselves.

Network coding changed this, recommending more muddled activities than essentially reproducing and transmitting data parcels could be performed on the nodes that make up a given network. This has prompted quick progress and has animated the utilization of new numerical apparatuses, in fields, for example, polynomial math, matroid theory, geometry, chart theory, combinatorial theory, and improvement, among others. Even though NC is a muddled theme to talk about without a critical numerical foundation, this research goes for a progressively casual crowd and, in this research, we will take measures to lessen the intricacy of the examined scientific theory, while simultaneously attempting to catch the different interior subtleties. In this presentation, we will likely offer a concise portrayal of what NC includes, and consequently explain the various highlights that characterize this innovation. To accomplish this, we give some information on the NC and its basic theory in this research, talk about the mainstream coding schemes right now being used in the Section and quickly investigate the network encoding dependent on the physical layer in the Section, concentrating on Network Encoding physics (PNC), as it is progressively utilized in wireless networks.

- **Theory Behind Network Coding**

The consequences of this research are communicated as the hypothesis of the most extreme cut of the base stream in the theory of the network. This hypothesis expresses that in a stream network, where information streams starting with one node then onto the next, the most extreme measure of stream from the source to the sink (greatest stream) is equivalent to the base limit that would not enable any stream to go from the source-sink when it is cut/expelled from the net with a particular goal in mind (least cut). [3] Demonstrated that when operations were permitted in moderate nodes, the most extreme multicast speed was equivalent to the base sliced from the source to every beneficiary essentially, if every one of the collectors have a similar least cut from the source, NC will enable all nodes to at the same time arrive at the base cutting limit.

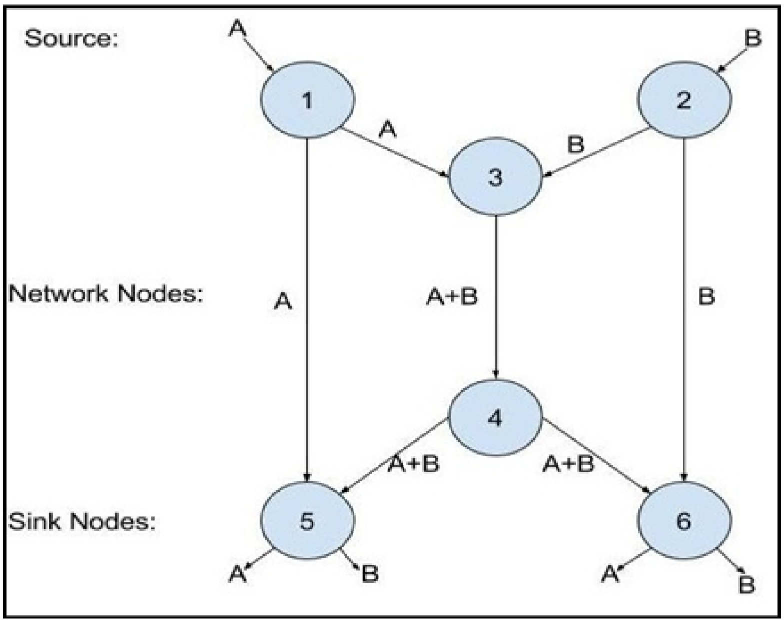

Figure 4: Butterfly Network

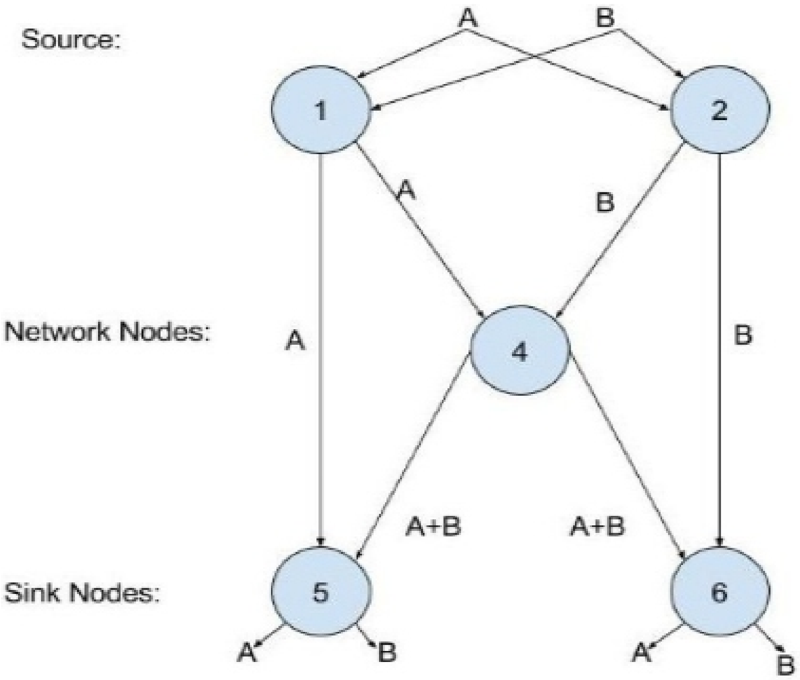

Figure 5: Modified wireless butterfly network

In this model, node 5 will get both A and A + B, from which it can decode B by subtracting these two qualities. Node 6 would utilize a similar technique to decode An in the wake of accepting both B and A + B. From this simple model; we can see that few other coding techniques could be connected to a variable number of bundles, in different network configurations.

The wireless network butterflies modified as shown in Figure 5 is different from the network to the original butterfly in the sense that the packet transmissions may be transmitted from the source node to more than one node. Therefore, transmissions are represented using hyperc arcs, rather than arcs. We now have a brief and hopefully overview of the underlying theory and the nature of network coding.

- **Network Coding Schemes**

The way nodes code and decode the packets they transmit / receive can have a big impact on the resulting network performance. Most of the NC schemes in use today are based on algebraic theory. While the previous schemes, such as the traditional XOR coding scheme and the deterministic linear network coding scheme were deterministic in nature, the most common schemes in use today are not deterministic, which means that they are free from the constraint of having feedback information on packages for every package sent by all receivers. In this research we will see some common coding schemes, namely Random Linear Network Coding (RLNC), Triangular Network Coding (TNC) and Opportunistic Network Coding (ONC).

- **Random Linear Network Coding**

Here, once again, we will use a network model for a general communication system consisting of sources, network nodes and sinks connected by channels that could have losses. We can represent a system of this type as a directed graph G = {V, E} where the vertices V represent the different nodes of the network and the set of edges E consists of arcs between the nodes and denotes the connections in the network [4]

- **Triangular Network Coding**

In order to intrinsically resolve the problem with RLNC described above, where the receivers that obtain an insufficient number of packets cannot recover the original packets, the triangular network coding has been proposed in [5]. The package coding scheme based on triangular schemes is performed in two phases. Thus, the packets are coded bit by bit, in which the bits of "0" are added in such a way as to generate a triangular model, known as triangulation, as shown in Figure 6.

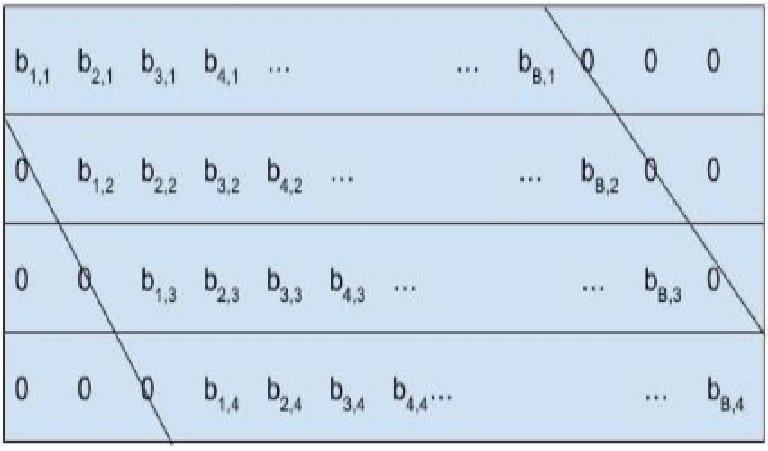

Figure 6: Triangular Pattern

- **Opportunistic Network Coding**

In ONC, tracking nodes can browse all nearby transmissions and store the data packets listened to, regardless of whether they are for them or not [6]. As such, the sensor nodes know the packets listened to and routed by each neighbouring node and can perform network coding operations based on this information.

5. APPLICATIONS OF NC TO WIRELESS NETWORKS

It is unlikely that the incorporation of network coding in the physical layer will be practical in the near future, for a variety of reasons detailed in [7]. However, it is quite feasible to build a network coding in overlapping networks. In overlapping networks, nodes are applications that run on computers and edges are the transport level connections between computers. Overlapping networks can be based on infrastructure, as shown by content distribution networks such as Akamai.

- **File download**

Downloading files from a server to a client computer is one of the most common tasks that occur in network communication. While the downloaded file is traditionally unicast from the server to the client, if we ignore the delay, this can also be seen as a multicast of the file from the server to a large group of clients using a proportionally large amount of buffering.

- **Video on Demand, Live Media Broadcast, and Instant Messaging (IM)**

Video on demand can be considered a specialized way to download files in which the parts of the downloaded file should arrive in order and should be decoded almost in real time, taking into account a small delay. The network encoding can be applied in this case by dividing the file into fragments, which can be downloaded sequentially. A similar technique can be used with live media streaming.

- **Wireless Mesh Networks**

In addition to an application layer overlay network, another convenient place where network encryption can be applied is a link-level network, such as a wireless mesh network. Mesh networks consist of mesh routers, which provide access to an existing infrastructure and mesh client, which provide multiple hop connectivity to mesh routers and use the connectivity provided by other client meshes.

- **Network Coding Meets Multimedia**

Although each node in the network transmits only messages in a traditional communication system, the recent network coding (NC) paradigm proposes to implement a simple network processing with combinations of packets in the nodes. NC extends the concept of "encoding" of a message beyond the encoding of the source (for compression) and encoding of the channel (for protection against errors and losses). It has been shown to increase network performance compared to the implementation of traditional networks, reduce the delay and provide robustness to transmission errors and network dynamics.

6. CONCLUSION

Wireless sensor networks (WSN) have drawn the attention of the research community over the most recent couple of years, driven by an abundance of hypothetical and pragmatic difficulties. This developing interest can be to a great extent credited to new applications empowered by vast scale networks of little devices equipped for collecting information from the physical environment, performing simple processing on the extricated data and transmitting it to remote locations. Critical outcomes around there over the most recent couple of years have introduced a flood of common and military applications. Starting today, most conveyed wireless sensor networks measure scalar physical wonders like temperature, pressure, stickiness, or location of items.

We examined in this research, we consider wireless sensor networks, in which very small sensors can be extended to surfaces and can collect energy from the environment, to form detection surfaces through a network of integrated communication. The sensor nodes are generally equipped with a radio transceiver, a microcontroller, a memory unit and a set of transducers with which they can acquire and process data. To reduce the size of each node and the power requirements, the transceiver oscillator is replaced by a resonant circuit on the chip. However, the central frequency of the resonant circuit is random, which means that each node chooses a random channel to transmit and another random channel to receive. Nodes can self-organize to form a multi-hop network and transmit data to a receiving node.The performance between any two nodes is constant if only routing is used, but grows linearly in the number of channels if the network coding is used and the radio frequency intervals are chosen optimally. The reason network coding is of great help here is that randomly combined packets can find their way to the destination without the need to explicitly inform the nodes where their destinations are located.

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Interval Graph with Consecutive Cliques of Size 3 - Signed Roman Domination

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ABSTRACT

The theory of Graphs is an important branch of Mathematics that was developed exponentially. Domination in graphs is rapidly growing area of research in graph theory today. It has been studied extensively and finds applications to various branches of Science & Technology.

Interval graphs have drawn the attention of many researchers for over 40 years. They form a special class of graphs with many interesting properties and revealed their practical relevance for modeling problems arising in the real world.

In this paper a study of signed Roman domination in an interval graph with consecutive cliques of size 3 is carried out.

Keywords: Signed Roman dominating function, Signed Roman domination number, Interval family, Interval graph.

Mathematical Subject Classification: 05C69.

1. INTRODUCTION

The major development of graph theory has occurred in recent years and inspired to a larger degree and it has become the source of interest to many researchers due to its applications to various branches of Science & Technology.

Domination in graphs introduced by Ore [11] and Berge [3] has become an emerging area of research in graph theory today. Many graph theorists, Allan, R.B. and Laskar, R.[2], Cockayne and Hedetniemi [4, 5] and others have contributed significantly to the theory of dominating sets, domination numbers and other related topics. Haynes et al. [7, 8] presented a survey of articles in the wide field of domination in graphs.

Recently, dominating functions in domination theory have received much attention. A purely graph – theoretic motivation is given by the fact that the dominating function problem can be seen, in a clear sense, as a proper generalization of the classical domination problem. Similarly edge dominating functions are also studied extensively.

Let (V, E) be a graph. A subset D of V is said to be a dominating set of G if every vertex in $V - D$ is adjacent to a vertex in D. The minimum cardinality of a dominating set is called the domination number and is denoted by (G).

We consider finite graphs without loops and multiple edges.

2. SIGNED ROMAN DOMINATING FUNCTION

The concept of Signed dominating function was introduced by Dunbar et al. [6]. There is a variety of possible applications for this variation of domination. By assigning the values −1 or +1 to the vertices of a graph we can model such things as networks of positive and negative electrical charges, networks of positive and negative spins of electrons and networks of people or organizations in which global decisions can be made.

The Roman dominating function of a graph G was defined by Cockayne et.al [5]. The definition of a Roman dominating function was motivated by an article in Scientific American by Ian Stewart [9] entitled "Defend The Roman Empire!" and suggested by even earlier byReVelle [12]. Domination number and Roman domination number in an interval graph with consecutive cliques of size 3 are studied byC. Jaya Subba Reddy, M.Reddappa and B.Maheswari [10].

The concept of signed Roman dominating function was introduced by Ahangar et al. [1]. They presented various lower and upper bounds on the signed Roman domination number of a graph and characterized the graphs which have these bounds.The minimal signed Roman dominating functions of corona product graph of a path with a star is studied by Siva Parvati [13].

Let (V, E) be a graph. A signed Roman dominating function on the graph G is a function $f: V \to \{-1,1,2\}$, which satisfies the following two conditions:

(i) For each $u \in V$, $\sum_{v \in N[u]} f(v) \geq 1$;

(ii) Each vertex u for which $f(u) = -1$ is adjacent to at least one vertex v for which $f(v) = 2$.

Thevalue $f(V) = \sum_{u \in v} f(u)$ is called as the weight of the function f, and it is denoted by $w(f)$. The signed Roman domination number of G,denoted by $\gamma_{sR}(G)$ is the minimum weight of a signed Roman dominating function of$G$.

Each signed Roman dominating function f on G is uniquely determined by the ordered partitions $(V_{-1}, V_1, V_2)$of $V(G)$, where $V_i = \{v \in V / f(v) = i\}$ for $i = -1, 1,2$. Then$w(f) = -|V_{-1}| + |V_1| + 2|V_2|$.

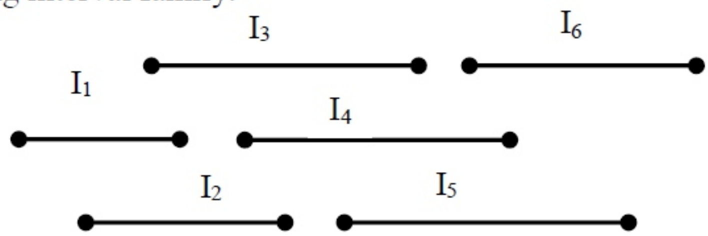
There exists a 1-1 correspondence between the functions $f : V \to \{-1, 1, 2\}$ and the ordered partitions$(V_{-1}, V_1, V_2)$of$V$. Thus we write $f = (V_{-1}, V_1, V_2)$.

3. INTERVAL GRAPH

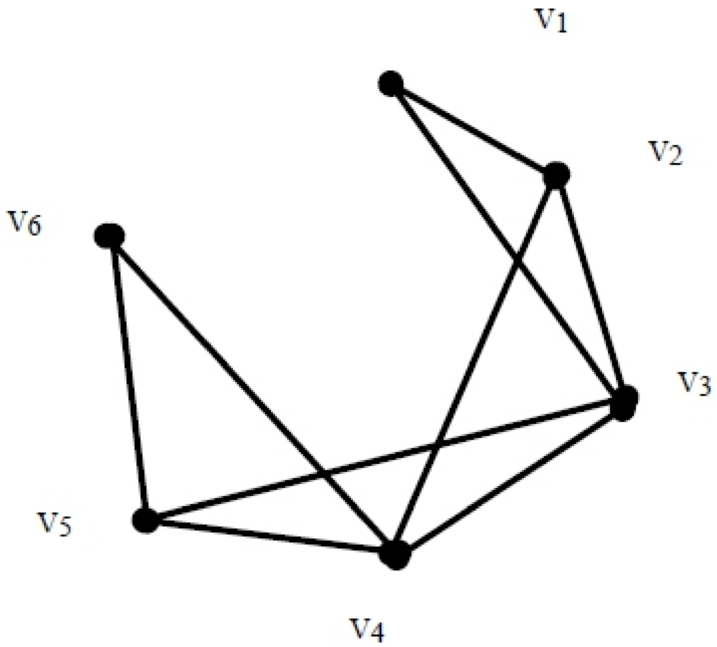
Let $I = \{I_1, I_2, I_3, \ldots\ldots\ldots I_n\}$ be an interval family, where each I_i is an interval on the real line and $I_i = [a_i, b_i]$ for$i = 1, 2, 3, \ldots\ldots n$. Here a_i is called the left end point and b_i is called the right end point of I_i. Without loss of generality, we assume that all end points of the intervals in are distinct numbers between 1 and 2n. Two intervals $i = [a_i, b_i]$ and $j = [a_j, b_j]$ are said to intersect each other if either $a_j < b_i$ or $a_i < b_j$. The intervals are labelled in the increasing order of their right end points.

Let $G\ V, E$ be a graph. G is called an interval graph if there is a 1-1 correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect. If i is an interval in I the corresponding vertex in G is denoted by v.

Consider the following interval family.



The corresponding interval graph is



Interval graph

In what follows we consider interval graphs of this type. That is the interval graph which has consecutive cliques of size 3. We denote this type of interval graph by G.

Thesigned Roman domination is studied in the following for the interval graph G.

4. RESULTS

Theorem 4.1: Let G be the Interval graph with n vertices, where n $\geq$ 6. Then the signed Roman domination number of G is

$\gamma_{sR}\,\mathcal{G} = 2k + 2$ for $n = 5k + 1$, $5k + 3$, $5k + 5$

$= 2k + 3$ for $n = 5k + 2$, , $5k + 4$,

where $k = 1,2,3$ .... respectively.

Proof: Let $\mathcal{G}$ be the interval graph with n vertices, where $n \geq 6$.

Let the vertex set of be $\{v_1, v_2, v_3, v_4 .... v_n\}$.

Case 1: Suppose $n = 5k + 1$, where $k = 1,2,3 \ldots \ldots ..$.

Let $: V \rightarrow \{-1, 1, 2\}$ and let (V_{-1}, V_1, V_2) be the ordered partition of V induced by f where $V_i = \{v \in V / f\ v = \ \}$ for $i = -1, 1,2$. Then there exist a 1-1 correspondence between the functions $f : V \rightarrow \{-1, 1, 2\}$ and the ordered pairs (V_{-1}, V_1, V_2) of V. Thus we write $f = V_{-1}, V_1, V_2$.

Let $V_1 = \{v_1, v_4, v_6, ... v_{n-10}, v_{n-7}, v_{n-5,}, v_{n-2}\}$;

$V_2 = v_3, v_8, v_{13}, \ldots \ldots \ldots \ldots \ldots .. v_{n-13}, v_{n-8,}, v_{n-3}, v_n$;

$V_{-1} = v_2, v_5, v_7, \ldots \ldots \ldots \ldots \ldots .., v_{n-9}, v_{n-6,}, v_{n-4}, v_{n-1}$.

It was shown in [10] that V_2 is a minimumdominating set of $\mathcal{G}$. Further the set V_2 dominates V_{-1}. That is , every vertex u such that $f\ u = -1$ is adjacent to some vertex v with $f\ v = 2$.

Therefore $f = V_{-1}, V_1, V_2$ becomes a signed Roman dominating function of $\mathcal{G}$.

Now $V_1 = 2k, V_2 = 2k + 1, V_{-1} = 2k$.

$$\text{Therefore} \sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$$

$$= -2k + 2k + 2k + 2 = 2\ k+2.$$

Let $g = (V'_{-1}, V'_1, V'_2)$ be a signed Roman dominating function of $\mathcal{G}$, where V'_2 dominates V'_{-1}. Then $g\ V = \sum_{v\in V'} g(v) = \sum_{v\in V'_{-1}} g(v) + \sum_{v\in V'_1} g(v) + \sum_{v\in V'_2} g(v)$

$$= -\ '_{-1} + \ V'_1 + 2\,V'_2$$

Since V_2 is a minimum dominating setof $\mathcal{G}$, we have $V_2 \leq V'_2$. This implies that $g\ V = -V'_{-1} + V'_1 + 2\,V'_2 \geq -V_{-1} + V_1 + 2\,V_2 = f(V)$.

Therefore (V) is a minimum weightof $\mathcal{G}$, where $f(V_{-1}, V_1, V_2)$ is a signed Roman dominating function .

Thus $\gamma_{sR}\,\mathcal{G} = 2k + 2$.

Case 2: Suppose $n = 5k + 2$, where $k = 1,2,3 \ldots \ldots ..$.

Now we proceed as in Case1.

Let $V_1 = \{v_1, v_4, v_6, \ldots\ldots\ldots\ldots v_{n-8}, v_{n-6}, v_{n-3,}, v_{n-1}\}$;

$V_2 = v_3, v_8, v_{13}, \ldots \ldots \ldots \ldots \ldots .. v_{n-14}, v_{n-9,}, v_{n-4}, v_n$;

$V_{-1} = v_2, v_5, v_7, \ldots \ldots \ldots \ldots \ldots .., v_{n-10}, v_{n-7,}, v_{n-5}, v_{n-2}$.

ClearlyV_2 is a minimumdominating set of $\mathcal{G}$. Here we observe that the setV_2dominates V_{-1}.

Therefore $f = V_{-1}, V_1, V_2$ is a signed Roman dominating function of $\mathcal{G}$.

Now $V_1 = 2k + 1, V_2 = k + 1, V_{-1} = 2k.$

Therefore $\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$.

$= -2k + 2k + 1 + 2k + 2 = 2\ k+3.$

If $g = (V'_{-1}, V'_1, V'_2)$ is a signed Roman dominating function of $\mathcal{G}$, then it follows as in Case 1, that (V) is a minimum weightof$\mathcal{G}$ for the Roman dominating function $f\ V_{-1}, V_1, V_2$.

Thus$\gamma_{sR}\,\mathcal{G} = 2k + 3.$

Case 3: Suppose $n = 5k + 3$, where $k = 1,2,3 \ldots \ldots ..$.

Now we proceed as in Case1.

Let $V_1 = \{v_1, v_4, v_6, \ldots\ldots\ldots\ldots v_{n-9}, v_{n-7}, v_{n-4,}, v_{n-2}\}$;

$V_2 = v_3, v_8, v_{13}, \ldots \ldots \ldots \ldots \ldots .. v_{n-15}, v_{n-10,}, v_{n-5}, v_n$;

$V_{-1} = v_2, v_5, v_7, \ldots \ldots \ldots \ldots \ldots .., v_{n-8}, v_{n-6,}, v_{n-3}, v_{n-1}$.

We have seen [10], that $V_2$is a minimumdominating set of $\mathcal{G}$. Here we observe that the setV_2dominates V_{-1}.

Therefore $f = V_{-1}, V_1, V_2$is a signed Roman dominating function of $\mathcal{G}$.

Now $V_1 = 2k + 1, V_2 = k + 1, V_{-1} = 2k + 1.$

Therefore $\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$.

$= -2k - 1 + 2k + 1 + 2k + 2 = 2\ k+2.$

If $g = (V'_{-1}, V'_1, V'_2)$ is a signed Roman dominating function of $\mathcal{G}$, then it follows as in Case 1, that (V) is a minimum weightof$\mathcal{G}$ for the signed Roman dominating function $f\ V_{-1}, V_1, V_2$.

Thus $\gamma_{sR}\mathcal{G} = 2k + 2$.

Case 4: Suppose $n = 5k + 4$, where $k = 1,2,3 \ldots \ldots ..$.

Now we proceed as in Case1.

Let $V_1 = \{v_1, v_4, v_6, \ldots \ldots \ldots \ldots \ldots .. v_{n-8}, v_{n-5}, v_{n-3,}, v_n\}$;

$V_2 = \quad _3, v_8, v_{13}, \ldots \ldots \ldots \ldots \ldots .. v_{n-16}, v_{n-11,}, v_{n-6}, v_{n-1}$;

$V_{-1} = \quad _2, v_5, v_7, \ldots \ldots \ldots \ldots \ldots .., v_{n-9}, v_{n-7,}, v_{n-4}, v_{n-2}$.

Obviously V_2 is a minimumdominating set of . Here we observe that the set V_2dominates V_{-1}.

Therefore $f = V_0, V_1, V_2$ is a signed Roman dominating function of .

Now $V_1 = 2k + 2, V_2 = k + 1, V_{-1} = 2k + 1$.

Therefore $\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$.

$$=-2k - 1 + 2k + 2 + 2k + 2 = 2\ k+3.$$

If $g = (V'_{-1}, V'_1, V'_2)$ is a signed Roman dominating function of $\mathcal{G}$, then it follows as in Case 1, that $f(\ )$ is a minimum weightof$\mathcal{G}$for the signed Roman dominating function $f\ V_{-1}, V_1, V_2$.

Hence $\gamma_{sR}\mathcal{G} = 2k + 3$.

Case 5: Suppose $n = 5k + 5$, where $k = 1,2,3 \ldots \ldots ..$.

Now we proceed as in Case1.

Let$V_1 = \{v_1, v_4, v_6, \ldots\ldots\ldots\ldots\ldots\ldots\ldots v_{n-9}, v_{n-6}, v_{n-4,}, v_{n-1}\}$;

$V_2 = v_3, v_8, v_{13}, \ldots \ldots \ldots \ldots \ldots .. v_{n-17}, v_{n-12,}, v_{n-7}, v_{n-2}$;

$V_{-1} = v_2, v_5, v_7, \ldots \ldots \ldots \ldots \ldots .., v_{n-8}, v_{n-5,}, v_{n-3}, v_n$.

Clearly V_2 is a minimumdominating set of $\mathcal{G}$and the setV_2 dominates V_{-1}.

Therefore $f = V_{-1}, V_1, V_2$ is a signed Roman dominating function of . Now $V_1 = 2k + 2, V_2 = k + 1, V_{-1} = 2k + 2$.

If $g = (V'_{-1}, V'_1, V'_2)$ is a signed Roman dominating function of $\mathcal{G}$, then it follows as in Case 1, that $f(\ )$ is a minimum weightof$\mathcal{G}$for the signed Roman dominating function $f\ V_{-1}, V_1, V_2$.

Hence $\gamma_{sR}\ G = 2k + 2$.

Theorem 4.2: Let $\mathcal{G}$ the an interval graph with n vertices, where $2 < n < 6$.

Then $\gamma_{sR} G = 1$ for $n = 4$

$= 2$ for $n = 3, 5$.

Proof: Let $\mathcal{G}$ be the interval graph with n vertices, where $2 < n < 6$.

Case 1: Suppose $n = 3$. Let $v_1, v_2, {}_3$ be the vertices of $\mathcal{G}$.

$$\text{Let } V_1 = v_1 ;\ V_2 = \{v_2\} ;\ V_{-1} = v_3$$

Obviously V_2 is a minimumdominating set of , and V_2 dominates V_{-1}. Therefore $f = V_{-1}, V_1, V_2$ is a signed Roman dominating functionof .

And $\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$.

$$= -1 + 1 + 2\text{x}1 = 2.$$

Thus $\gamma_{sR}\mathcal{G} = 2$.

Case 2: Suppose $n = 4$. Let v_1, v_2, v_3, v_4 be the vertices of . Let

$V_1 = \{v_2\}; V_2 = \{v_3\};\ V_{-1} = v_1, v_4$.

Clearly V_2 is a minimumdominating set of $\mathcal{G}$ and V_2dominates V_{-1}.

Therefore $f = V_{-1}, V_1, V_2$ is a signed Roman dominating function of .

And $$\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v) .$$

$$= -2 + 1 + 2\text{x}1 = 1.$$

Thus $\gamma_{sR}\mathcal{G} = 1$.

Case 3: Suppose $n = 5$. Let v_1, v_2, v_3, v_4, v_5 be the vertices of .

Let $V_1 = \{v_2, v_5\}$; $V_2 = v_3 ;\ V_{-1} = v_1, v_4$.

Again V_2 is a minimumdominating set of $\mathcal{G}$ and the set V_2dominates V_{-1}.

Therefore $f = V_{-1}, V_1, V_2$ is a signed Roman dominating function of .

Therefore $\sum_{v\in V} f(v) = \sum_{v\in V_{-1}} f(v) + \sum_{v\in V_1} f(v) + \sum_{v\in V_2} f(v)$.

$$= -2 + 2 + 2 \times 1 = 2.$$

Thus $\gamma_{sR}\mathcal{G} = 2$.

Theorem 4.3: Let $\mathcal{G}$ the an interval graph with n vertices , where $n \geq 6$. Then $\gamma_{sR}\ \ \mathcal{G}\ \ = \gamma\ \ + 1$ for $n = 5k\ + 1,\ 5k\ + 2,$ and $5k\ + 4,$ where $k =$ $1, 2, 3, \ldots \ldots \ldots$ respectively.

Proof : Let $\mathcal{G}$ be the interval graph with n vertices , where $n \geq 6$.

Then by [10], we have

$\gamma_R \mathcal{G} = 2k + 2$, for $n = 5k + 2, 5k + 4$, where $k = 1, 2, 3 \ldots \ldots \ldots \ldots$

$= 2k + 1$, for $n = 5k + 1$, where $k = 1, 2, 3 \ldots \ldots \ldots \ldots$

Now by Theorem 4.1, we have

$\gamma_{sR} \mathcal{G} = 2k + 3$, for $n = 5k + 2, 5k + 4$, where $k = 1, 2, 3 \ldots \ldots \ldots \ldots$

$= 2k + 2$, for $n = 5k + 1$, where $k = 1, 2, 3 \ldots \ldots \ldots \ldots$

For $n = 5k + 2, 5k + 4$, where $k = 1, 2, 3 \ldots \ldots \ldots \ldots$

$$\gamma_{sR}\ \ \mathcal{G}\ \ = 2\ \ + 3$$
$$= (2\ \ + 2) + 1 = \gamma_R\ \ \mathcal{G}\ \ + 1$$

Again for $n = 5k + 1$, where $k = 1, 2, 3 \ldots \ldots \ldots \ldots$

$$\gamma_{sR}\ \ \mathcal{G}\ \ = 2\ \ + 2$$
$$= (2\ \ + 1) + 1 = \gamma_R\ \ \mathcal{G}\ \ + 1$$

Theorem 4. 4: Let $\mathcal{G}$ be the Interval graph with n vertices , where $n \geq 8$.Then $\gamma_{sR} \mathcal{G} =$ γ for $n = 5k + 3, 5k + 5$, where $k = 1, 2, 3,$.................... respectively.

Proof : Let $\mathcal{G}$ be the interval graph with n vertices , where $n \geq 8$.

Suppose $n = 5k + 3, 5k + 5$, where $k = 1, 2, 3, \ldots \ldots \ldots.$

Then $\gamma_{sR} \mathcal{G} = 2k + 2$ and $\gamma_R \mathcal{G} = 2k + 2$,(by [10])

Hence $\gamma_{sR} \mathcal{G} = \gamma_R \mathcal{G}$.

Theorem 4.5: Let $\mathcal{G}$ be the interval graph with n vertices , where $n \geq 6$. Then $\gamma_{sR} \mathcal{G} =$ $2\ \gamma\ \ \mathcal{G}$,for$= 5k + 1, 5k + 3, 5k + 5$,where $k = 1,2,3$ ....respectively.

Proof: Let $\mathcal{G}$ be the interval graph with n vertices , where $n \geq 6$.

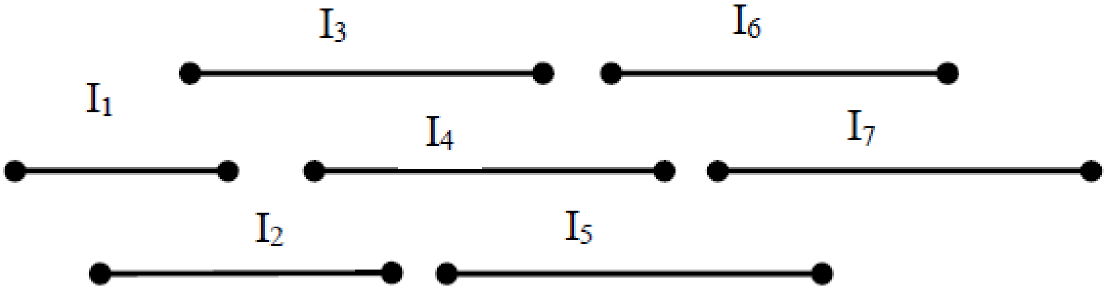
Suppose $n = 5k + 1, 5k + 3, 5k + 5$ where $k = 1,2,3$ .... respectively.

Then by Theorem 4.1, the signed Roman domination number is

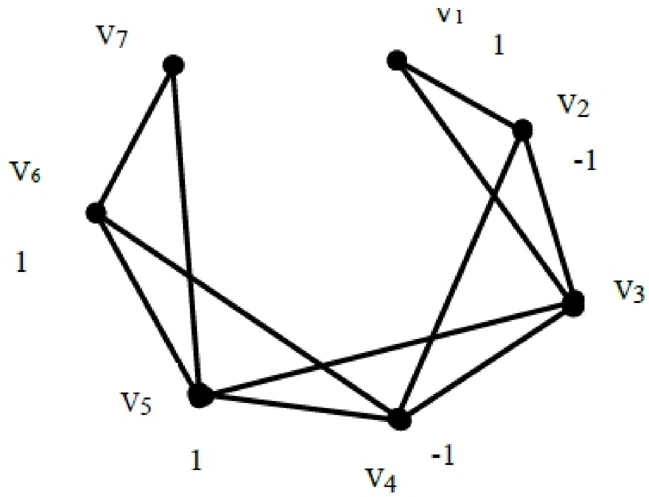
$$\gamma_{sR}\ \ \mathcal{G}\ \ = 2\ \ + 2$$
$$= 2(k + 1) = 2\gamma \mathcal{G} (\text{ by[10]})$$
$$\text{Thus } \gamma_{sR} \mathcal{G} = 2\gamma \mathcal{G}.$$

5. ILLUSTRATIONS

Illustration 1: n =7



Interval family



Interval graph

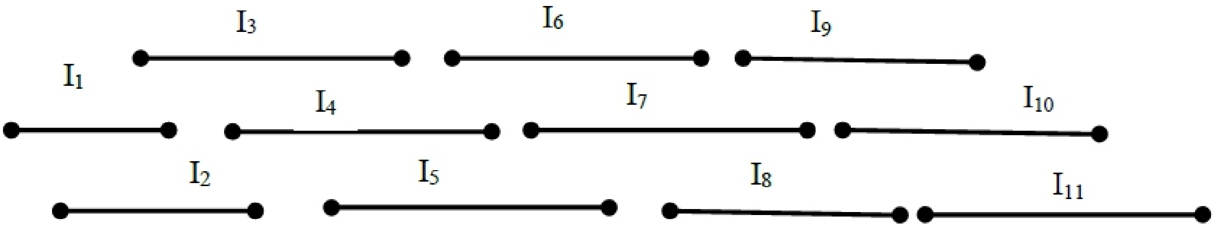
$D = \{v_3, v_7\}$and $\gamma G = 2$.

$V_1 = \{v_1 , v_5, v_6\}; V_2 = v_3 , v_7 ; V_{-1} = v_2, v_4$

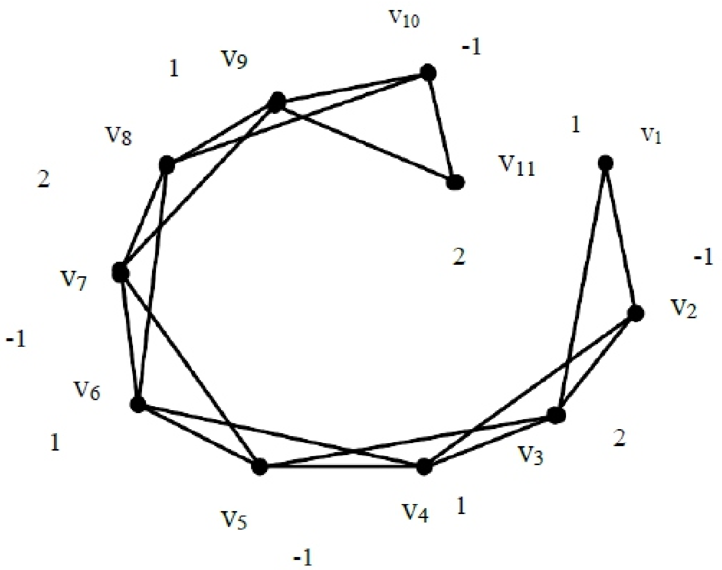
$\sum_{v \in V} f(v) = V_{-1} . -1 + V_1 . 1 + V_2 . 2 = -1\ 2 + 1\ 3 + 2(2) = 5 = f(\ )$

Therefore $\gamma_{sR} G = 5$.

Illustration 2: n =11



Interval family



Interval graph

$D = \{v_3, v_8, v_{11}\}$and $\gamma G = 3.$

$V_1 = \{v_1, v_4, v_6, v_9\}$; $V_2 = v_3 , v_8, v_{11}$; $V_{-1} = v_2, v_5, v_7, v_{10}$

$$\sum_{v \in V} f(v) = V_{-1} \,.\, -1 + V_1 \,.\, 1 + V_2 \,.\, 2 = -1\ 4 + 1\ 4 + 2(3) = 6 = f(\,)$$

Therefore $\gamma_{sR} G = 6.$

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On Neutrosophic Soft Compact Topological Spaces

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A B S T R A C T

In this paper, the concept of almost and near compactness on neutrosophic soft topological space have been introduced along with the investigation of their several characteristics. That's shown that the neutrosophic soft continuous image of neutrosophic soft almostly compact is neutrosophic soft almostly compact and it's properties developed here.

Keywords: Neutrosophic soft sets; Compactness on neutrosophic soft topological space; Neutrosophic soft continuous.

1. INTRODUCTION

After the introduction of the concept of a fuzzy set by Zadeh in his classic paper [1]. C.L.Chang [2] has defined fuzzy topological spaces. In 1983, Atannasov [3] introduced the notion of intuitionistic fuzzy sets. Soft sets theory was proposed by Molodtsov [4] in 1999, as a new mathematical tool for handling problems which contain uncertainties. Maji et al [5] gave the first practical application of soft sets in decision-making problems. Shabir and Naz [6] presented soft topological spaces and defined some concepts of soft sets on this spaces and separation axioms. Moreover, topological structure on fuzzy soft set was defined by Çoker [7], Tanay and Kandemir [8], Varol and Aygün [9]. Turanlı and Es [10] defined compactness in intuitionistic fuzzy soft topological spaces.The concept of neutrosophic set(NS) was first introduced by Smarandache [11,12] which is generalization of classical sets, fuzzy set, intuitionistic fuzzy set etc.The concept of connectedness and compactness on neutrosophic soft topological space defined by Bera and Mahapatra [13].

2. PRELİMİNARİES

Hereafter, we recall some necessary definitions and theorems related to neutrosophic soft set, neutrosophic soft topological space for the sake of completeness.

Definition 2.1.[11] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A is characterized by a truth-member function TA, an indeterminacy-membership function IA , and a falsity-membership function FA . TA(x), IA(x) and FA(x) are real Standard or non Standard subsets of]-0,1+[.That is TA, IA , FA :X →]-0,1+[. There is no restriction on the sum of TA(x), IA(x) , FA(x) and so , -0≤ sup TA(x)+sup IA(x)+ FA(x)≤3+ .

Definition 2.2. [4] Let U be an initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Then for A E, a pair (F,A) is called a soft set over U, where F:A→ P(U) is a mapping.

Definition 2.3. [5] Let U be an initial universe set and E be a set of parameters. Let NS(U) denote the set of neutrosophic sets (NSs) of U. Then for A E, a pair (F,A) is called a neutrosophic soft set (NSS) over U, where F:A→ NS(U) is a mapping.

Definition 2.4. [14] Let U be an initial universe set and E be a set of parameters. Let NS(U) denote the set of neutrosophic sets (NSs) of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function fNrepresenting a mapping fN:E→ NS(U) where fN is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parametrized family of some elements of the set NS(U) and therefore it can be written as a set of ordered pairs, N={(e,{<x,TfN(e)(x), IfN(e)(x), FfN(e)(x)>:xϵU}):eϵE} where TfN(e)(x), IfN(e)(x), FfN(e)(x) ϵ[0,1], respectively the truth-membership, indeterminacy-membership , falsity-membership function obvios.

Example2.5.[15] Let U={h1,h2,h3} be a set of houses and E={e1(beautiful), e2(wooden), e3(costly)} be a set of parameters with respect to which the nature of houses are described.

Let
fN(e1)={<h1,(0.5,0.6,0.3)>, <h2,(0.4,0.7,0.6)>, <h3,(0.6,0.2,0.3)>};
fN(e2)={<h1,(0.6,0.3,0.5)>, <h2,(0.7,0.4,0.3)>, <h3,(0.8,0.1,0.2)>};
fN(e3)={<h1,(0.7,0.4,0.3)>, <h2,(0.6,0.7,0.2)>, <h3,(0.7,0.2,0.5)>};

Then N={[e1, fN(e1)], [e2, fN(e2)], [e3, fN(e3)] }is an NSS over (U,E).

Definition 2.6. [14]
1.The complement of a neutrosophic soft set N is denoted by Nc and is defined by Nc={(e,{<x,FfN(e)(x),1- IfN(e)(x), TfN(e)(x)>:xϵU}):eϵE},
2. Let N1 and N2 be two NSSs over the common universe (U,E). Then N1 is said to be the neutrosophic soft subset of N2 iffor each eϵE and for each xϵU, TfN1(e)(x)≤ TfN2(e)(x), IfN1(e)(x)≥ IfN2(e)(x), FfN1(e)(x)≥ FfN2(e)(x).

We write N1 N2 and then N2 is the neutrosophic soft superset of N1.

Definition 2.7.[14]
1.Let N1 and N2 be two NSSs over the common universe (U,E). Then their union is denoted by N1 ᴗ N2=N3 and is defined as:
N3={(e,{<x,TfN3(e)(x), IfN3(e)(x), FFN3(e)(x)>:xϵU}):eϵE} where TfN3(e)(x)= TfN1(e)(x)◊ TfN2(e)(x), IfN3(e)(x)= IfN1(e)(x)* IfN2(e)(x), FfN3(e)(x)= FfN1(e)(x)*FfN2(e)(x).

2.Their intersection is denoted by N1 ∩ N2=N4 and is defined as:
N4={(e,{<x,TfN4(e)(x), IfN4(e)(x), FFN4(e)(x)>:xϵU}):eϵE} where TfN4(e)(x)= TfN1(e)(x)*TfN2(e)(x), IfN4(e)(x)= IfN1(e)(x)◊ IfN2(e)(x), FfN4(e)(x)= FfN1(e)(x)◊FfN2(e)(x).

Definition 2.8. [13]
1. Let M and N be two NSSs over the common universe (U,E). Then M-N may be defined as, for each eϵE and for each xϵU,
M-N={<x,TfM(e)(x)*FfN (e)(x), IfM(e)(x)◊(1- IfN (e)(x)), FfM(e)(x) ◊TfN(e)(x) > };

2. A neutrosophic soft set N over (U,E) is said to be null neutrosophic soft set if TfN(e)(x)=0, IfN(e)(x)=1, FfN(e)(x)=1 for each eϵE and for each xϵU. It is denoted by ɸu.

A neutrosophic soft set N over (U,E) is said to be absolute neutrosophic soft set if Tf N(e)(x)=1, IfN(e)(x)=0, Ff N(e)(x)=0 for each e∈E and for each x∈U.It is denoted by 1u. Clearly, ϕuc =1u, 1uc =ϕu.

Definition 2.9. [13] Let NSS(U,E) be the family of all neutrosophic soft sets over U via parameters in E and τu   NSS(U,E). Then τu is called neutrosophic soft topology on (U,E) if the following conditions are satisfied.
(i) ϕu, 1u ∈ τu ,
(ii) The intersection of any finite number of members of τu also belongs to τu.
(iii) The union of any collection of members of τu belongs to τu.

Then the triple (U,E, τu) is called a neutrosophic soft topological space. Every member of τu is called τu-open neutrosophic soft set. An NSS is called τu-closed iffit's complement is τu-open.

Definition 2.10. [13] Let (U,E, τu) be a neutrosophic soft topological space over (U,E) and M∈NSS(U,E) be arbitrary. Then the interior of M is denoted by Mo or int(M) and is defined as:
Mo= {N1: N1 is neutrosophic soft open and N1   M}.

Definition 2.11.[13] Let (U,E, τu) be a neutrosophic soft topological space over (U,E) and A ∈ NSS(U,E) be arbitrary. Then the closure of A is denoted by Ā or cl(A) and is defined as:
Ā=∩{N1: N1 is neutrosophic soft closed and A   N1}.

Theorem 2.12. [13] Let (U,E, τu) be a neutrosophic soft topological space over (U,E) and A ∈ NSS(U,E). Then , (Ā)c=(Ac)o and (Ao)c=$\overline{(Ac)}$.

Proposition 2.13. [13] Let N1 and N2 be two neutrosophic soft sets over (U,E). Then,
(i) (N1∪N2)c =N1c∩N2c ,
(ii) (N1∩ N2)c =N1c∪N2c .

Definition 2.14. [13] Let (U,E, τu) be a neutrosophic soft topological space and M∈ τu. A family Ω={Qi :i ∈Γ} of neutrosophic soft sets is said to be a cover of M if M   Qi.

If every member of that family which covers M is neutrosophic soft open then it is called open cover of M. A subfamily of Ω which also covers M is called a subcover of M.

Definition 2.15. [13] Let (U,E, τu)be a neutrosophic soft topological space and M∈ τu.Suppose Ω be an open cover of M. If Ω has a finite subcover which also covers M then M is called neutrosophic soft compact.

Definition 2.16. [13] Let φ : U→ V and ψ :E→E be two functions where E is the parameter set each of the crisp sets U and V. Then the pair (φ, ψ) is called an NSS function from (U, E) to (V, E). We write, (φ, ψ) : (U,E) → (V,E).

Definition 2.17. [13] Let (M,E) and (N,E) be two NSSs defined over U and V, respectively and (φ, ψ) be an NSS function from (U,E) to (V,E). Then,
(1) The image of (M,E) under (φ, ψ), denoted by (φ, ψ) (M,E), is an NSS over V and is defined as:
(φ, ψ) (M,E)=(φ(M),ψ(E))={ <ψ(a),fφ(M)(ψ(a))>:a∈E} where for each b∈ ψ(E) and y∈V.

$\max\varphi(x)=y \max \psi (a)=b[Tf(M)(a)(x)]$, if $x\epsilon\varphi\text{-}1(y)$,
$T\varphi(M)(b)(y)=\{ 0$, otherwise.
$\min\varphi(x)=y \min \psi (a)=b[If(M)(a)(x)]$, if $x\epsilon\varphi\text{-}1(y)$, $I\varphi(M)(b)(y)=\{ 1$,
otherwise.
$\min\varphi(x)=y \min \psi (a)=b[Ff(M)(a)(x)]$, if $x\epsilon\varphi\text{-}1(y)$,
$F\varphi(M)(b)(y)=\{ 1$, otherwise.

(2) The pre-image of (N,E) under (φ, ψ), denoted by (φ, ψ)-1 (N,E), is an NSS over U and is defined by:
(φ, ψ)-1 (N,E)=(φ-1(N),ψ-1(E)) where for each aϵ ψ-1(E) and xϵU.
$T\varphi\text{-}1(N)(a)(x)=TfN(\psi(a))(\varphi(x))$,
$I\varphi\text{-}1(N)(a)(x)=IfN(\psi(a))(\varphi(x))$,
$F\varphi\text{-}1(N)(a)(x)=FfN(\psi(a))(\varphi(x))$.
If ψ and φ are injective(surjective), then (φ, ψ) is injective(surjective).

Definition 2.18. [13] Let (U,E,τu) and (V,E,τv) be two neutrosophic soft topological spaces. (φ, ψ) : (U,E,τu) → (V,E,τv) is said to be a neutrosophic soft continuous mapping if for each (N,E)ϵ τv, theinverse image(φ, ψ)-1 (N,E)ϵ τu i.e., the inverse image of each open NSS in (V,E,τv) is also open in (U,E,τu).

Theorem 2.19. [13] Let (U,E,τu) and (V,E,τv) be two neutrosophic soft topological spaces. Also let, (φ, ψ) : (U,E,τu) → (V,E,τv) be a neutrosophic soft continuous mapping. If (M,E) is neutrosophic soft compact in (U,E,τu), then (φ, ψ)(M,E) is so in (V,E,τv).

3. NEUTROSOPHIC SOFT ALMOST COMPACTNESS AND NEUTROSOPHIC SOFT NEAR COMPACTNESS

Here, the Notion of almost compactness and near compactness on neutrosophic soft topological space is developed with some basic theorems.

Definition 3.1.
(a) A neutrosophic soft topological space (U,E,τu) is called neutrosophic soft almost compact iff every open cover of (U,E,τu) has a finite subcollection whose closures cover (U,E,τu), or equivalently, every open cover contains a finite subcollection whose closures form a cover of (U,E,τu).
(b) A neutrosophic soft topological space (U,E,τu) is called neutrosophic soft nearly compact iff every open cover of (U,E,τu) has a finite subcollection such that the interiors of closures of neutrosophic soft sets in this subcollection covers (U,E,τu).

Example 3.2. Let U={h1,h2}, E={e1,e2} and τu={ϕu, 1u, N1, N2,N3,N4}, where N1, N2,N3,N4 beingneutrosophic soft sets are defined as following:
fN1(e1)={<h1,(1,0,1)>,<h2,(0,0,1)>};
fN1(e2)={<h1,(0,1,0)>,<h2,(1,0,0)>};
fN2(e1)={<h1,(0,1,0)>,<h2,(1,1,0)>};
fN2(e2)={<h1,(1,0,1)>,<h2,(0,1,1)>};
fN3(e1)={<h1,(1,1,1)>,<h2,(0,1,1)>};
fN3(e2)={<h1,(0,1,0)>,<h2,(0,1,1)>};
fN4(e1)={<h1,(1,1,0)>,<h2,(1,1,0)>};
fN4(e2)={<h1,(1,0,0)>,<h2,(0,1,1)>};

Here N1∩ N1= N1,N1∩ N2= ϕu, N1∩ N3= N3, N1∩ N4= N3, N2∩ N2= N2, N2∩ N3= ϕu, N2∩ N4= N2, N3∩ N3= N3, N3∩ N4= N3, N2∩ N4= N4, and N1∪ N1= N1, N1∪ N2= ϕu, N1∪ N3= N1, N1∪N4= 1u, N2∪ N2= N2, N2∪ N3= N4, N2∪N4= N4, N3∪N3= N3, N3∪N4= N4, N4∪N4= N4;

Corresponding t-norm and s-norm are defined as a*b=max{a+b-1,0} and a◊b=min{a+b,1}. Then τu is a neutrosophic soft topology on (U,E) and so (U,E,τu) is a neutrosophic soft topological space over (U,E) [13].

The family {N1, N2,N3,N4} is an open cover of (U,E,τu). Since cl(N1∪N2)=cl(N1∪N2)=1u, (U,E,τu) is neutrosophic soft almost compact topological space. Also,sinceint(cl(N1∪N2))=int(cl(N1∪N2))=1u, (U,E,τu) is neutrosophic soft nearly compact topological space.

It is clear that in neutrosophic soft topological spaces we have the following implications: neutrosophic soft compact→ neutrosophic soft nearly compact→ neutrosophic soft almost compact.

Theorem 3.3. A neutrosophic soft topological space (U,E,τu) is called neutrosophic soft almost compact iff each family Ω={Qi :i ϵI} of neutrosophic soft open sets in (U,E,τu) having the finite intersection property we have ∩iϵI cl(Qi)≠ ϕu.

Proof .Let (U,E,τu) be an almost compact neutrosophic soft topological space. Consider Ω={Qi :i ϵI} be a family of neutrosophic soft open sets in (U,E,τu) having the finite intersection property. Suppose the ∩iϵI cl(Qi)= ϕu.Then we have iϵI [cl(Qi)]c=∩iϵIint(Qic)= 1u. Since (U,E,τu) almost compact neutrosophic soft topological space, there exists a finite subfamily {Qic :i = 1,2,…,n} such that n cl(int(Q c))= 1. Hence n cl([(Q)]c)= n [int(cl (Q))]c= 1 =>∩ n int(cl(Q))= ϕ .But from Q =int(Q) int(cl(Qi)), we see that ∩ n Q = ϕ which in contradiction with the finite intersection property of the family.

Next assume that (U,E,τu) is not almost compact. Then, a neutrosophic soft open cover of {Qi :i ϵI}, say,of (U,E,τu) has no finite subcover i.e., in=1 cl (Qi)≠ 1u. Since [cl(Qi)]c=int(Qic), consists of neutrosophic soft open sets in (U,E,τu) and having the finite intersection property. Then by hypothesis, ∩i=n1cl([cl(Qi)]c) ≠ ϕu => in=1 [cl([cl
(Qi)]c)]c≠ 1u => n int(cl (Qi))≠ 1u which is in contradiction with n
Qi = 1u since Qi int(cl(Qi)) for each i=1,2,…,n.

Definition 3.4. A neutrosophic soft set N1 is called a neutrosophic soft regular open set iff N1=int(cl(N1)); a neutrosophic soft set N2 is called a neutrosophic soft regular closed set iff N2= cl(int(N2)).

Theorem 3.5.In a neutrosophic soft topological space (U,E,τu) the following conditions are equivalent:

(I) (U,E,τu) is neutrosophic soft almost compact.

(ii) For each family Ω={Qi :i ϵI} of neutrosophic soft regular closed sets such that ∩iϵI Qi= ϕu, there exists a finite subfamilyΩ1={Qi :i =1,2,…,n} such that ∩ n Qi = ϕu.

(iii) ∩iϵIcl(Qi)≠ ϕu holds for each family Ω={Qi :i ϵI} of neutrosophic soft regular open sets having the finite intersection property.

(iv) Each neutrosophic soft regular opencover of (U,E,τu) contains a finite subfamily whose closures cover (U,E,τu).

Proof. The prof of this theorem follows a similar pattern to Theorem 3.3.

Definition 3.6. Let (U,E,τu) and (V,E,τv) be two neutrosophic soft topological spaces. Then (φ, ψ) : (U,E,τu) → (V,E,τv) is said to be a neutrosophic soft almost continuous mapping if for each (N,E) neutrosophic soft regular open set of (V,E,τv) , theinverse image(φ, ψ)-1 (N,E)ϵ τu i.e., the inverse image of each neutrosophic soft regular open set in (V,E,τv) is neutrosophic soft open in (U,E,τu).

Theorem 3.7. Let (U,E,τu) and (V,E,τv) be two neutrosophic soft topological spaces and (φ, ψ) : (U,E,τu) → (V,E,τv) a neutrosophic soft almost continuous surjection mapping. If (M,E) is neutrosophic soft almost compact in (U,E,τu), then (φ, ψ) (M,E) is so in (V,E,τv).

Proof. Let {(Ni,E) : i ϵI} be a neutrosophic soft open cover of (φ, ψ) (M,E) i.e., (φ, ψ) (M,E) iεı(Ni,E). Since (φ, ψ) is neutrosophic soft almost continuous, {(φ, ψ)-1int(cl((Ni,E))): iϵI} is a neutrosophic soft open cover of (M,E) . Since (M,E) is almost compact, there exists a finite subcover {(φ, ψ)-1(Ni,E): i=1,2,…,n} such that (M,E) n cl(((φ, ψ)-1(int(cl(N ,E))))=1 Hence (φ, ψ) (M,E) (φ, ψ)[n cl((φ, ψ)-1(int(cl(N ,E))))]= n (φ, ψ)[cl(φ, ψ)-1(int(cl(N ,E))))]=f(1)=1 . But from int(cl(N ,E)) cl(N ,E) and from the neutrosophic soft almost continuity of f,

(φ, ψ)(cl((φ, ψ)-1int(cl((Ni,E)))) (φ, ψ)((φ, ψ)-1 cl((Ni,E)))) cl (Ni,E) for each i=1,2,…,n, i.e., n cl (N ,E)=1 . Hence, (φ, ψ) (M,E) is neutrosophic soft almost compact also.

Definition 3.8. Let (U,E,τu) and (V,E,τv) be two neutrosophic soft topological spaces. Then (φ, ψ) : (U,E,τu) → (V,E,τv) is said to be a neutrosophic soft weakly continuous mapping if for each (N,E) neutrosophic soft open set of (V,E,τv) ,
(φ, ψ)-1 (N,E) int ((φ, ψ)-1(cl(N,E))).

Theorem 3.9. Let (U,E,τu) and (V,E,τv) be two neutrosophic soft topological spaces and (φ, ψ) : (U,E,τu) → (V,E,τv) a neutrosophic soft weakly continuous surjection mapping. If (M,E) is neutrosophic soft compact in (U,E,τu), then (φ, ψ) (M,E) is neutrosophic soft almost compact in (V,E,τv).

Proof. The proof is similar to Theorem 3.7.

Definition 3.10. Let (U,E,τu) and (V,E,τv) be two neutrosophic soft topological spaces. Then (φ, ψ) : (U,E,τu) → (V,E,τv) is said to be a neutrosophic soft strongly continuous mapping if for each (M,E) neutrosophic soft set of (V,E,τv) , (φ, ψ)[cl(M,E)] (φ, ψ) (M,E).

Theorem 3.9. Let (U,E,τu) and (V,E,τv) be two neutrosophic soft topological spaces and (φ, ψ) : (U,E,τu) → (V,E,τv) a neutrosophic soft strongly continuous surjection mapping. If (M,E) is neutrosophic soft almost compact in (U,E,τu), then (φ, ψ) (M,E) is neutrosophic soft compact in (V,E,τv).

Proof. By using a similar technique of the proof of Theorem 3.7, the theorem holds.

Corollary 3.12. Let (U,E,τu) and (V,E,τv) be two neutrosophic soft topological spaces and (φ, ψ) : (U,E,τu) → (V,E,τv) a neutrosophic soft strongly continuous surjection mapping. If (M,E) is neutrosophic soft nearly compact in (U,E,τu), then (φ, ψ) (M,E) is neutrosophic soft compact in (V,E,τv).

4. CONCLUSION

In this paper, the concepts of Neutrosophic soft topological spaces are introduced and studied. Some interesting properties are al so established. The results in this work can be extended to the Neutrosophic connectedness properties.

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Anti-magic Labeling for Boolean Graph of Cycle B(C_n) (n>4)

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ABSTRACT

A graph G is anti-magic if there is a labeling of its edges with 1, 2, . . . , |E| such that the sum of the labels assigned to edges incident to distinct vertices are different. A conjecture of Hartsfield and Ringel states that every connected graph different from K2 is anti-magic. Our main result validates this conjecture for Boolean graph of cycleCn(n>4) is anti-magic.

Keywords : Boolean graph BG(G), Anti-magic Labeling.

INTRODUCTION:

Suppose $G = (V, E)$ is a graph. For each vertex v of G denoted by $E_G\,(V)$, the set of edge of G incident to . We shall write (V) for $E_G\,(V)$ Let $f : E \rightarrow \{1, 2, \ldots, |E|\}$ be a bijective mapping. The vertex-sum $\varphi_f\,(v)$ at v is defined as $\varphi_f\; v \; = {}_{e\in(v)}\; f(e)$.For any two distinct vertices u, v of G,$\varphi_f\,(v) \neq \varphi_f\,(u)$ gives an anti-magic labeling of G. A graph G is called anti- magic if G has an anti-magic labeling. The problem of anti-magic labeling of graphs was introduced by Hartsfield and Ringel [4]. They conjectured that all graphs with no single edge component are anti-magic. Graph Labeling has many applications in coding theory, X - ray crystallography, radar, astronomy, circuit design, communication network addressing, and data base management .

Conjecture 1: [4]Every connected graph different from K_2 is anti-magic.

This conjecture is still open. Interestingly, the graph K2 can be regarded as a tree on two vertices. Thus, if we restrict ourselves to trees, the above conjecture holds. Hartsfield and Ringel proved that paths, cycles and complete graph Kn, (n>3) are anti-magic. Recently, Alon et al. [1] have proved that the conjecture is true for some classes of dense graphs. They have shown that all dense graphs with (n>4) vertices and minimum degree $\Omega(\log n)$ are anti-magic. They also proved that if G is a graph $(n \geq 4)$ vertices and the maximum degree $\Delta(G) \geq 4n - 2$, then G is anti-magic and all complete bipartite graphs except K_2 are anti-magic. Anti-magic labeling of the Cartesian product of graphs was studied in [7]; if G is a regular anti-magic graph then for any graph H, the Cartesian product H X G is anti-magic. It was proved in [4] that 2- regular graphs are anti-magic and proved in [6] that 3-regular graphs are anti-magic. As a consequence, if G is 2-regular or 3-regular then for any graph H, H X G is anti-magic. In this paper, we extend anti-magic labeling to Boolean Graph of cycle.

Definition 1: Boolean graph BG (G) is a graph with vertex set $V(G) \cup E(G)$ and two vertices in B(G) are adjacent if and only if they correspond to two adjacent vertices of G or to a vertex and non - incident edge of G.

Theorem 1: The Boolean graph of cycle$B(C_n)$, $(n \geq 4)$ is anti-magic.

Proof: Let C be a cycle with the vertices $v_1, v_2, v_3, \dots, v_n$. By the definition of Boolean graph $B(C_n)$ the vertex set is given by

$$V\ BG\ C_n = v\ ; 1 \leq i \leq n \cup \{u_j\ ; 1 \leq j \leq n\}$$

and the edge set is given by

$$E\ BG\ C_n = v_i\ v_{i+1};\ 1 \leq i \leq n-1 \cup \{u_j\ u_{j+1}\ ; 1 \leq j \leq n-1\}$$

We discuss Boolean graph of cycl in two cases.

Case (a): $n \equiv 0 \pmod 2$

Label the vertices of $B(C_n)$using the functionf : $E \to N$ as follows: $f(v_i$

$v_{i+1}) = i$; $i = 1, 2, \dots, n-1$ & $f(v_1, v_n) = n$

$f(u_j\ u_{j+1}) = n + j$; $j = 1, 2, \dots, n-1$ & $f(u_1, u_n) = 2n$

$f(v_i\ u_j) = (n-2)(i+1) + j + 3$ if $i < j$

and $f(v_i u_j) = (n-1)(i-1) + (n-2)j + 3$ if $i > j$

The induced function $f^* : V \to N$, such that $f^*(v) = \sum_{u \in nbd(v)} f(v_i u_j)$

We consider the when labels of vertices are distinct.

Subcase (i): when $i = 1$ where $i < j$.

$$f^*(v_i) = f(v_i\ v_{i+1}) + f(v_i v_n) + \sum_{j=2}^{n-1} f(v_i u_j)$$

$$f^*(v_1) = f(v_1\ v_2) + f(v_1 v_n) + \sum_{j=2}^{n-1} [(n-2)(i+1) + j + 3]$$

$$f^*(v_1) = 1 + n + (n-2)(1+1)(n-2) + 3(n-2) + \frac{n(n-1)}{2} - 1$$

$$= 1 + n + 2(n-2)^2 + 3(n-2) + \frac{n(n-1)}{2} - 1$$

$$= \frac{1}{2}[2n + 2\,(2n^2 - n - 4n + 2) + n^2 - n]$$

$$f^*(v_1) = \frac{1}{2}\;[5n^2 - 9n + 4]$$

Subcase (ii): When i = 2where i< j

$$f^*(v_i) = \sum_{i=1}^{2} f(v_i v_{i+1}) + \sum_{j=3}^{n} f(v_i u_j)$$

$$= f(v_1\, v_2) + f(v_2\, v_3) + \sum_{j=3}^{n}[(n-2)(i+1) + j + 3]$$

$$= 1 + 2 + (n-2).\,(n-2)\,(i+1) + 3\,(n-2) + \frac{n(n+1)}{2} - 3$$

$$f^*(v_2) = 3(n-2)^2 + 3\,(n-2) + \frac{n^2+n}{2}$$

$$= \frac{1}{2}[7n^2 - 17n + 12]$$

Sub case (iii): When i = 3, 4, …, n–1

$$f^*(v_i) = f(v_{i-1}\, v_i) + f(v_i\, v_{i+1}) + \sum_{\substack{j=1 \\ j\neq i-1,i}}^{n} f(v_i u_j)$$

$$= (i-1) + i + \sum_{\substack{j=1 \\ i>j}}^{i-2} f(v_i u_j) + \sum_{\substack{j=i+1 \\ i<j}}^{n} f(v_i u_j)$$

$$= 2i-1 + \sum_{j=1}^{i-2}[(n-1)(i-1) + (n-2)j + 3] + \sum_{j=i+1}^{n}[(n-2)(i+1) + j + 3]$$

$$= 2i-1 + (n-1)\,(i-1)\,(i-2) + 3(i-2) + (n-2)\,\frac{(i-2)(i-1)}{2}$$

$$+ (n-i)(n-2)(i+1) + 3(n-i) + \frac{n(n+1)}{2} - \frac{i(i+1)}{2}$$

$$f^*(v_i) = \frac{1}{2}[(n-1)i^2 + (2n^2 - 15n + 19)i + (3n^2 + 9n - 22)]$$

Sub case (iv): When i = n

$$f^*(v_n) = f(v_1 v_n) + f(v_{n-1} v_n) + \sum_{\substack{j=1 \\ i>j}}^{n-2} f(v_i u_j)$$

$$= n + (n-1) + \sum_{j=1}^{n-2} [(n-1)(i-1) + (n-2)j + 3]$$

$$= 2n - 1 + (n-1)(n-2)(i-1) + 3(n-2) + (n-2), \frac{(n-2)(n-1)}{2}$$

$$= \frac{1}{2}[(2n^2 - 6n + 4)i + n^3 - 7n^2 + 24n - 22]$$

We consider the case when labels of edges are distinct.

Subcase (v): When j = 1 where i>j

$$f^*(u_j) = f(u_j u_{j+1}) + f(u_j u_n) + \sum_{i=j+2}^{n} f(v_i u_j)$$

$$= (n+j) + 2n + \sum_{i=j+2}^{n} [(n-1)(i-1) + (n-2)j + 3]$$

$$= 3n + j + (n-1)\left[\frac{n(n+1)}{2} - \frac{(j+1)(j+2)}{2}\right] + [(n-2)j - n + 4](n-j-1)$$

$$= \frac{1}{2}[6n + 2j + (n-1)(n^2 + n - j^2 - 3j - 2) + 2(n - j - 1)(nj - 2j - n + 4)]$$

$$= \frac{1}{2}[(5-3n)j^2 + (2n^2 - 7n + 1)j + n^3 - 2n^2 + 13n - 6]$$

Subcase (vi): When j = 2, 3, …, n–2

$$f^*(u_j) = f(u_{j-1}u_j) + f(u_j\, u_{j+1}) + \sum_{\substack{i=1 \\ i\neq j,j+1}}^{n} f(v_i u_j)$$

$$= f(u_{j-1}u_j) + f(u_j\, u_{j+1}) + \sum_{\substack{i=1 \\ i<j}}^{j-1} f(v_i u_j) + \sum_{\substack{i=j+2 \\ i>j}}^{n} f(v_i u_j)$$

$$= (n + j-1) + (n+j) + \sum_{i=1}^{j-1}[(n-2)(i+1) + j + 3] + \sum_{i=j+2}^{n}[(n-1)(i-1) + (n-2)j + 3]$$

$$= 2n + 2j - 1 + (n-2)\frac{(j-1)j}{2} + (n+j+1)(j-1) + (n-1)\left[\frac{n(n+1)}{2} - \frac{(j+1)(j+2)}{2}\right]$$

$$+[(n-2)j - n + 4](n-j-1)$$

$$= \frac{1}{2}[(4n + 4j - 2) + (n-2)(j-1)j + 2(j-1)(n + j + 1) +$$

$$(n-1)[(n(n+1) - (j+1)(j+2)] + 2(n-j-1)[(n-2)j - n + 4]$$

$$= \frac{1}{2}[(5 - 2n)j^2 + (2n^2 - 6n + 5)j + n^3 - 2n^2 + 9n - 10]$$

Subcase (vii): When $j = n-1$ where $i<j$

$$f^*(u_j) = f(u_{j-1}u_j) + f(u_j\, u_{j+1}) + \sum_{\substack{i=1 \\ i<j}}^{j-1} f(v_i u_j)$$

$$= (2n - 2) + (2n-1) + \sum_{i=1}[(n-2)(i+1) + j + 3]$$

$$= 4n-3 + (n-2) + \frac{(j-1)j}{2} + (n+1+j)(j-1)$$

$$= \frac{1}{2}[nj^2 + (n+2)j + 6n - 8]$$

Subcase (viii): When $j = n$ where $i<j$

$$f^*(u_j) = f(u_{j-1}u_j) + f(u_1 u_j) + \sum_{\substack{i=2 \\ i<j}}^{j-1} f(v_i u_j)$$

$$f^*(u_j) = (n+j-1) + 2n + \sum_{i=2}^{j-1}[(n-2)(i+1) + j + 3]$$

$$= 3n + j-1 + (n-2)\left\lceil \frac{(j-1)j}{2} - 1 \right\rceil + (n+1+j)(j-2)$$

$$f^*(u_j) = \frac{1}{2}[nj^2 + (n+2)j - 2]$$

Case (b): $n \equiv 1 \pmod 2$

Let us label the vertices of $B(C_n)$ using the function $f : E \to N$ as follows: $f(v_i v_{i+1}) = 2i-1$, $i = 1, 2, \ldots, n-1$

$f(v_1 v_n) = 2n - 1$

$f(u_j u_{j+1}) = 2j$; $j = 1, 2, \ldots, n-1$

$f(u_1 u_n) = 2n$

$f(v_i u_j) = (n-2)(i+1) + j + 3$ if $i < j$

and $f(v_i u_j) = (n-1)(i-1) + (n-2)j + 3$ for $i > j$

The induced function $f^* : V \to N$ such that $f^*(v) = \sum_{u \in nbd(v)} f(v_i u_j)$

We consider the when the labels are distinct.

Subcase (i): When $i = 1$ where $i < j$

$$f^*(v_i) = f(v_i v_{i+1}) + f(v_i v_n) + \sum_{\substack{j=2 \\ i<j}}^{n-1} f(v_i u_j)$$

$$= (2i-1) + (2n-1) + \sum_{j=2}^{n-1}[(n-2)(i+1) + j + 3]$$

$$= (2i-1) + (2n-1) + [(n-2)(i+1) + 3](n-2) + \frac{(n-1).n}{2} - 1$$

$$= \frac{1}{2}[(2n^2 - 8n + 12)i + 3n^2 + n - 10]$$

Subcase (ii): When $i = 2$ where $i < j$

$$f^*(v_i) = f(v_{i-1} v_i) + f(v_i v_{i+1}) + \sum_{\substack{j=3 \\ i<j}}^{n} f(v_i u_j)$$

$$= 2(i-1) - 1 + 2i - 1 + \sum_{j=3}^{n}[(n-2)(i+1) + j + 3]$$

$$= 4i-4 + [(n-2)(i+1) + 3](n-2) + \left\lceil \frac{n(n+1)}{2} - 1 - 2 \right\rceil$$

$$= \frac{1}{2}[8i - 8 + (2n-4)(ni + n - 2i + 1) + (n^2 + n) - 6]$$

$$= \frac{1}{2}[8i - 8 + 2n^2 i + 2n^2 - 4ni + 2n - 4ni - 4n + 8i - 4 + n^2 + n - 6]$$

$$= \frac{1}{2}[(2n^2 - 8n + 16)i + 3n^2 - n - 18]$$

Subcase (iii): When $i = 3, 4, 5, \ldots, n-1$

$$f^*(v_i) = f(v_{i-1} v_i) + f(v_i v_{i+1}) + \sum_{\substack{j=1 \\ j \neq i-1, i}}^{n} f(v_i u_j)$$

$$= 2(i-1) - 1 + 2i - 1 + \sum_{\substack{j=1 \\ i>j}}^{i-2} f(v_i u_j) + \sum_{\substack{j=i+1 \\ i<j}}^{n} f(v_i u_j)$$

$$= 4i - 4 + \sum_{j=1}^{i-2}[(n-1)(i-1) + (n-2)j + 3] + \sum_{j=i+1}^{n}[(n-2)(i+1) + j + 3]$$

$$= 4i - 4 + [(n-1)(i-1)+3](i-2) + (n-2)\frac{(i-2)(i-1)}{2} + [(n-2)(i+1)+3](n-i) + \left\lceil \frac{n(n+1)}{2} - \frac{i(i+1)}{2} \right\rceil$$

$$= \frac{1}{2}[8i - 8 + (2i-4)(ni - n - i + 4) + (n-2)(i^2 - 3i + 2) + (ni + n - 2i + 1)(2n - 2i) + n^2 + n - i^2 - i)$$

$$f^*(v_i) = \frac{1}{2}[(n-1)i^2 + (2n^2 - 15n + 23)i + 3n^2 + 9n - 28].$$

Subcase (iv): When $i = n$

$$f^*(v_i) = f(v_{i-1} v_i) + f(v_1 v_i) + \sum_{\substack{j=1 \\ i>j}}^{n-2} f(v_i u_j)$$

$$= 2(i-1) - 1 + 2n - 1 + \sum_{j=1}^{n-2}[(n-1)(i-1) + (n-2)j + 3]$$

$$= 2i - 3 + 2n - 1 + [(n-1)(i-1) + 3](n-2) + (n-2)\left\lfloor\frac{(n-2)(n-1)}{2}\right\rceil$$

$$= \frac{1}{2}[(4n + 4i - 8) + (2n - 4)[ni - n - i + 4] + (n-2)(n^2 - 3n + 2)$$

$$= \frac{1}{2}[(2n^2 - 6n + 8)i + n^3 - 7n^2 + 24n - 28].$$

We consider the case when the labels of edges are distinct.

Sub case (v): When $j = 1$ where $i > j$.

$$f^*(u_j) = f(u_j u_{j+1}) + f(u_j u_n) + \sum_{\substack{i=j+2 \\ i>j}}^{n} f(v_i u_j)$$

$$= 2j + 2n + \sum_{i=j+2}^{n}[(n-1)(i-1) + (n-2)j + 3]$$

$$= 2j + 2n + (n-1)\left\lfloor\frac{n(n+1)}{2} - \frac{(j+1)(j+2)}{2}\right\rceil + [(n-2)j - n + 4](n-j-1)]$$

$$= \frac{1}{2}[4j + 4n + (n-1)(n^2 + n - j^2 - 3j - 2) + 2(n-j-1)(nj - 2j - n + 4)]$$

$$f^*(u_j) = \frac{1}{2}[(5-3n)j^2 + (2n^2 - 7n + 3)j + n^3 - 2n^2 + 11n - 6]$$

Subcase (vi): When $j = 2, 3, \ldots, n-2$

$$f^*(u_j) = f(u_{j-1}u_j) + f(u_j u_{j+1}) + \sum_{\substack{i=1 \\ i\neq j, j+1}}^{n} f(v_i u_j)$$

$$= f(u_{j-1} u_j) + f(u_j u_{j+1}) + \sum_{\substack{i=1 \\ i<j}}^{j-1} f(v_i u_j) + \sum_{i=j+2}^{n} f(v_i u_j)$$

$$= 2(j-1) + 2j + \sum_{i=1}^{j-1}[(n-2)(i+1) + j + 3] + \sum_{i=j+2}^{n}[(n-1)(i-1) + (n-2)j + 3]$$

$$= 2j - 2 + 2j + (n-2)\frac{(j-1)j}{2} + (n+j+1)(j-1) + (n-1)\left\lfloor\frac{n(n+1)}{2} - \frac{(j+1)(j+2)}{2}\right\rceil + [(n-2)j - n + 4](n-j-1)$$

$$= \frac{1}{2}\ [8j - 4 + (n-2)\ (j^2 - j) + 2\ (j-1)\ (n+j+1) + (n-1)\ (n^2 + n - j^2 - 3j - 2) + 2\ (n-j-1)\ (nj - 2j - n + 4]$$

$$= \frac{1}{2}[(5 - 2n)j^2 + (2n^2 - 6n + 9)j + n^3 - 2n^2 + 5n - 12].$$

Subcase (vii): When $j = n-1$ where $i<j$

$$f^*\ (u_j) = f(u_{j-1}u_j) + f\ (u_j\ u_{j+1}) + \sum_{\substack{i=1 \\ i<j}}^{j-1} f(v_i u_j)$$

$$= 2(j-1) + 2j + \sum_{i=1}^{j-1}[(n-2)(i+1) + j + 3]$$

$$= 4j - 2 + (n-2)\ \frac{(j-1)j}{2} + (n+1+j)\ (j-1)$$

$$= \frac{1}{2}[8j - 4 + (n - 2)\ j\ (j-1) + 2\ (j-1)\ (n+1+j)]$$

$$= \frac{1}{2}[nj^2 + (n+10)j - 2n - 6].$$

Subcase (viii): When $j = n$ where $i<j$

$$f^*\ (u_j) = f(u_{j-1}u_j) + f\ (u_1u_j) + \sum_{\substack{i=2 \\ i<j}}^{j-1} f(v_i u_j)$$

$$= 2(j-1) + 2n + \sum_{i=2}^{j-1}[(n-2)(i+1) + j + 3]$$

$$= 2n + 2j - 2 + (n-2)\ \frac{(j-1)j}{2} - 1 + (n+j+1)\ (j-2)$$

$$= \frac{1}{2}[4n + 4j - 4 + (n-2)\ (j^2 - j - 2) + 2\ (j-2)\ (n+j+1)$$

$$= \frac{1}{2}[nj^2 + (n+4)j - 2n - 4]$$

As a whole the labeling of all the vertices and the edges of the Boolean graph of cycle is anti- magic.

$$\therefore B(C_n)\text{is anti-magic.}$$

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Applications of M -Dimensional Flexible Fuzzy Soft Algebraic Structures

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A B S T R A C T

In this paper, we introduce the concept of m-dimensional structures on flexible fuzzy soft subgroups, and investigate some of its properties. we also obtain the characterisations of normal flexible fuzzy soft subgroup with illustrative examples.

Keywords: Soft set, relation, fuzzy soft set, pre-image, flexible fuzzy subset, m-level subset, m-dimensional flexible subgroup, cosset.

Section-1: INTRODUCTION:

In classical mathematics, the notion of exact solution of a mathematical model is defined. However, in general, mathematical models are quite complicated and it becomes an arduous task to define exact solution of these models. As a result, the notion of approximate solutions is introduced. Such introduction included the emergence of soft set theory where an approximate description of the object is provided. In fact, in soft set theory, there is no restriction on the parameterization tools which makes it very convenient and easily applicable in real life. Thus, soft set approach has come to be recognised as fundamentally important. Aktas and Cagman [1] introduced the basic conceptsof soft groups, soft subgroups, normal soft subgroups and soft homomorphism and discussed their basic properties. Jun [4] also in another paper, introduced the notion of soft p- ideals , p- idealistic soft BCI- algebras and discussed their basic properties. The algebraic structures of soft set theory have also been studied extensively. Feng et.al [2, 3] considered the algebraic structure of semi ring and introduced the notion of soft semi ring. Some basic algebraic properties of soft semi ring and some related notions such as soft ideals, idealistic soft semi rings and soft semi ring homomorphism were defined and investigated with illustrative examples .Jun [5] applied the notion of soft sets to the theory of BCK/BCI- algebras and introduced the notion of soft BCK/ BCI- algebras, soft sub algebras and then derived their basic properties. It was proved that soft equality relation is a congruence relation with respect to some operations. The notions of soft sub rings, soft ideal of a soft ring, idealistic soft rings and soft ring homomorphism were introduced with some corresponding example. Atagun and Sezgin [13] introduced and studied some sub structures such as soft sub rings and soft ideals of a ring, soft subfield of a field and soft sub module of a module with several illustrative examples. Complex intuitionstic flexible fuzzy soft interior ideals and M-structures defined various algebraic structure in [15,16] . By introducing the concept of normalistic soft group, normalistic soft group homomorphism, and establishing that the normalistic soft group isomorphism is an equivalence relation on normalistic soft groups which defined in [1]. On flexible fuzzy subgroups with flexible fuzzy order discussed by [14-16] .Maji et.al [8,9,10] introduced the notion of fuzzy soft sets. In 2011, Neog and Sut [12] put forward some propositions regarding fuzzy soft set theory. In this paper, we introduce the concept of m- dimensional structures on flexible fuzzy soft subgroup, and investigate some of its properties. we also obtain the various structures of flexible fuzzy soft subgroup with illustrative examples.

SECTION-2 PRELIMINARIES:

We review basic definitions that we are necessary for this paper.

Definition 2.1:[18]: A fuzzy set μ in a universe X is a mapping μ : X →[0,1]. Definition 2.2:[9] Let U be any Universal set, E set of parameters and AÍ E. Then a pair (K,A) is called soft set over U, where K is a mapping from A to 2U , the power set of U.

Example 2.3: Let X={c1,c2,c3} be the set of three cars and E = {costly(e1), metallic colour(e2), cheap(e3)}be the set of parameters, where A={e1,e2} Ì E. Then (K,A)={K(e1)={c1,c2,c3},K(e2)={c 1,c2} } is the crisp soft set over X.

Definition 2.4:[11]: Let U be an initial universe. Let P (U) be the power set of U, E be the set of all parameters and A⊆ E. A soft set (f_A, E) on the universe U is defined by the set of order pairs (f_A, E) = {(e, f_A (e)): e ∈ E, f_A∈ P(U)} where f_A : E → P(U) such that f_A (e) = ϕ if e ∉ A. Heref_A is called an approximate function of the soft set.

Example 2.5: Let U = {$u_{1,2}$,u_3,u_4} be a set of four shirts and E = {white(e_1),red(e_2),blue (e_3)} be a set of parameters. If A = {e_{1} $_{,2}$} ⊆ E. Let f(e_1) = {u_1, u_2 ,u_3, u_4} and f_A(e_2)= {$u_{1,2}$,u_3}. Then we write the soft set (f , E) = {(e_1, {u_1, u_2 ,u_3, u_4}), (e_2,{ u_1,u_2,u_3})} over U which describe the "colour of the shirts" which Mr. X is going to buy. We may represent the soft set in the following form: U = { (e_1,u_1), (e_2,u_1), (e_1, u_2), (e_2,u_2), (e_1,u_3), (e_2, u_3), (e_1, u_4)}

Definition 2.6:[7] Let U be the universal set, E be the set of parameters and AÌ E. Let K(X) denote the set of all fuzzy subsets of U. Then a pair (K,A) is called fuzzy soft set over U, where K is a mapping from A to K(U).

Example 2.7: Let U={c1,c2,c3} be the set of three cars and E={costly(e1),metallic color(e2), cheap(e3)} be the set of parameters, where A={e1,e2} Ì E. Then (K,A)={K(e1)={c1/0.6,c2/0.4,c3/0.3} , K(e2)={ c1/0.5,c2/0.7,c3/0.8}} is the fuzzy soft set over U denoted by KA.

Definition 2.8:[7] Let KA be a fuzzy soft set over U and a be a subset of U. Then upper a - inclusion of KA denoted by KaA = { xÎA /K(x) ≥ a }. Similarly KaA = { xÎ A / K(x) ≤ a } is called lower a-inclusion of KA.

Definition 2.9:[11] Let KA and GB be fuzzy soft sets over the common universe U and ψ : A ® B be a function. Then fuzzy soft image of KA under ψ over U denoted by ψ(KA) is a set- valued function, where ψ(KA):B® 2U defined by ψ(KA) (b)={È{K(a) / aÎA and ψ (a)=b} , if ψ-1(b)¹ φ} for all bÎ B, the soft pre-image of GB under ψ over U denoted by ψ-1(GB) is a set- valued function, whereψ-1(GB) : A ® 2U defined by ψ-1(GB)(b) = G(ψ(a)) for all a Î A. Then fuzzy soft anti-image of KA under ψ over U denoted by ψ(KA) is a set-valued function, where ψ(KA):B ® 2U defined byψ-1(KA)(b) ={Ç{K(a) / aÎA and ψ (a) = b} , if ψ-1(b) ¹ φ} for all b Î B.

Definition 2.10:[14] Let X be a set. Then a mapping μ: X→ P*([0,1]) is called flexible subset of X, where P*([0,1]) denotes the set of all non empty subset of [0,1]

Definition 2.11:[14] Let X be a non empty set .Let μ and l be two flexible fuzzy subset of X. Then the intersection of μ and λ denoted by μÇl and defined by μÇl={min{a,b}/ aÎμ(x),bÎl(x)} for all xÎX. The union of μ and λ denoted by μÈl and defined by μÈl={max{a,b}/ aÎ μ(x),bÎl(x)} for all xÎX.

Definition 2.12 [14] Let U be an initial universe, E be the set of all parameters and A ⊂ E. A pair (F, A) is called a flexible fuzzy soft set over U where F: A →P(U) is a mapping from A into P(U) , where P(U) denotes the collection of all subsets of U.

Example 2.13:Consider the example 2.5, here we cannot express with only two real numbers 0 and 1, we can characterized it by a membership function instead of crisp numbers 0 and 1, which associate with each element a real number in the interval [0,1].Then

$(f_A, E) = \{f(e_1) = \{(u_1, 0.7), (u_2, 0.5), (u_3, 0.4), (u_4, 0.2)\},$

$f_A(e_2) = \{(u_1, 0.5), (u_2, 0.1), (u_3, 0.5)\}\}$ is the fuzzy soft set representing the "colour of the shirts" which Mr. X is going to buy.

Definition 2.14: An m- dimensional flexible fuzzy soft set (or a P*[0,1]m- set) on X is a mapping A: X→P* [0,1]m. we denote the set of all m-dimensional flexible fuzzy soft sets on X by m(X).

Note that [0,1]m (m-power of [0,1]) is considered a posset with the point wise order ≤ , where m is an arbitrary ordinal number (we make an appointment that m = [n/n < m] when m > 0), ≤ is defined by x ≤ y ↔ ui(x) ≤ ui(y) for each i ∈ m (x,y ∈ P*[0,1]m), and ui: P*[0,1]m → [0,1] is the i- th projection mapping (i ∈ m). Also, 0 = (0,0,0, ……… ,0) is the smallest element in P*[0,1]m and 1 = (1,1,............. , 1) is the largest element in P*[0,1]m

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SECTION-3: M-DIMENSIONAL FLEXIBLE FUZZY SOFT SUBGROUP

Definition-3.1: An m-dimensional flexible fuzzy soft set A on a group G is called an m- dimensional flexible fuzzy soft subgroup if the following conditions hold:

(MFFSG1) inf {A(x*y)} ≥ min {infA(x), infA(y)}and inf {A(x-1)} ≥ inf {A(x)}, (MFFSG2) sup {A(x*y)} ≤ max {sup A(x), sup A(y)} and sup A(x-1) ≤ sup A(x). That is

(MFFSG1) inf {xi ₒ A(x*y)} ≥ min {inf (xi ₒ A(x)), inf (xi ₒ A(y))}and inf {xi ₒ A(x-1)} ≥ inf {xi ₒ A(x)},

(MFFSG2) sup {xi ₒ A(x*y)} ≥ min {sup (xiₒ A(x)), sup (xi ₒ A(y))}and sup {xiₒ A(x-1)} ≥ sup {xi ₒ A(x)}, for all x, y ∈ G, i = 1,2,3 ----------, m. we denote the set of all m-dimensional flexible fuzzy soft subgroup of a group G by Fm(G).

Example-3.2: Let S = {e,a,b,ab}be the non-cyclic group of order 4. We define an m- dimensional flexible fuzzy subset A : S → P*[0,1]m by
(0.7, 0.7, …… ,0.7), if q = ei
A(q) =
(0.2, 0.2 ……. ,0.2), otherwise.

By direct calculations, It is easy to see that A is an m-dimensional flexible fuzzy soft subgroup of S.

Now we state the following lemma's without proof.

Lemma-3.3: Let A ∈ Fm(G).Then for all x ∈ G
(i) inf A(e) ≥ inf A(x) and inf A(x) = inf A(x-1)
(ii) sup A(e) ≤ sup A(x) and sup A(x) = sup A(x-1).

Lemma-3.4: Let A be an m-dimensional flexible fuzzy soft subgroup of G. Then A# = { x /x ∈G , inf A(x) = inf A(e)} and A# = { x /x ∈ G , inf A(x) ≥ {0,0,0,…............ , 0} are subgroups of G.

Definition-3.5: Let A1 and A2 be two m-dimensional flexible fuzzy soft subsets of a group. Then the intersection is defined as inf (A1 ∩ A2) (xy-1) = min { inf A1(xy-1), inf A2(xy-1)}

Theorem-3.6: Let A1 and A2 be two m-dimensional flexible fuzzy soft subsets of a group. Then A1 ∩ A2 is an m-dimensional flexible fuzzy soft subgroup of G.

Proof: Here we show that
inf (A1 ∩ A2) (x*y-1) ≥ min {inf (A1 ∩ A2) (x) , inf (A1 ∩ A2)(y-1)}
By definition -3.5, we see that
inf (A1 ∩ A2)(x*y-1) = min { inf A1(x*y-1), inf A2(x*y-1)}
≥ min { inf (A1(x), A1(y-1)), inf (A2(x), A2(y-1))}
= min { inf (A1(x), A2(x)), inf (A1(y-1), A2(y-1))}
= min {inf (A1 ∩ A2)(x), inf (A1 ∩ A2)(y-1)}.

Hence A1 ∩ A2 is an m-dimensional flexible fuzzy soft subgroup of G.

Definition-3.7: Let A ∈ m(U). For t ∈ P*[0,1]m, the set At = { x ∈ U / A(x) ≥ t} is called an m- level subset of an m-dimensional flexible fuzzy soft subset A. Note that At is an ordinary subset of U.

Theorem-3.8: Let G be a group and let A be an m- dimensional flexible fuzzy soft subgroup of G. Then the m-level subset At, for t ∈ P*[0,1]m, t ≤ A(e), is a subgroup of G, where e is the identity of G.

Proof: Since At = {x ∈ G / A(x) ≥ t}, then At is non-empty. e ∈ At for all t ∈ P*[0,1]m.

Let x,y ∈ At. Then A(x) ≥ t and A(y) ≥ t. Since A ∈ Fm(G), inf A(x*y) ≥ min {inf A(x), inf A(y)}. This means that A(x*y) ≥ t. Hence x*y ∈ At. Let x ∈ At implies A(x) ≥ t. Since A is an m-dimensional flexible fuzzy soft subgroup of G, inf A(x-1) ≥ inf A(x) and hence A(x) ≥ t. This implies that x-1 ∈ At . Therefore, At is a subgroup of G.

Example 3.9: Let Q = {e, a,b,ab} be the klein 4-group. we define an m-dimensional flexible fuzzy soft subgroup A: Q → P*[0,1]m of S by A (e) = t0, A(a) = t1, A(b) = A(ab) = t2, where t0> t1> t2 for all t0 , t1, t2 ∈ P*[0,1]m. Since A is an m-dimensional flexible fuzzy soft subgroup of S. we note that, At0 = {e}, At1 = {e,a} and At2 = { e,a,b,ab } are the subgroups of S.

Theorem-3.10: Let G be a group and let A be an m-dimensional flexible fuzzy soft subset of G such that At is a subgroup of G for al t ∈ P*[0,1]m, t ≤ A(e) . Then A is an m-dimensional flexible fuzzy soft subgroup of G.

Proof: Suppose that x,y ∈ At and let A(x) = t1 and A(y) = t2. Then x ∈ At1, y ∈ At1. We assume that t1 ≤ t2. Then it implies At2 is subset of At1. So y ∈ At1.Thus x,y ∈ At1 and since At1 is a subgroup of G, by hypothesis, x*y ∈ At1.Therefore, inf A(x*y) ≥ t1 = min {inf A(x),

inf A(y)}. Again, let x ∈ G and A(x) = t. Then x ∈ At1. Since At is a subgroup of G, x-1 ∈ At. Therefore A(x-1) ≥ t. Hence A(x-1) ≥ A(x). Thus. A is an m-dimensional flexible fuzzy soft subgroup of G.

Example 3.11: Let S3 = { e ,a,a2,b,ab,a2b}be the symmetric group with 6 elements. we define an m-dimensional flexible fuzzy subset A : S3 → P*[0,1]m by
A(a) = A(b) = t1, A (e) = A(a2b) = t0 , A(a2) = A(ab) = t2, for all t0, t1, t2 ∈ P*[0,1]m, where t0 > t1 > t2. From the theorem-3.10, it is easy to clear that A is not an m-dimensional flexible fuzzy soft subgroup of S3 because At1 = {e, a, b, a2b} is not a subgroup of S3.

Definition 3.12: Let G be a group and A be an m-dimensional flexible fuzzy soft subgroup of G. The subgroups At = { x ∈ G / A(x) ≥ t } for t ∈ P*[0,1]m are called m-dimensional level subgroups of A.

We now state the following theorem without its proof.

Theorem 3.13: Every subgroup H of a group G can be realized as an m-dimensional level subgroup of some m- dimensional flexible fuzzy soft subgroup of G.

Definition 3.14: Let A ∈ Fm(G). Then A is called an m-dimensional commutative flexible fuzzy soft subset of G if and only if A(x*y) = A(y*x) for all x, y ∈ X.

Definition 3.15: Let A ∈ Fm(G). Then A is called an m-dimensional normal flexible fuzzy soft subgroup of G if it is an m-dimensional commutative flexible fuzzy subset of G.

Let Nm(G) denotes the set of all m-dimensional normal flexible fuzzy soft subgroups of G. Example 3.16: Let Φs = {± 1, ± i , ± j , ± k } be a group of quaternious with 8 elements. we define an m-dimensional flexible fuzzy soft subgroup A : Φs → P*[0,1]m of Φs by A(1) = t0, A(-1) = A(± i) = t1 , A(± j) = A(±k) = t2, for all t0> t1> t2 and t0, t1, t2 ∈ P*[0,1]m. Then A is an m-dimensional flexible fuzzy soft subgroup of Φs.

Remark 3.17: Every m-dimensional flexible fuzzy soft subgroup of G is normal if G is an abelian group.

Example 3.18: Let Q = { e, a,b,ab} be the klein 4-group. We define an m-dimensional flexible fuzzy soft subgroup A: Q → P*[0,1]m by A (e) = A(ab) = t0 , A(a) = A(b) = t1, for all t0> t1 and t0 , t1 ∈ P*[0,1]m. Since Q is an abelian group, so from the above remark, A is an m-dimensional normal flexible fuzzy soft subgroup of Q.

Remark 3.19: If A1,A2 ∈ Fm(G) and A1, A2 does not belong to Nm(G), then A1 ∩ A2 is not an m-dimensional normal flexible fuzzy soft subgroup of G.

4. PROPERTIES OF M-DIMENSIONAL NORMAL FLEXIBLE FUZZY SUBGROUPS

Definition 4.1: Let A1,A2 ∈ Fm(G) and A1 is subset of A2 . Then A1 is called an m- dimensional normal flexible fuzzy soft subgroup of an m-dimensional flexible fuzzy soft subgroup A2 , if inf A(xyx-1) ≥ min {inf A(y), inf A(x)} for all x, y ∈ G.

From the above definition we see that

(1) If A1 ∈ Nm(G), A2 ∈ Fm(G), and A1 is subset of A2, then A1 is m-dimension normal flexible fuzzy soft subgroup of A2.
(2) Every m-dimensional flexible fuzzy soft subgroup is an m-dimensional normal flexible fuzzy soft subgroup of itself.

Definition 4.2: Let A be an m-dimension flexible fuzzy soft subgroup of a group G. For any x ∈ G, we define a map Åξ : G → P*[0,1]m by Åx(x) = A(xy-1) for all y ∈ G.

Theorem 4.3: If A1 ∈ Nm(G) and A2 ∈ Fm(G), then A1 ∩ A2 is an m-dimensional normal flexible fuzzy soft subgroup of A2.

Proof: Clearly, A1 ∩ A2 ∈ Fm(G) and A1 ∩ A2 is subset of A2. By definition- 4.2,
(A1 ∩ A2)(xyx-1) = min { inf A1(xyx-1), inf A2(xyx-1)}
= min {inf A1(y), inf A2(xyx-1)}
≥ min { inf A1(y) , inf(A2(x), A2(y), A2(x-1))}
= min { inf (A1 ∩ A2)(y), A2(x)}
For all x, y ∈ G. Therefore, A1 ∩ A2 is an m-dimensional normal flexible fuzzy soft subgroup of G.

Theorem 4.4: Let A1,A2, A3 ∈ Fm(G) be such that A1 and A2 are m-dimensional normal flexible fuzzy soft subgroups of A3. Then A1 ∩ A2 is an m-dimensional normal flexible fuzzy soft subgroup of A3.

Proof: Since A1, A2 ∈ Fm(G), it follows that A1 ∩ A2 ∈ Fm(G) and A1 ∩ A2 is subset of A3.

Now
(A1 ∩ A2)(xyx-1) = inf (A1(xyx-1), A2(xyx-1)}
≥ inf {inf (A1(y), A3(x)), inf (A2(y), A3(x))}
≥ inf {(A1 ∩ A2)(y), A3(x)}.

Hence A1 ∩ A2 is an m-dimensional normal flexible fuzzy soft subgroup of A3.

Theorem 4.5: If $A1'$ is an m-dimensional normal flexible fuzzy soft subgroup of A1 and A1, A2 $\in$ Fm(G), then $A1' \cap A2$ is an m-dimensional normal flexible fuzzy soft subgroup of $A1 \cap A2$.

Proof: Clearly, $A1' \cap A2 \in Fm(G)$ and $A1' \cap A2$ is subset of $A1 \cap A2$. By definition- 4.2,
$(A1' \cap A2)(xyx\text{-}1) = \min \{ \inf(A1'(y)), \inf A2(xyx\text{-}1)\}$
$\geq \min \{ \inf A1'(y), \inf(A2(x), A2(y), A2(x\text{-}1))\}$
$= \min \{ \inf(A1' \cap A2)(y), \inf(A1 \cap A2(x)\}$

For all x, y $\in$ G. Therefore, $A1' \cap A2$ is an m-dimensional normal flexible fuzzy soft subgroup of $A1 \cap A2$.

Now we state the following theorem without its proof

Theorem 4.6: Let A $\in$ Nm(G). Then A# = { x/ x $\in$ G, A(x) = A(e)} and A# = { x/ x $\in$ G , 1A(x) $\geq$ (0, 0, ………, 0)} are normal flexible fuzzy soft subgroups of G.

Theorem 4.7: Let A be an m-dimensional flexible fuzzy soft subgroup of a group G. Then A is an m-dimensional normal flexible fuzzy soft subgroup of G if and only if A([x, y]) $\geq$ A(x) for all x, y $\in$ G.

Proof: Suppose that A is an m-dimensional normal flexible fuzzy soft subgroup of G. Then A(x-1y-1xy)
$\geq$ inf (A(y-1xy), A(x-1))
= inf (A(x), A(x)) = A(x).

Now suppose that A satisfies the relation A([x,y]) $\geq$ A(x) for all x, y $\in$ G. Then for x, z $\in$ G, we have
A(x-1zx) = A(zz-1x-1zx)
$\geq$ inf (A(z), A([z,x])) = A(z).

Thus

A(x-1zx) $\geq$ A(z) for all z,x $\in$ G. --- (1)

Again, we get

A(z) = A(x.x-1zxx-1) $\geq$ inf (A(x), A(x-1zx)) --------------------------------------- (2)

Now if inf (A(x), A(x-1zx)) = A(x), then we get that A(z) $\geq$ A(x) for all x, z $\in$ G.Then A is a constant function, and there is nothing to prove. So we assume that inf (A(x), A(x-1zx)) = A(x-1zx). Then (2) gives that A(z) $\geq$ A(x-1zx) for all x,z $\in$ G. By this inequality with (1),we have A(x-1zx) = A(z) for all x,z $\in$ G. Hence A is constant on the conjugate classes of G. Example 4.8: Let D8 be a dihedral group of order 8 given by D8 = { e,a,a2,a3,b,ab,a2b,a3b } . where we have a4 = b2 = e and a3b = ba. Define A: D8→P*[0.1]m by setting A(e) = A(a3b) = 1, A(a) = A(a2) = A(a3) = A(b) = A(ab) = A(a2b) = 0.3.

It is easy to see that A $\in$ Fm(D8) and A does not belong to Nm(D8) because A(a3.b) $\neq$ A(b.a3). Now the cossets of A in D8 is given by
Ae = { 1, 0.3, 0.3. 0.3, 0.3, 0.3, 0.3, 1}
Aa = { 0.3, 1, 0.2,0.3, 0.3, 0.3, 1, 0.3}
Aa2= {0.3.0.3, 1, 0.3, 0.3, 0.3, 0.3, 1}
Aa3= { 0.3,0.3,0.3,1, 0.3, 0.3, 0.3,1}

Ab = {1, 0.3,0.3,0.3, 0.3, 0.3, 1, 0.3}
Aab ={0.3,0.3,0.3,0.3,0.3, 1, 0.3, 1}
Aa2b = {0.3,0.3, 0.3, 0.3, 1,0.3,1,0.3}
Aa3b = {1,0.3, 0.3,0.3,0.3,0.3,0.3,1}.

Here Ab = Aa3 , Aa2 = Aab. But Ab o Aa2 = Aa2b ≠ Aa3 o Aab = Ab. Hence A is m-dimensional normal flexible fuzzy soft subgroup.

CONCLUSION: Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as computer science, information science, topological spaces and so on. This provides sufficient motivation to researchers to review various concepts and results on algebraic structures and in the broader framework of soft set setting.

.These include smoothness of functions, game theory, operations research, Reimann and Perron integrations, probability theory and measure theory. In this article, we discuss the concept of m-dimensional structures on flexible fuzzy soft subgroup, and investigate some of its properties. The characterisations of various structures of normal flexible fuzzy soft subgroups related to cossets with illustrative examples.

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