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Aims and Scope

The Journal of Mathematical Modelling and Applied Computing is an Indian research journal, which publishes top-level original and review papers, short communications and proceedings on Interdisciplinary Integrative Forum on Modelling, Simulation and Scientific Computing in Engineering, Physical, Chemical Biological, Medical, Environmental, Social, Economic and Other Systems using Applied Mathematics and Computational Sciences and Technology.

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A STUDY OF GRAPH THEORY AND ITS APPLICATIONS IN VARIOUS FIELD OF MATHEMATICS

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Abstract:

Graph theory is a branch of Mathematics deals with the study how networks can be encoded and their properties can be measured. Graph theory is becoming most significant as it is most frequently used too many other areas of Mathematics, science and Engineering etc. It is being actively used in varies fields such as biochemistry, Communication networks and Coding theory), Algorithms and Computation and operations research etc. The powerful combinatorial methods found in graph theory have also been used to prove fundamental results in other areas of pure mathematics. This paper, besides giving a general outlook of these facts, includes new graph theoretical proofs of Fermat's Little Theorem. In this paper we also discussed the timetabling problem and the assignment of frequencies in GSM mobile phone networks.

Introduction

The origins of graph theory can be traced to Leonhard Euler who devised in 1735 a problem that came to be known as the "Seven Bridges of Konigsberg". In this problem, someone had to cross once all the bridges only once and in a continuous sequence, a problem the Euler proved to have no solution by representing it as a set of nodes and links. This led the foundation of graph theory and its subsequent improvements.
IKeeping in view of above applications, we will discuss new graph theoretical proofs of Fermat's Little Theorem. In this paper we will also studing the timetabling problem and the assignment of frequencies in GSM mobile phone networks. This work is new and very useful for the

studing the another applications of graph theory. This type of work has not been previously discussed in the literature.

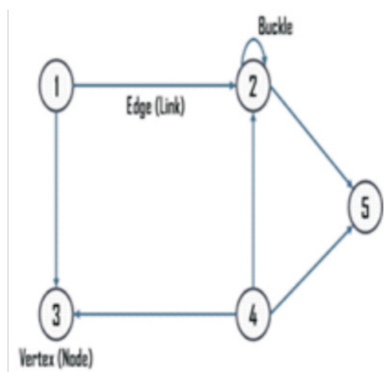
Fundamental Concept of Graph Theory

Graph:
A graph *G* is a set of vertex (nodes) *v* connected by edges (links) *e*. Hence we can write *G=(v , e)*.

Vertex A
node which will be denoted by *v* is an meeting point point of a graph. It is the concept of a location such as a city, an administrative division, a road intersection or a transport terminal.

Edge
An edge which will be denoted by *e* is a link between two nodes. The link (*i* , *j*) is of initial extremity *i* and of terminal extremity *j*. A link is the direction that is commonly represented as an arrow.

Some Basic Classifications of Graph Representation of a Network



This simple graph has the following definition: G = (v,e)
where,
v = (1,2,3,4,5)
e = (1,2), (1,3), (2,2), (2,5), (4,2), (4,3), (4,5)

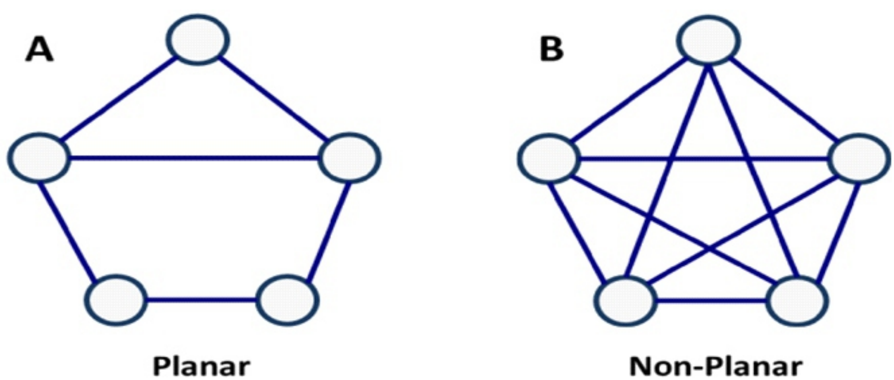
Sub-Graph.
A sub-graph is a subset of a graph *G* where *p* denotes the number of sub-graphs. For example $G' = (v', e')$ can be a distinct sub-graphs of *G*. For example, the road transportation network of a urban is a sub-graph of a regional transportation network, which is itself a sub-graph of a state transportation network and which is also a sub-graph of country and so on.

Buckle (Loop or self edge). A link that makes a node correspond to itself is a buckle.

Planar Graph:
A graph where all the intersections of two edges are a node. Since this type of graph is located within a plane, its topology is two-dimensional. This is most difficult in the case of power grids, road and railway networks, whereas large care must be inferred to the definition of nodes.

Non-Planar Graph:
A graph where there are no nodes at the intersection of at least two edges. This type of graph implies a third dimension in the topology of the graph since there is the possibility of having a movement "passing over" another movement such as for air and maritime transport.

Geometrical Representation of Planar Graph and Non-Planar Graph:



Graph **A** is planar because there is no link is overlapping with another. Graph **B** is non-planar because several links are overlapping.

Simple Graph:
A graph that includes only one type of link between its vertices. For example a rail network simple graph.

Multigraph :
A graph that includes several types of links between its vertices. Some vertices may be connected to one link type while others can be connected to more than one that are running in parallel. For example a graph depicting a rail network with different links between vertices serviced by either or both modes is a multigraph.

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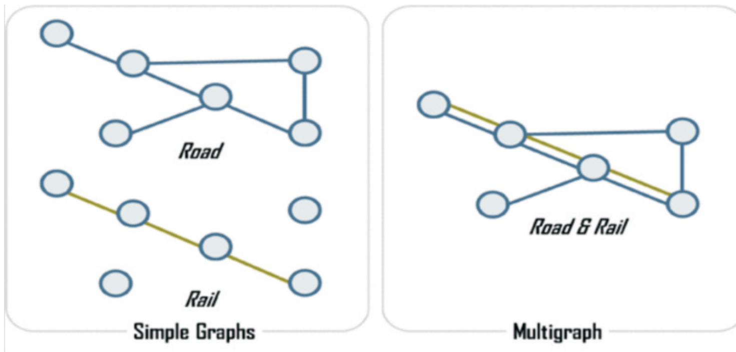
Geometrical Representation of Simple Graph and Multigraph:

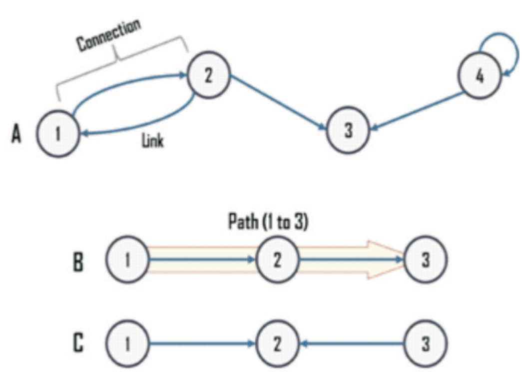


The multigraph is a combination of the two simple graphs as we cleared in the above definition of simple graph.

Links and their Structures A transportation network enables flows of people, freight or information, which are occurring along its links. Graph theory must thus offer the possibility of representing movements as linkages, which can be considered over several aspects:

Connection: A set of two nodes as every node is linked to the other. Considers if a movement between two nodes is possible, whatever its direction. Knowing connections makes it possible to find if it is possible to reach a node from another node within a graph.

Path: A sequence of links that are traveled in the same direction. For a path to exist between two nodes, it must be possible to travel an uninterrupted sequence of links. Finding all the possible paths in a graph is a fundamental attribute in measuring accessibility and traffic flows.



Here, On graph **A**, there are 5 links [(1,2), (2,1), (2,3), (4,3), (4,4)] and 3 connections [(1-2), (2-3), (3-4)]. On graph **B**, there is a path between 1 and 3, but on graph **C** there is no path between 1 and 3.

Chain. A sequence of links having a connection in common with the other. Direction does not matter.

Length of a Link, Connection or Path:

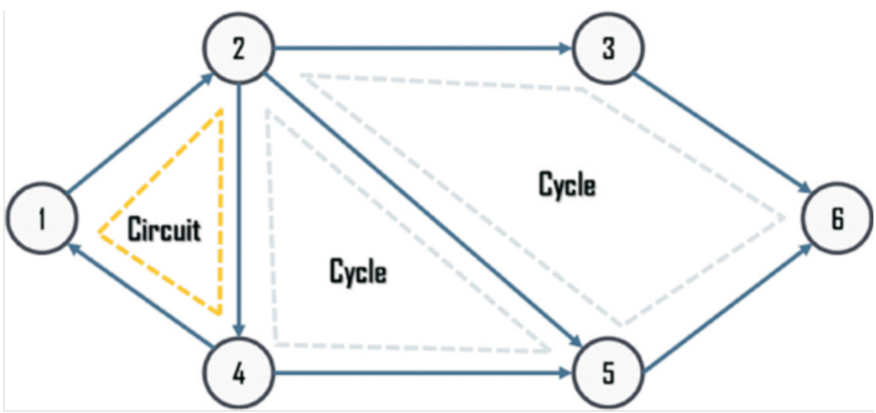
Refers to the label associated with a link, a connection or a path. This label can be distance, the amount of traffic, the capacity or any attribute of that link. The length of a path is the number of links (or connections) in this path.

Cycle

Refers to a chain where the initial and terminal node is the same and that does not use the same link more than once is a cycle.

Circuit:

A path where the initial and terminal node corresponds. It is a cycle where all the links are traveled in the same direction. Circuits are very important in transportation because several distribution systems are using circuits to cover as much territory as possible in one direction (delivery route)



On this graph, 2-3-6-5-2 is a cycle but not a circuit. 1-2-4-1 is a cycle and a circuit.

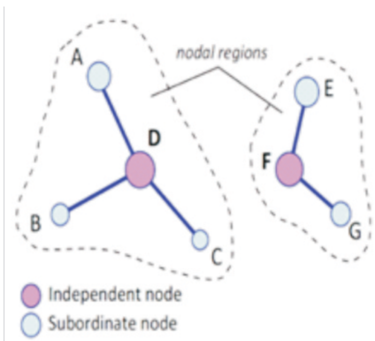
Clique. A clique is a maximal complete subgraph where all vertices are connected.

Cluster. Also called community, it refers to a group of nodes having denser relations with each other than with the rest of the network. A wide range of methods are used to reveal clusters in a network, notably they are based on modularity measures (intra- versus inter-cluster variance).

Ego network: For a given node, the

ego network corresponds to a sub-graph where only its adjacent neighbors and their mutual links are included.

Nodal region: A nodal region refers to a subgroup (tree) of nodes polarized by an independent node (which largest flow link connects a smaller node) and a number of subordinate nodes (which largest flow link connects a larger node). Single or multiple linkage analysis methods are used to reveal such regions by removing secondary links between nodes while keeping only the heaviest links.

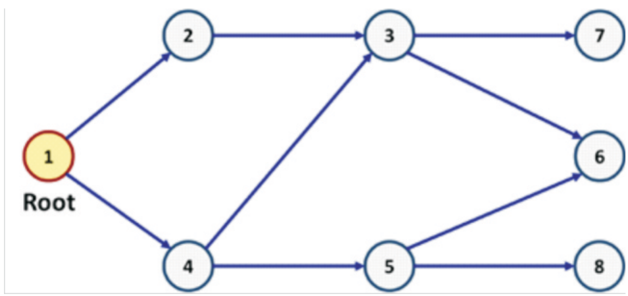


	A	B	C	D	E	F	G	Total
A		15	5	***35***	5	10	5	75
B			5	***15***	1	5	5	31
C				***10***	1	5	1	17
D					20	25	10	55
E						***40***	15	55
F							***20***	20
G								0
Total	0	15	10	55	27	90	56	253

Nodal Region **A** refers to a subgroup (tree) of nodes polarized by an independent node (which largest flow link connects a smaller node) and a number of subordinate nodes (which largest flow link connects a larger node). Single or multiple linkage analysis methods are used to reveal such regions by removing secondary links between nodes while keeping only the heaviest links. D and F are independent nodes because their largest flow is directed towards smaller nodes. A,B,C and E,G are subordinate nodes because their largest flow is directed towards larger nodes (D and F). This algorithm can be also applied to directed graphs and may extend to secondary links (e.g. for each node, including up to 50% of its total traffic) so as to avoid losing too much information. Such methods proposed by Nystuen and Dacey (1961) are also labeled single linkage analysis (largest flows only) and multiple linkage analysis (largest flows over a certain threshold). They are often used to reveal functional regions based on flow patterns among localities.

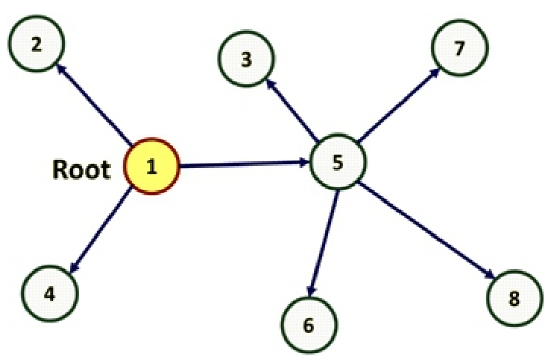
Dual Graph. A method in space syntax that considers edges as nodes and nodes as edges. In urban street networks, large avenues made of several segments become single nodes while intersections with other avenues or streets become links (edges). This method is particularly useful to reveal hierarchical structures in a planar network.

Root. A node *r* where every other node is the extremity of a path coming from *r* is a root. Direction has an importance. A root is generally the starting point of a distribution system, such as a factory or a warehouse.



Node 1 is the only root of this graph because every other node is part of a path originating from node 1.

Trees. A connected graph without a cycle is a tree. A tree has the same number of links than nodes plus one, . ($e = v-1$). If a link is removed, the graph ceases to be connected. If a new link between two nodes is provided a cycle is created. A branch of root *r* is a tree where no links are connecting any node more than once. River basins are typical examples of tree-like networks based on multiple sources connecting towards a single estuary. This structure strongly influences river transport systems.



This graph is a tree having node 1 as a root.

Applications of Graph Theory

Graph theory is rapidly moving into the main stream of mathematics mainly because of its applications in various fields which include biochemistry, electrical engineering algorithms and computations and operations research (scheduling) etc.

There are so many vast applications of Grapy theory. A few of them had been discussed in this manuscript:

1. Fermat's (Little) Theorem

There are many proofs of Fermat's Little Theorem. The first known proof was communicated by Euler in his letter of March 6, 1742 to Goldbach. The graph theoretic proof given below together with some number theoretic results, was used to prove Euler's generalization to non-prime modulus.

Theorem (Fermat). Let a be a natural number and let p be a prime such that a is not divisible by p. Then, $a^p - a$ is divisible by p.

Proof. Consider the graph $G = (V, E)$, where the vertex set V is the set of all sequences $(a_1, a_2, ..., a_p)$ of natural numbers between 1 and a (inclusive), with $a_i \neq a_j$ for some $i \neq j$. Clearly, V has $a^p - a$ elements. Let $u = (u_1, u_2, ..., u_p)$, $v = (u_p, u_1, ..., u_{p-1}) \in V$. Then, we say $uv \in E$. With this assumption, each vertex of G is of degree 2. So, each component of G is a cycle of length p. Therefore, the number of components is $(a^p - a) / p$. That is, $p \mid (a^p - a)$.

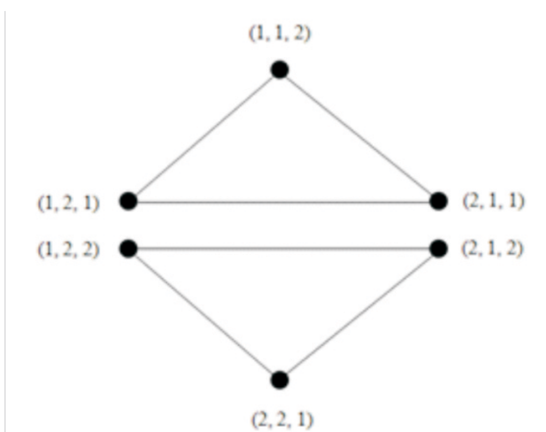


Figure . *The graph G for a* = 2 *and p* = 3

2.The Timetabling Problem

If in a college there are n professors x_1, x_2, …, x_n and n subjects y_1, y_2, …, y_m to be taught. Given that professor x_i is required (and able) to teach subject y_j for p_{ij} periods ($p = [p_{ij}]$ is called the teaching requirement matrix), the college administration wishes to make a timetable using the minimum possible number of periods. This is known as the timetabling problem and can be solved using the following strategy. Construct a bipartite multigraph G with vertices x_1, x_2, …, x_n, y_1, y_2,…, y_m such that vertices x_i and y_j are connected by p_{ij} edges. We presume that in any one period each professor can teach at most one subject and that each subject can be taught by at most one professor. Consider, first, a single period. The timetable for this single period corresponds to a matching in the graph and, conversely, each matching corresponds to a possible assignment of professors to subjects taught during this period. Thus, the solution to the timetabling problem consists of partitioning the edges of G into the minimum number of matchings.

3.Map Coloring and GSM Mobile Phone Networks

Given a map drawn on the plane or the surface of a sphere, the famous four color theorem asserts that it is always possible to properly color the regions of the map such that no two adjacent regions are assigned the same color, using at most four distinct colors . For any given map, we can construct its dual graph as follows. Put a vertex inside each region of the map and connect two distinct vertices by an edge if and only if their respective regions share a whole segment of their boundaries in common. Then, a proper vertex coloring of the dual graph yields a proper coloring of the regions of the original map.

The Groupe Spécial Mobile (GSM) was created in 1982 to provide a standard for a mobile telephone system. The first GSM network was launched in 1991 by Radiolinja in Finland with joint technical infrastructure maintenance from Ericsson. Today, GSM is the most popular standard for mobile phones in the world, used by over 2 billion people across more than 212 countries. GSM is a cellular network with its entire geographical range divided into hexagonal cells. Each cell has a communication tower which connects with mobile phones within the cell. All mobile phones connect to the GSM network by searching for cells in the immediate vicinity.

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Conclusion: **Graph theory** deals with the study how networks can be encoded and their properties can be measured. Graph theory is very broad area and there are so many applications of Grapy theory. The powerful combinatorial methods found in graph theory have also been used to prove fundamental results in other areas of pure mathematics. This paper, besides giving a general outlook of these facts, includes new graph theoretical proofs of Fermat's Little Theorem. In this paper we also discussed the timetabling problem and the assignment of frequencies in GSM mobile phone networks

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MATHEMATICAL MODELLING OF TRANSPORT OF DECAYING CONTAMINANTS IN GROUNDWATER RESULTING FROM INSTANTANEOUS SPILL

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Abstract

The growing emergence of groundwater contamination and the increased level of water scarcity problems require appropriate management of groundwater resources in term of both quantity and quality. For better management plans, it is imperative to develop mathematical modelling tools that enable to obtain the response of the considered groundwater system to hydraulic and environmental excitations and to predict resulting changes in groundwater quality in space and time. In this paper, one dimensional finite difference contaminant transport model is presented for an instantaneous spill which incorporates advection, dispersion, diffusion and decay mechanisms of contaminant migration in groundwater system. The developed model is capable of simulating the migration of contaminant species that are characterized by non-linear degradation or decay involving biological or chemical processes. The simulation results were compared with available analytical model, and were found to be in excellent agreement with that obtained from analytical solutions for a wide range of field conditions with regard to dispersion and source definition. The developed numerical model can be used for the forecasting of contaminant dispersion in laboratory and field for the quantitative description of the time-space distribution of the contaminant and to investigate the effect of non-linearity, which will help in addressing a number of real life groundwater quality problems.

Keywords: Groundwater Hydrology, Contaminant Transport, Numerical Modelling, Finite Difference Method, Decay, Biodegradation

1. Introduction

Water is one of man's basic and precious resources. Contamination of water either on surface or in ground is crucial problem. . In most of groundwater contaminant transport investigations it is not practical to monitor all aspects of the groundwater flow and solute distributions. Information between and beyond monitoring locations and in the future are needed to understand the site and make informed decisions. The role of groundwater models in the study of groundwater flow and transport has long been a topic of interest for water resources people. The growing emergence of groundwater contamination and the increased level of water scarcity problems require appropriate management of groundwater resources in term of both quantity and quality. For better management plans, it is imperative to develop mathematical modelling tools that enable to obtain the response of the considered groundwater system to hydraulic and environmental excitations and to predict resulting changes in groundwater quality in space and time.

The finite difference method is a well known numerical method that has been applied to advection dispersion equation (Akram et al. 1999). The concept of linear "caricature" isotherm and its usefulness in obtaining exact analytical solutions were introduced for concentration profiles under nonequilibrium conditions (Manorajan 1995). An analytical solution for solute diffusion in a semi-infinite two-layer porous medium for arbitrary boundary and initial conditions obtained by Liu and Ball (1998) using the Green's function approach. An improved FDM has been developed by Hossain and Yonge (1999) to provide oscillation free results with the introduction of minimum artificial dispersion. A one-dimensional theory of contaminant migration through a saturated deforming porous media is developed by Smith (2000) based on a small and large strain analysis of a consolidating soil and conservation of contaminant mass. Analytical one-dimensional solutions are obtained by Pang and Hunt (2001) for continuous and pulse contaminant sources in a semi-infinite saturated porous medium when the dispersion coefficient increases linearly with distance downstream. Serrano (2001) used the method of decomposition for obtained series of solutions for the non-linear equation of advection and diffusion. These expressions permit an accurate forecasting of contaminant propagation under non-linearity in laboratory or field investigations at early or prolonged times after the spill. This paper presents the practical scenario of an instantaneous spill, and that of a constant concentration boundary condition for situations of non-linear decay, non-linear Freundlich isotherm, and non-linear Langmuir isotherm. Khebchareon and Saenton (2005) present an initial development of a one-dimensional numerical solution of mass transfer behavior of the entrapped dense non-

aqueous phase liquid (DNAPL) in the subsurface environment where the system of equations is solved implicitly.
Many analysts worked on the non linearity problem of decay and sorption. But number of analyst considered only one or two parameter of non linearity in there model either in analytical or numerical. In this model proposed combine effect of decay and sorption on account for the solution of governing equation of contaminant transport.

2. Contaminant Transport Mechanisms

There are three main physical processes effecting contaminant transport namely advection, dispersion and diffusion. In addition, chemical processes that effect transport are decay and sorption. Advection is the mass transport caused by the bulk movement of flowing ground water. The driving force is the hydraulic gradient. In highly permeable materials such as sand and gravel, advection is the most important transport process, and each transport prediction will only be as accurate as the flow description. Advective flow becomes more complex when the density and/or the viscosity of water change with solute concentration. Diffusion is the net flux of the solutes from a zone of higher concentration to a zone of lower concentration. Diffusion over geological time, however, can have a significant impact. The effect of diffusion will normally be masked by the effect of advection in groundwater zones with high flow velocities. Dispersive spreading, within and transverse to, the main flow direction causes a gradual dilution of the contamination plume. Dispersive spreading will lead to increase in plume uniformity with travel distance. The combination of dispersion and diffusion termed as hydrodynamic dispersion.
Degradation process also decreases the source of contamination with time. Reactions of the first order are applied to describe radioactive decay or simple degradation processes. Reactions of the first order are a linear and do not change the characteristics of the transport equation. Sorption refers to adsorption and desorption. Adsorption describes the adhesion of molecules or ions to the grain surface in the aquifer. The release from the solid phase is called desorption. Adsorption causes diminution of concentrations in the aqueous phase and a retardation of contaminant transport compared to water movement. The degree of sorption depends on a number of factors, including the concentration and the characteristics of the contaminant, the soil type and its composition, the pH value of water, and the presence of other water solutes. These factors are in time and space, resulting in a variation of retardation in the natural environment. The rate of adsorption onto the solid material as related to the concentration in the groundwater is expressed by

adsorption kinetics. The relationship between the concentration of a solute in adsorbed phase and in the adjacent water phase at equilibrium is an adsorption isotherm (C.W. Fetter, 1993 and F.W. Schwartz, 1988).

3. Governing equation for contaminant transport

Assumptions taken in to considerations in model development are that the soil is homogeneous and isotropic, the porosity of soil is constant, saturated hydraulic conductivity is constant, ground water pore velocity is constant, one-dimensional flow is taken, hydrodynamic dispersion coefficient is constant, Freundlich and Langmuir parameters are constant and retardation factor is constant.
The one-dimensional advective-dispersive equations in an infinite aquifer subject an instantaneous point source and linear biological or radioactive decay.

$$\frac{\partial C}{\partial t} - D\frac{\partial^2 C}{\partial x^2} + u\frac{\partial C}{\partial x} + aC = 0 \text{ ----------}$$

$$\text{------------------ } -\infty < x < \infty, 0 < t$$

$$C(\pm\infty, t) = 0$$

The one-dimensional advective-dispersive equations in an infinite aquifer subject an instantaneous point source and non-linear biological or radioactive decay.

$$\frac{\partial C}{\partial t} - D\frac{\partial^2 C}{\partial x^2} + u\frac{\partial C}{\partial x} + aC^b = 0 \text{ ----------}$$

$$\text{------------ } -\infty < x < \infty, 0 < t$$

$$C(\pm\infty, t) = 0$$

4. Analytical solution for decaying contaminant species transport

4.1 *Analytical Solution for Linear Decaying Contaminant Species Transport*

The one-dimensional advective-dispersive equations in an infinite aquifer subject an instantaneous point source and linear biological or radioactive decay.

$$\frac{\partial C}{\partial t} - D\frac{\partial^2 C}{\partial x^2} + u\frac{\partial C}{\partial x} + aC = 0$$

$$-\infty < x < \infty, C(x,0) = 0$$

$$C(\pm\infty, t) = 0, \quad C(x,0) = C_i\delta(x) \qquad (1)$$

The solution to equation (1)

$$C = \sum_{n=0}^{\infty} C_n = \frac{C_i e^{-\left[\left((x-ut)^2/4Dt\right)-at\right]}}{\sqrt{4\pi Dt}} \qquad (2)$$

which is the well-known solution to the advective– dispersive equation with linear decay.

Decomposition series converge to the exact solution to the differential equation. In many instances, however, the closed-form solution may not be identified. This is especially true in many nonlinear equations. While a closed-form solution is mathematically desirable, the series solution constitutes an accurate model of interest to the practicing hydrologist.

4.2 *Analytical Solution for Non Linear Decaying Contaminant Species Transport*

In cases of non-linear biological or radioactive decay, equation (1) becomes

$$\frac{\partial C}{\partial t} - D\frac{\partial^2 C}{\partial x^2} + u\frac{\partial C}{\partial x} + aC^b = 0 \qquad (3)$$

solving the equation, we arrive at the closed-form expression

$$C(x,t) \approx C_0(x,t)e^{\left[2atC_0(x,t)^{b-1}\right]/(b+1)}, \quad (4\text{-}22)$$

b>0

iC>1

For the case of linear decay, b=1, and equation (4) is identical to the exact solution of equation (1), that is equation (2).

Thus, equation (4) is a useful, simple, and stable expression for practical applications in the forecasting of contaminant propagation under non-linear decay during early or prolonged-time simulations and a full range of values in the physical parameters.

4.3 Governing equation for contaminant transport

(a) The one-dimensional advective-dispersive equations in an infinite aquifer subject constant point source and linear biological or radioactive decay.

$$\frac{\partial C}{\partial t} - D\frac{\partial^2 C}{\partial x^2} + u\frac{\partial C}{\partial x} + aC = 0 \text{ ------------------------}$$

$$\text{----------} -\infty < x < \infty, 0 < t$$

$$C(\pm\infty, t) = 0$$

5 Solution of governing equation

Following assumptions are used for developing numerical model of contaminant transport equation:

(1) The porous medium is homogeneous and isotropic.

(2) The solute transport, across any fixed plane, due to microscopic velocity variations in the flow tubes, may be quantitatively expressed as the product of dispersion coefficient and the concentration gradient.

(3) The flow in the medium is unidirectional and the average velocity is taken to be constant throughout the length of the flow field.

(4) The FDM is approximations the higher order terms in Taylor's Series

are neglected.
(5) Contaminant is conservative i.e. decay is not considered for sorption cases. Also contaminant is assumed to be non reactive.
(6) Contaminant is non conservative i.e. decay is considered in case of decay and combine effect of decay and sorption. Also contaminant is assumed to be non reactive.
(7) Retardation process is considered in sorption cases.
(8) No other process like pumping, recharge etc. is considered.

5.1 Implicit scheme

The principal models of contaminant transport in groundwater are advection and dispersion. Retardation and degradation can significantly impact the transport. Extensive research has been and is being carried out on the numerical aspect of simulating advective-dispersive transport. A large volume of literature is available on finite difference models (FDMs) and finite element models (FEMs) for simulating advective-dispersive transport, in general and advective-dispersive transport in groundwater, in particular. The FDM for simulating contaminant transport in groundwater is based on either first-order or second-order approximation of the advective term. The first-order approximation of the advective term results in a stable algorithm at the expense of introducing unacceptably large artificial dispersion. The second-order accurate central difference approximation of the advective term, on the other hand, leads to oscillatory results. Oscillations are usually eliminated by adapting up-winding schemes. Adaptation of upwind FDMs, however, can introduce large artificial dispersion.
The implicit formulation for governing equation is obtained by the replacing the space derivative with its finite difference analog at the (j+1) time level i.e. (t+1). The time derivative is then replaced by a backward difference approximation relative to the j+1 time level i.e. t+1.
To illustrate, let us consider an one dimensional partial differential equation of second order of following type

$$\frac{\partial \Phi}{\partial t} = \frac{\partial^2 \Phi}{\partial x^2}$$

$$\frac{\Phi_i^{j+1} - \Phi_i^j}{\Delta t} = \frac{\Phi_{i+1}^{j+1} - 2\Phi_i^{j+1} + \Phi_{i-1}^{j+1}}{(\Delta x)^2}$$

The truncation error is the approximation is again $o\left((\Delta t)+(\Delta x)^2\right)$.

The above equation forms a set of

simultaneous linear algebraic equations with the unknowns F_i^{j+1}. The unknowns are thus given implicitly. This set of simultaneous equations is solved for the whole aquifer domain at a particular time level. In this passion, the solution is marched forward in time by solving the system of equations at each time level. The beauty of this method is that it is unconditionally stable.

Finite difference by implicit scheme is given by

$$\left[\frac{\partial C}{\partial t}\right]=\left[\frac{C_i^{t+1}-C_i^t}{\Delta t}\right]$$

$$\left[\frac{\partial C}{\partial x}\right]=\left[\frac{C_i^{t+1}-C_{i-1}^{t+1}}{\Delta x}\right]$$

$$\left[\frac{\partial^2 C}{\partial x^2}\right]=\left[\frac{C_{i+1}^{t+1}-2C_i^{t+1}+C_{i-1}^{t+1}}{(\Delta x)^2}\right]$$

5.2 Numerical solution for decaying contaminant species transport

The one-dimensional advective-dispersive equations in a semi infinite aquifer subject to a general non-linear sorption isotherm of the form The one-dimensional advective-dispersive equations in a semi infinite aquifer subject to constant point source and linear biological or radioactive decay.

$$\frac{\partial C}{\partial t}-D\frac{\partial^2 C}{\partial x^2}+u\frac{\partial C}{\partial x}+aC=0$$

The above equation may be solved for a variety of boundary and initial conditions. However, the following boundary and initial conditions were considered.

B.C.

$$C(x=0,t>0)=C_0$$

$$C(x=L,t\geq 0)=0$$

I.C.

$$C(0<x\leq L,t=0)=0$$

I.C.

$$C(0<x\leq L,t=0)=0$$

Equation (1) can be discretized as follows by employing second-order accurate central difference approximation of the advective term

$$\frac{\partial C}{\partial t}=\frac{C_i^j-C_i^{j-1}}{\Delta t}$$

$$\frac{\partial C}{\partial x}=\frac{C_{i+1}^j-C_{i-1}^j}{2\Delta x}$$

$$\frac{\partial^2 C}{\partial x^2}=\frac{C_{i+1}^j-2C_i^j+C_{i-1}^j}{\Delta x^2}$$

Where $C_i^{\perp}$ is contaminant concentration at i-th space step and j-th step, $\Delta\ x$ is space step and Δt is time step.

So the 1-D advection-dispersion equation becomes, in FDM form:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - u\frac{\partial C}{\partial x} - aC$$

So we have

$$\left(-k1 - k2\right)C_{i-1}^{j} + \left(1 + k3 + 2k2\right)C_i^j + \left(k1 - k2\right)C_{i+1}^{j} = C_i^{j-1}$$

$$P * C_{i-1}^{j} + Q * C_i^j + R * C_{i+1}^{j} = C_i^{j-1}$$

Where $P = \left(-k1 - k2\right)$, $Q = \left(1 + k3 + 2k2\right)$ and $R = \left(k1 - k2\right)$

Where i ranges from 2 to (n-1) [excluding the boundaries x=0 and x=nΔx].

In matrix form :

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & . & . & . & . & 0 & 0 & 0 & 0 & 0 \\ P & Q & R & 0 & 0 & 0 & . & . & . & . & 0 & 0 & 0 & 0 & 0 \\ 0 & P & Q & R & 0 & 0 & . & . & . & . & 0 & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & P & Q & R & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & 0 & P & Q & R \\ 0 & 0 & 0 & 0 & 0 & 0 & . & . & . & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1^j \\ C_2^j \\ C_3^j \\ . \\ . \\ . \\ C_{n-2}^j \\ C_{n-1}^j \\ C_n^j \end{bmatrix} = \begin{bmatrix} C_0 \\ C_2^{j-1} \\ C_3^{j-1} \\ . \\ . \\ . \\ C_{n-2}^{j-1} \\ C_{n-1}^{j-1} \\ C_n^{j-1} \end{bmatrix}$$

$$\left[M^F\right]\left[C^j\right] = \left[R^F\right]$$

The matrix $[M^F]$ is tridiagonal and is constant. At each time step, systems of equations are solved for concentrations at the nodes by forward and backward substitutions using Gauss Elimination Technique.

6. Results and discussion

6.1 Contaminant concentration distribution for decaying species

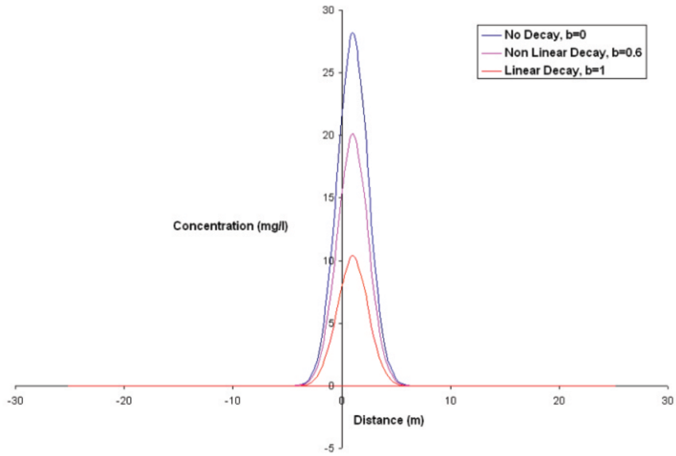

Figure 1 **Contaminant concentration distribution subject to linear decay, non linear decay and no decay after an instantaneous spill**

Figure 1 shows the concentration vs. distance profile at one month after the spill when no decay is present, according to equation (2); when linear decay is present, according to equation (2) setting b=1; and when non-linear decay is present, according to equation (4) setting b=0.6.

This comparison is quantitative and can be used to assess the effect of the non-linear parameter b, since the parameter a has dimensions affected by b. Yet, the graph shows the physical bounds of the plume.

In general, non-linear decay scales down the concentration profile, the degree of which is controlled by the magnitude of b. Since the concentrations are greater than 1 and b<1, most of the linear plume is scaled up with respect to the linear decay plume.

6.2 Results obtained by numerical model

6.2.1 Contaminant concentration distribution species for decaying and sorbing species at different time

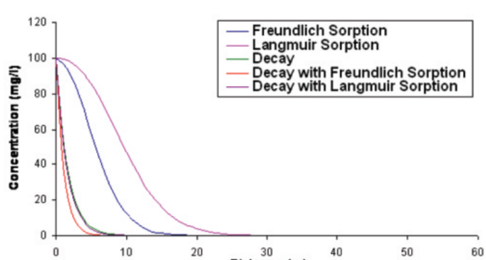

Figure 2 **Concentration Distribution at 10 month**

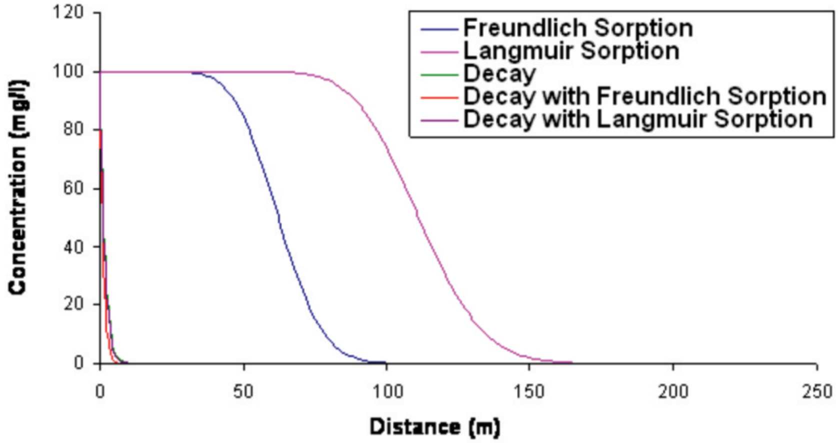

Figure 3 **Concentration Distribution at 120 month**

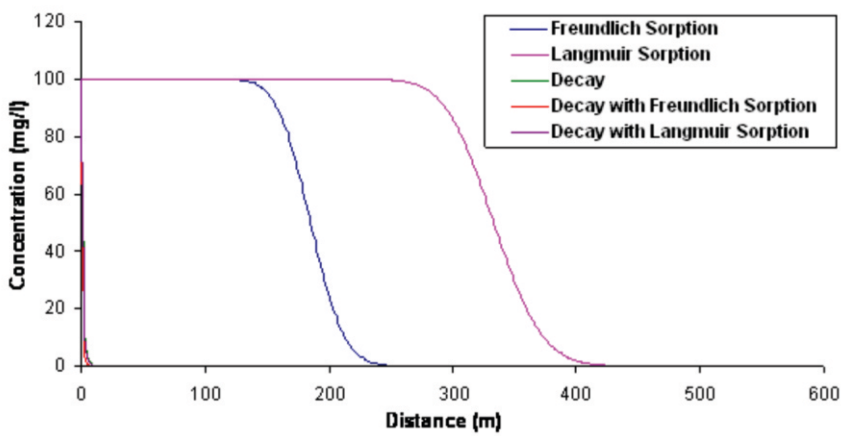

Figure 4 **Concentration Distribution at 360 month**

Figures 2,3 and 4 shows concentration distribution of contaminant species at different time in longitudinal direction.

It is observed that at initial time contaminant species rapidly reduces in domain due to sorption and decay. Contaminant specie dose not travel more distance As time increases plot clearly difference between decay, Freundlich sorption and Langmuir sorption. Contaminant species diminish due to decay. In case of Freundlich sorption and Langmuir sorption solute particle travel long distance in the direction of ground water velocity and slowly reduces its movement and existance. Because of high value of Freundlich retardation factor contaminant species velocity less than velocity of species due to Langmuir sorption.

At the initial few months it is observed that front of concentration distribution curve is smooth and concentration slow decrease. However, as time increases front become sharp. It indicates that after traveling certain distance in long time concentration reduces rapidly.

6.2.2 *Contaminant concentration distribution species for decaying and sorbing species at different distance*

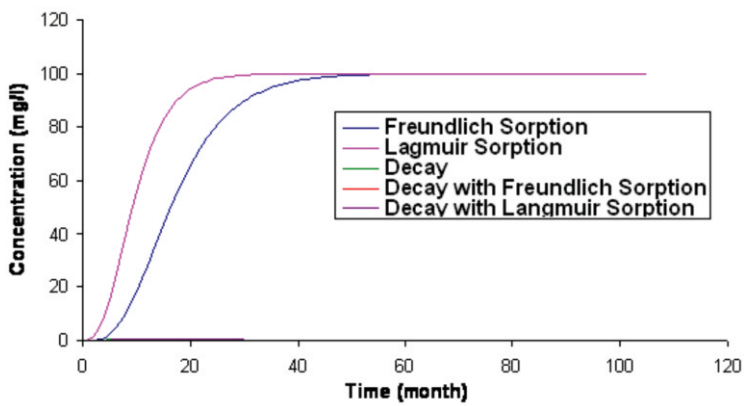

Figure 5 **Concentration Distribution at 1 meter**

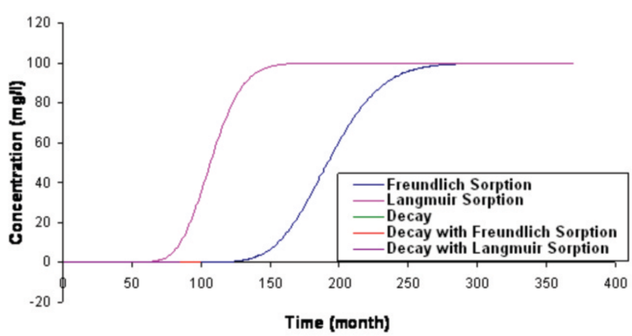

Figure 6 **Concentration Distribution at 100 meter**

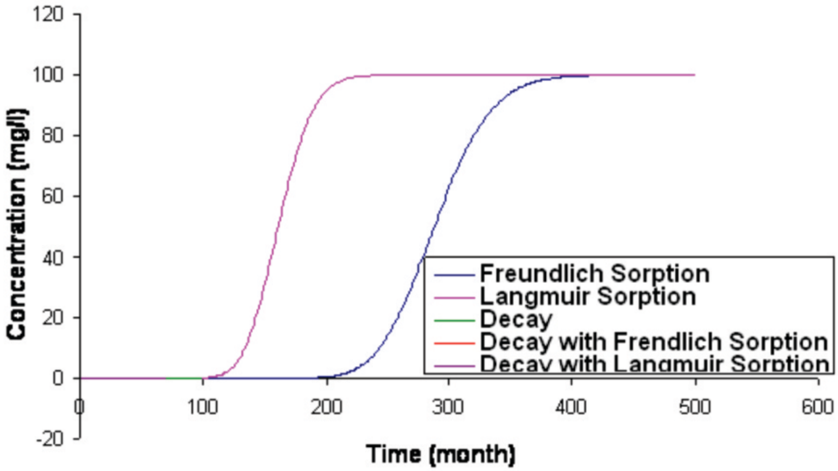

Figure 7 **Concentration Distribution at 150 meter**

Figures 5, 6 and 7 shows concentration distribution of contaminant species at different distance for continuous time.

It is observed that plume of contaminant species reaches early period in case of Langmuir sorption, shows that contaminant species velocity in the longitudinal direction of ground water is high. In case of Freundlich sorption, plume takes more time to reach at particular distance than Langmuir sorption. It indicates that contaminant species velocity more affected due Freundlich sorption than Langmuir sorption.

Contaminant species almost diminishes due to decay. When we take combine effect of decay and sorption, decay is always dominant than sorption. Because of that species concentration reduces rapidly in aquifer.

7. Conclusions

Using the method of decomposition, series solutions were constructed for the advective–dispersive transport equation in aquifers subject to nonlinear decay, non-linear Freundlich sorption, or nonlinear Langmuir sorption. Using the concept of double decomposition, the series were used to obtain analytical simulant solutions, which are closed-form expressions of part of the parent series. The analytic simulants were tested for numerical accuracy with respect to the parent non-linear series solution with an excellent agreement.

Plumes undergoing non-linear decay experience a profile re-scaling with respect to that of linear decay, the degree of which is controlled by the magnitude of the non-linear parameter b. The direction of the scaling (scaling up or scaling down with respect to the linear decay plume) is controlled by the magnitude of b (whether greater or less than 1). When values of $b<1$ produce plumes that experience less decay (i.e., are scaled up) than that of the linear decay, whereas values of $b>1$ produce non-linear plumes that experience more decay (i.e., are scaled down) than that of the linear decay.

However, the approximate analytical models presented here are not capable of predicting the form of a contaminant plume when the initial concentration is large and at the same time a is large, or when > Ci. More research is needed on the identification of simple solutions for the later conditions.

The FDM predictions were found to be in excellent agreement with analytical solutions for a wide range of field conditions with regard to dispersion and source definition. The new developed numerical model can be used for the forecasting of contaminant dispersion under non-linear reactions, or for the quantitative description of the effect of non-linearity in the sorption parameters, on the time-space distribution of the contaminant. The solution for numerical values of state variable only at specified points in the space and time domains defined for the problem. The above FDM model solved by using implicit scheme is unconditionally stable. The proposed models are flexible, stable, and could be used for laboratory or field simulations at early or prolonged contamination scenarios.

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APPLIED MATHEMATICS IN SCIENCE AND TECHNOLOGY

Satywan Malik

Abstract:-

"Behind the artisan is the chemist behind the chemist is the physicist, behind the physicist is a mathematician". The significance of the Journal of applied Mathematics in Science and technology has been emphasized and discussed in details. Diversified applications of it in versatile field of Science have been sought.

Introduction:-

Leaving aside applied mathematics all the other subjects thought in the higher classes can be classified into two groups i.e. subject related to arts group and subject related to science and technology group like physics, chemistry, astronomy, botany, zoology and engineering .It nourishes and in turn gets nourished from Science as well as art that is why it terms as Science of all Sciences.

Need of Relationship among Different subject:-

Whatever may be the form of the society it has an educational Structure for realizing certain aim and objectives. Different subjects of the curriculum help in the realization of these set goals although the courses are different yet they have common goals. The commonness draws them nearer and in this way learning in particular subjects effects the learning of other subjects. It has direct or indirect. Applied mathematics plays a vital role in helping the learning of other subjects. It has direct or indirect relationship with almost all the subjects. Let us begin with the relationship of applied mathematics with science and technology. Relationship of Applied Mathematics with the Science and Technology.

1. Applied Mathematics and Physics: - If we take physics, we see that its study requires, the knowledge of applied Mathematics at every point. All the physical law's, laws of motion, laws of liver and pulley. Laws of refection and reflection, laws of magnetism. Laws of electric current, movement of earth and planets and laws of quantum energy can only be understood and applied with the help of the understanding of Mathematics.

the need of the numerical calculation's in dealing with problem in physics clearly reveal's the value of Mathematics in the learning physics .The lenses and other equipment's used in microscope, telescope, photographic camera movie only be made useful and workable with the help of intensity, power and arrangement decided by the basic principal of applied Mathematics. In this way what we study in physics can only be studied effectively with the proper use of applied mathematics.

Applied Mathematics and Chemistry:-

Study in chemistry is also helped by knowledge of applied Mathematics. The composition and properties of the different elements in chemistry can only be understood properly with Mathematics for Example the type of composition, no matter whether volumetric or gravimetric is decided by law ratio and proportion governed by mathematics. The study of compounds, mixtures laws of chemical combination and the study of molecular or atomic structures chemical names of formula and chemical equations are based on the law of Mathematic in the preparation of different gases and chemical product like bleaching powder , salts acids medicians and other daily used products we need exact measurement in terms of weight ratios other calculations .In this way, What we study in chemistry can only be studied effectively with the proper use of applied Mathematics .

Applied Mathematics and Botany – Zoology:-

In all the experiments and studies of botany and zoology, we take the help of applied Mathematics. The cellular construction of animals and vegetables heredity process of reproduction balanced diet and similar other topics need the knowledge of applied Mathematics.
In any organism if we try to study the anatemooini structure and pattern of definite growth development, we have to take the help of the subject applied Mathematics. The graphs and statistical concepts used in these branches also reveal the need of applied Mathematics.

Applied Mathematic Astronomy:-

Astronomy in one sense is totally based on the learning of applied Mathematics the complicated and intensive study connected with the movements of planets and satellites, their relative attraction, distances and study of their orbits can only be possible with the knowledge of applied Mathematics for an ordinary individual. It would be a great wonder to know that eclipses, tides and the rising and setting of planets sand stars happen at a fixed day and time but for a student of astronomy it is a usual phenomenon conducted through the rules of mathematics. In this way mathematics does not only help in the understanding of astronomy but also renders the astronomy a reliable help in the realization of their dreams to straight on the distant planets like Venus and mars.

Applied Mathematics and Medical Science;

In Medicines the diagnosis as well as remedial treatment is base on the knowledge of Mathematics, Temperature, Blood Pressure, deficiency and excess of minerals and other Substance. The Pressure or absence of undesirable Substances and parasites in the blood, urine and stool tests can only be detected and correctly measured with the adequate Knowledge of mathematics .In preparation of the Doses of Medicines one has to take into account the Mechanism of measurement. Which is not possible without Mathematics? Can we imagine that particular ingredient of the medicine may prove most fatal or injurious to a person if its ratio or quantity is Increased or Decreased a little both nurses and Compounders will feel handicapped in the proper look after of the Patients and preparation of the proper doses of mixture and medicines of they happen to be ignorant of Mathematics.

Applied Mathematics Engineering:-

Applied Mathematics is base of all the Engineering Survey and Measurement which help the Science of Engineering to construct large bridges, plan the network of canals and dam's, extend railway lines across the wide forest and Lofy Mountains control the floods and establish the heavy Industry in the Product of Mathematics. Wise Engineer's with help of the knowledge of Applied Mathematics at command are always in a position to serve. The society and country in any front of Engineering Mechanical, Electrical, Civil, Electronic & Computer Engineering Extra.

In this way, we can realize that Applied Mathematics in quite in dispensable in learning science subjects in actual. The relationship between Applied Mathematics and science is just like the relationship between the body and it soul Body (science) has no meaning without its soul (Mathematics) Soul may have its existence without body But in true sense, The Existence of soul will prove fruitful only When it carries body along with it. In the same way applied Mathematics or science cannot bring any fruitful result. If they are not integrated used in Combination. Whatever we see in the modern world of Science and Technology has its root in the progress and Improvement of Applied Mathematics. That is why "Bacon has said Mathematics is gateway and key of Sciences''

Conclusion:-

Bacon concludes Mathematics as gateway & Key to Science. Science is lame without Mathematics. Once the two join, their versatility & diversity becomes more fruitful to explain the matter in more detail.

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A BRIEF STUDY OF REAL VALUED CONTINUOUS FUNCTION AND ITS APPLICATIONS

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Abstract:

This manuscript the deals with the study of nature of real valued continuous functions, continuity between metric spaces, types of discontinuous function with examples and also discussed the applications of continuous functions .

Introduction :

A continuous function is a function for which small changes in the input result in small changes in the output. Otherwise, a function is said to be a discontinuous function. Continuity of functions is one of the core concepts of topology, which is treated in full generality below. This manuscript focuses on the special case where the inputs and outputs of functions are real numbers. Also this manuscript discusses the definition for the more general case of functions between two metric spaces.

Definition

A function is continuous when its graph is a single unbroken curve.That is not a formal definition, but it helps you understand the idea.

Here is a continuous function.

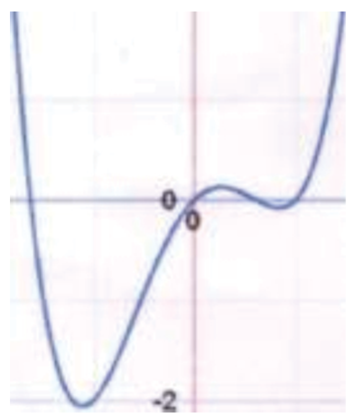


Figure 1: Continuous function

A function is *continuous at a point* if it does not have a hole or jump. It means the value of the function at a point *c* is equal to its limiting value along points that are nearby. Such a point is called a discontinuity. Otherwise, a function is discontinuous, at the points where the value of the function differs from its limiting value.

Definition of Continuity in Terms of Neighborhoods
A function f is continuous at a point c of its domain if, for any neighborhood $N_1(f(c))$ there is a neighborhood $N_2(c)$ such that $f(x)$? $N_1(f(c))$ whenever x Î N $_2$ (c).
This definition does not require any assumption on the nature of the domain. For instance, the function *f* is automatically continuous at every isolated point of its domain. As a specific example, every real valued function on the set of integers is continuous.

e-d de-(Epsilon–Delta) Definition of Continuous Functions

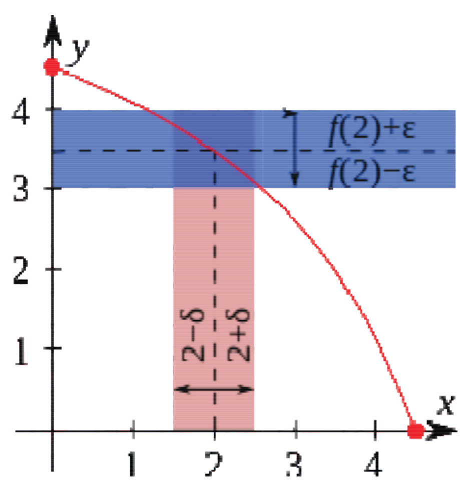


Illustration of the ε-δ-definition: for ε=0.5, c=2, the value δ=0.5 satisfies the condition of the definition.
By de- definition, f is said to be continuous at the point c if for given any number ε > 0, however small, there exists a number δ > 0 such that for all x in the domain of f with c − δ < x < c + δ, the value of f(x) satisfies

$$f(c)-\varepsilon < f(x) < f(c)+\varepsilon.$$

Alternatively

Continuity of f : I → R at d ∈means that for every ε > 0 there exists a δ > 0 such that for all x ∈:

|x-c| < δ→|f (x) - f (c) | < ε

More naturally, we can say that if we want to get all the f(x) values to stay in some small neighborhood around f(c).

Examples

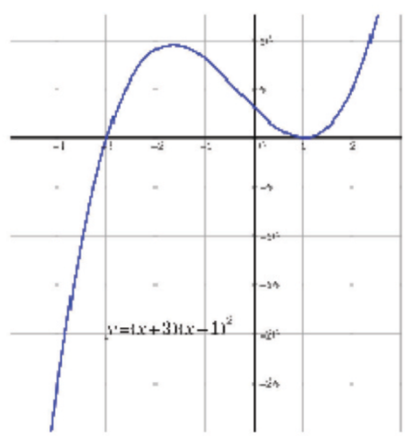


Figure 2 Graph of a cubic continuous function has no jumps or holes

All polynomial functions, such as $f(x) = x^3 + x^2 - 5x + 3$ (pictured), are continuous. This is a consequence of the fact that, given two continuous functions

f,g: I→R

defined on the same domain I, then the sum $f + g$ and the product fg of the two functions are continuous (on the same domain I)

Continuous Functions and Metric Space

The concept of continuous real-valued functions can be generalized to functions between metric spaces. Given two metric spaces (X, d_X) and (Y, d_Y) and a function

f:X→Y

then f is continuous at the point c in X (with respect to the given metrics) if for any positive real number ε, there exists a positive real number δ such that all x in X satisfying $d_X(x, c) < \delta$ will also satisfy $d_Y(f(x), f(c)) < \varepsilon$.

Types of Discontinuous Function

If a jump occurs in the graph of the function then the value of the function at a point differs from its limiting value, then the function is set to discontinuous at that point. There are four types of discontinuities.

(i) Jump discontinuity.

(ii) Point discontinuity

(iii) Essential discontinuity

(iv) Removable discontinuity

Jump Discontinuity

Jump Discontinuity occurs when the curve breaks at a particular point and start somewhere else in this case the right hand limit is not equal to the left hand limit

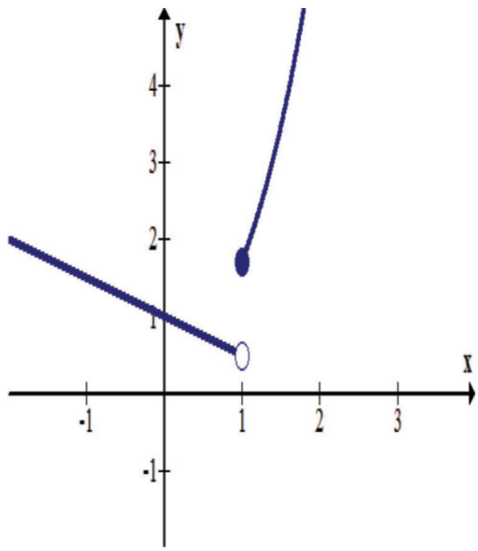


Figure 3: Graph of Jump discontinuous function

Point Discontinuity

Point Discontinuity occurs when the curve has a hole because function has a value that is of the curve at that point. In this case limit of f(x) is not equal to the value of the function at x.

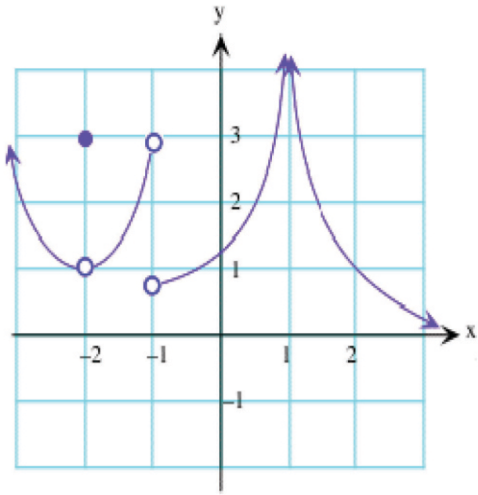


Figure 4: Graph of Point discontinuous function

Essential Discontinuity

Essential discontinuity occurs, when the curve has a vertical asymptote.

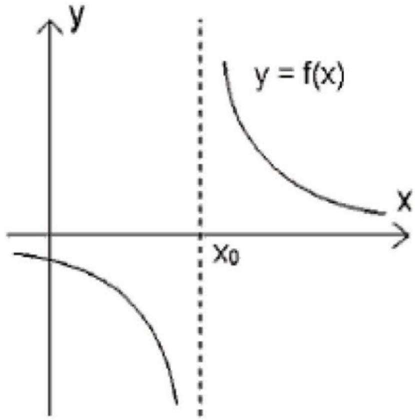


Figure 5: Graph of Essential discontinuous function

Removable Discontinuity

When the left hand limit is equal to the right hand limit but is not equal to the value of the function then this type of discontinuity occurs. And this type of discontinuity can be removed by redefining the function.

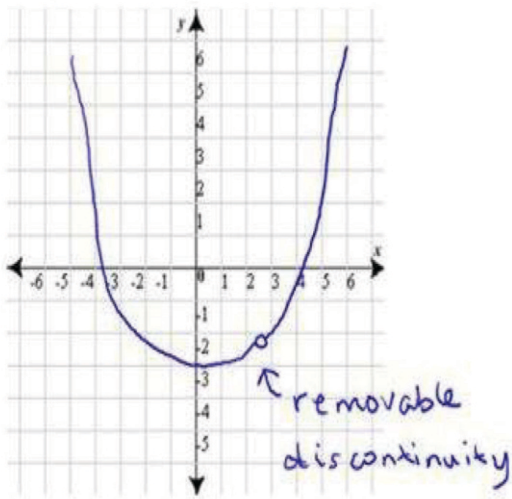
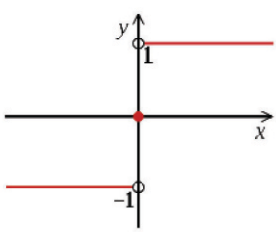


Figure 6: Graph of Removable discontinuous function

Examples of continuous function

(I) Plot of the signum function



The hollow dots indicate that sgn(x) is 1 for all x>0 and −1 for all x<0.

$$\operatorname{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

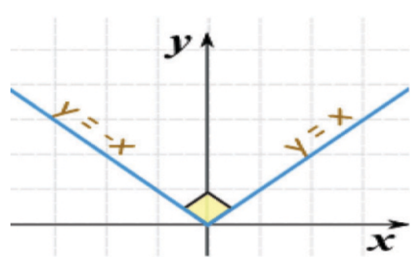
Signum is discontinuous at $x = 0$ but continuous everywhere else.

Another example: the function

$$f(x) = \begin{cases} \sin\left(\frac{1}{x^2}\right) \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases}$$

is continuous everywhere apart from $x = 0$. (2)

$$f(x) = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



At x=0 it has a very pointy change!

But it is still **defined** at x=0, because **f(0)=0** (so no "hole"), And the limit as you approach x=0 (from either side) is also **0** (so no "jump"), So it is in fact **continuous**.

(3) Example

$f(x) = x^2 - 2x + 1$

$\lim_{x\to c} f(x) = \lim_{x\to c} (x^2 - 2x + 1)$

$f(x) = c^2 - 2c + 1$

$f(x)=f(c)$

So, f is continuous at $x = c$

APPLICATIONS

Intermediate value theorem

Intermediate value theorem states that if real-valued function f is continuous on the closed interval $[a, b]$ and k is some number between $f(a)$ and $f(b)$, then there is some number c in $[a, b]$ such that $f(c) = k$. For example, if a child grows from 1 m to 1.5 m between the ages of two and six years, then, at some time between two and six years of age, the child's height must have been 1.25 m. As a consequence, if f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ differ in sign, then, at some point c in $[a, b]$, $f(c)$ must equal zero.

Extreme value theorem

This_ theorem states that if a function f is defined on a closed interval $[a,b]$ (or any closed and bounded set) and is continuous there, then the function attains its maximum, i.e. there exists $c \in [a,b]$ with $f(c) \geq f(x)$ for all $x \in [a,b]$. The same is true of the minimum of f.

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STATISTICAL ANALYSIS OF RAINFALL OVER SEONATH BASIN, CHHATTISGARH, INDIA

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Abstract

The variability in rainfall has been one of the most thought provoking issue for the Indian conditions. Over two-third population of the country having agriculture as their primary occupation and agriculture being too much dependent on rainfall, any anomalies in it significantly affects the Indian economy. Therefore it has become necessary to assess the potential impacts of erratic rainfall and hence, quantification of rainfall variability is necessary. This article presents the temporal variation of rainfall over Seonath river basin, Chhattisgarh, India using statistical non-parametric Mann-Kendall test and Sen's slope estimator test. The daily rainfall data for a period of 32 years (1980-2012) from 39 stations over the basin is collected from Water Resources Department, Chhattisgarh and Central Water Commission, India. The data is checked for autocorrelation and pre-whitening has been done to remove the effects of autocorrelation. The pre-whitened data for all the stations is used for detection of trend in annual rainfall using the statistical tests. The average value of rainfall over the whole basin was estimated by Thiessen polygon method and was analyzed for trend in monthly, seasonal and annual variation. The results reveal that most of the stations are showing an increasing trend for annual rainfall although very few of them are statistically significant (at 5% significance level). For the basin as a whole, a significant positive trend is observed in the month of July and that of negative for October. The trend of annual rainfall is clearly increasing for the whole basin during these 32 years.

Keywords: *rainfall, Mann-Kendall, Sen's slope, trend, monsoon*

1. Introduction

Climate change indicates a different behavior of the hydro-meteorological parameters comparing between two different periods. The climatic variability is not a very short span process. It takes years or decades to have a noticeable change in climate. Whenever the word 'Climate change' is coined, the changes in temperature and erratic rainfall come first into picture. The variation in rainfall and temperature has been arising as a challenging issue for the present generation and it will also remarkably affect the future. From the point of view of India, this may lead to severe detrimental conditions due to poor adaptation strategies and a very high population. Intense flooding and severe drought conditions may prevail in various parts of the country simultaneously (Gosain et al. 2006). This will be further accelerated by the rampant human interventions. But the matter to look into is that, be it a drought or a flood, the amount of natural precipitation will certainly govern these aspects to a significant extent. Moreover, in India, it matters for rainfall due to South-West monsoon i.e. rainfall during June-September.

2. Study Area and Data Used

Seonath river basin is situated in the fertile plains of Chhattisgarh Region. This Basin is situated between 20° 16' N to 22° 41' N Latitude and 80° 25' E to 82° 35' E Longitude. The basin occupies a large portion of the upper Mahanadi valley. Seonath river is a major tributary of Mahanadi river and it traverses a length of 380 km. The area of the basin is 30560 km^2. The basin receives about 1150 mm of mean annual rainfall, mostly in the monsoon season from late June to early October. The daily rainfall data of 39 Meteorological Stations under entire Seonath river basin for a period of 1981 to 2012 (32 years) were collected from State Data Centre, Department of Water Resources, Raipur (Chhattisgarh) and Central Water Commission Office, Bhubaneswar, to examine the temporal and spatial variability of rainfall. The location of the stations in the basin and their respective area is presented through Thiessen polygon map (Fig. 1). Thiessen polygon map was prepared in order to determine the rainfall over the whole basin, so that the trend can be obtained for the whole basin.

Figure 1 Location of rainfall stations in Seonath river basin (Thiessen Polygon map)

3. Methodology

In the present study, trend analysis has been done by using non-parametric Man- Kendall test and Sen's slope estimator test. These are statistical methods that are being used for studying the spatial variation and temporal trends of hydro climatic data series.

Trend detection in a series is largely affected by the presence of a positive or negative autocorrelation (Hamed and Rao, 1998; Serrano et al. 1999; Yue et al. 2003). With a positive autocorrelation in the series, possibility for a series of being detected as having trend is more, which may not be always true. On the other hand, this is reverse for negative autocorrelation in a series, where a trend is not detected. In this case, pre-whitening has been done using lag-1 autocorrelation. Then the results of various statistical methods applied to actual data and pre-whitened data are analyzed.

3.1 *Mann-Kendall Test*

Mann Kendall test is a statistical test widely used for the analysis of trend in climatological and hydrological time series. There are two advantages of using this test. First, it is a non-parametric test and does not require the data to be normally distributed. Second, the test has low sensitivity to abrupt breaks due to non-homogeneous time series.

The Mann-Kendall statistic *S* is given as

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} sgn(x_j - x_i)$$

(1)

The application of trend test is done to a time series x_i that is ranked from i = 1, 2 …n-1 and x_j, which is ranked from j = i+1, 2 ….n. Each of the data point x_i is taken as a reference point which is compared with the rest of the data points, x_j so that,

$$\operatorname{sgn}(x_j - x_i) = \begin{cases} +1, > (xj - xi) \\ 0, = (xj - xi) \\ -1, < (xj - xi) \end{cases}$$

(2)

It has been documented that when n ⩾ 8, the statistic S is approximately normally distributed with the mean.

$$E(S) = 0 \qquad (3)$$

The variance statistic is given as

$$Var(S) = \frac{n(n-1)(2n+5) - \sum_{i=1}^{m} t_i(i)(i-1)(2i+5)}{18}$$

(4)

where, t_i is considered as the number of ties up to sample I.

$$Z_{mk} = \begin{cases} \frac{S-1}{\sqrt{Var(S)}}, S > 0 \\ 0, S = 0 \\ \frac{S+1}{\sqrt{Var(S)}}, S < 0 \end{cases} \qquad (5)$$

Z_{mk} follows a standard normal distribution. A positive (negative) value of Z signifies an upward (downward) trend. A significance level α is also utilized for testing either an upward or downward monotone trend (a two-tailed test). If Z_{mk} appears greater than $Z_{\alpha}/2$ where α depicts the significance level, then the trend is considered as significant. Generally, Z_{mk} values are 1.645, 1.960 and 2.576

for significance level of 10%, 5% and 1% respectively.

3.2 *Sen's Slope Estimator Test*

The magnitude of trend is predicted by the Sen's slope estimator. Here, the slope (Ti) of all data pairs is computed as (Sen*, 1968)*

$$T_i = \frac{x_j - x_k}{j-k} \quad \text{For i = 1,2,3…..N} \qquad (6)$$

Where *xj* and *xk* are considered as data values at time j and k (j>k) correspondingly.

The median of these N values of Ti is represented as Sen's estimator of slope which is given as

$$Q_i = \begin{cases} T_{\frac{N+1}{2}}, & \text{N is odd} \\ \frac{1}{2}\left(T_{\frac{N}{2}} + T_{\frac{N+2}{2}}\right), & \text{N is even} \end{cases} \qquad (7)$$

Positive value of Q indicates an increasing trend and a negative value of Q_i gives a decreasing trend in the time series.

4. Results and Discussion

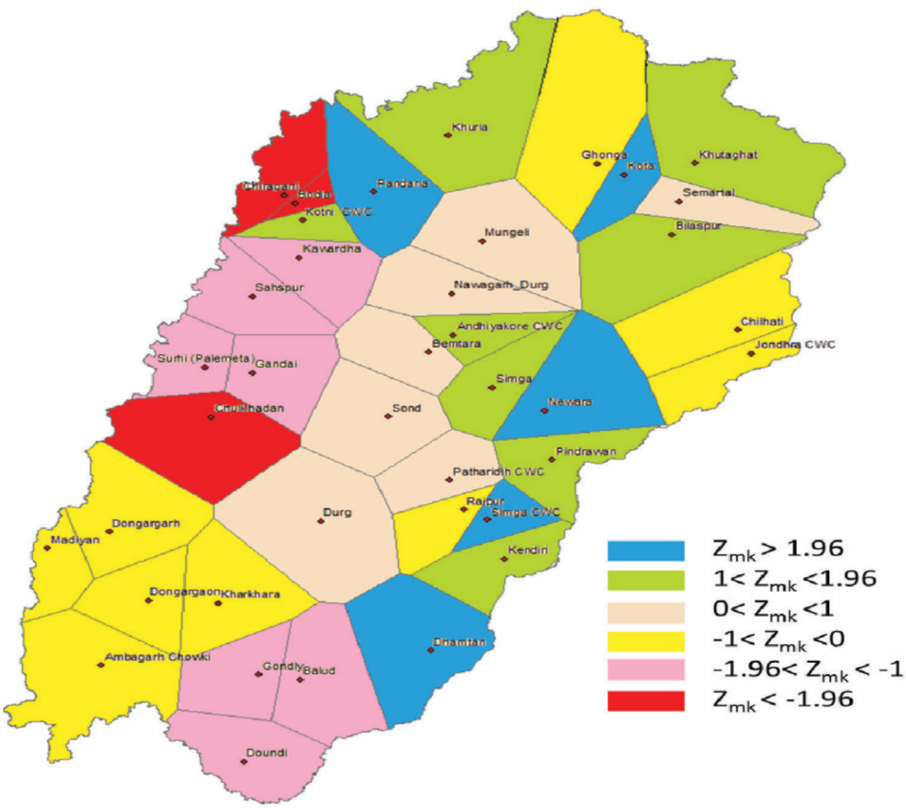

Figure 2 Mann-Kendall Test results for annual rainfall for pre-whitened data

The statistical tests are applied on the pre-whitened dataset. From fig. 2, it can be clearly observed that, out of 39 stations, 20 stations are showing a positive value of Z_{mk} whereas 19 stations are showing negative values. The Mann-Kendall co-efficient Z_{mk} value for 5% significance level is 1.96. Thus it can be observed that only 5 stations are showing a positive trend whereas only 3 stations are showing a negative trend at 5% significance level.

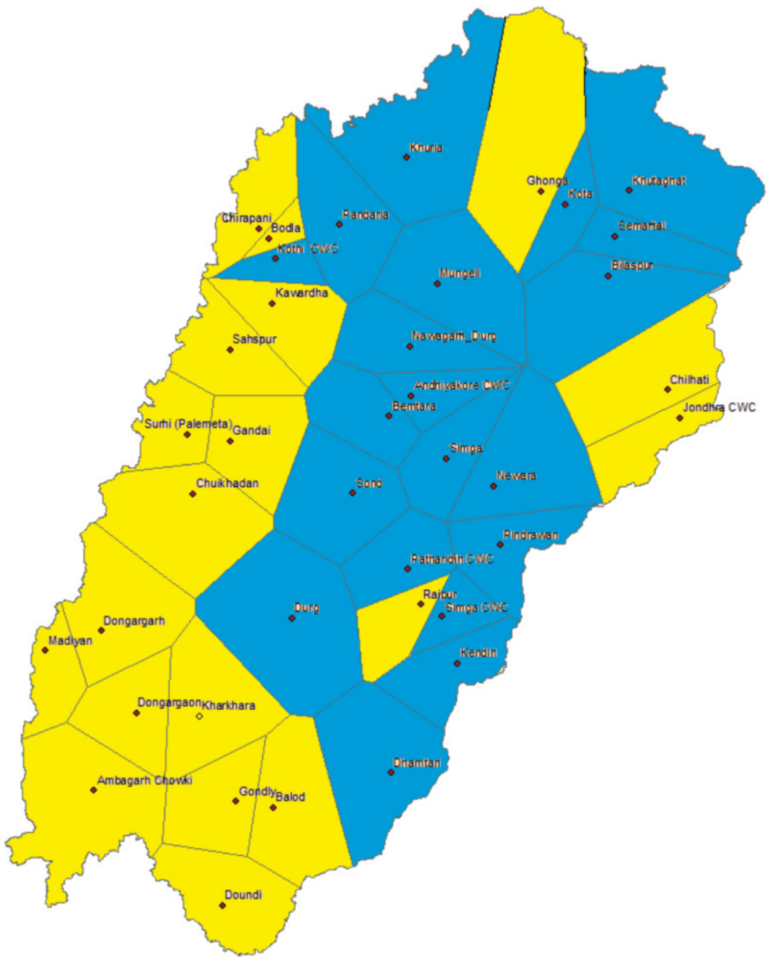

Figure 3 Sen's Slope Estimator Test results for annual rainfall for pre-whitened data

The results of Sen's slope estimator test are quite similar to that of Mann-Kendall test results. From fig. 3, it can be observed that, 20 stations are showing increasing trend in annual rainfall for pre-whitened data and the rest decreasing.

The trend of rainfall over different stations is determined using the non-parametric tests. But it is essential to determine the behavior of rainfall over the whole basin. The Thiessen polygon method was used to determine the rainfall over the whole Seonath river basin.

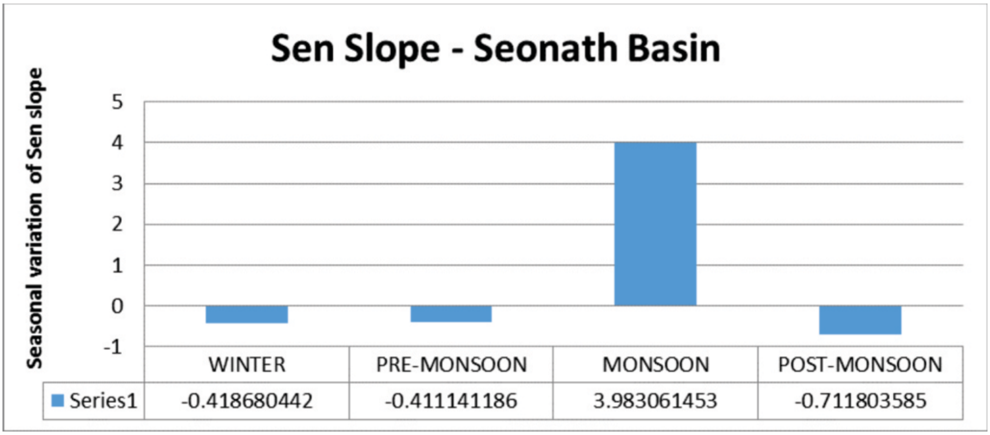

Figure 4 Sen's slope results for Seasonal rainfall

Table 1 Monthly Sen's slope results for rainfall

Month	Variance	Mean	Median	Standard Deviation	Sen's slope(β)	(% change α)
Jan	314.34833	13.30411	3.67972	17.72987	-0.03946	-0.094904329
Feb	117.38473	9.446052	5.39712	10.83442	-0.22121	-0.749369926
Mar	88.979965	8.214486	5.336704	9.432919	-0.02805	-0.109279824
Apr	29.140871	6.213002	4.745383	5.398229	0.024628	0.126847691
May	131.97939	10.84519	7.09259	11.48823	-0.17392	-0.51316768
Jun	3481.5742	150.3759	140.6504	59.00487	-0.33211	-0.070672857
Jul	9137.973	303.7481	294.1403	95.59275	3.528383	0.371716677
Aug	4333.2955	298.6511	303.0616	65.82777	-0.14358	-0.015384195
Sep	3056.0606	165.8706	163.2684	55.28165	1.059016	0.204307029
Oct	751.28343	41.5091	38.57057	27.40955	-0.72155	-0.556257041
Nov	373.96891	9.93016	1.533419	19.33828	0	0
Dec	164.88507	5.32481	1.092111	12.84076	-0.00489	-0.029405063

Fig. 4 shows seasonal rainfall, a high positive slope is observed i.e. Sen's slope value is 3.983 for monsoon season and negative for other seasons. From Table 1, it can be observed that the Sen's slope is positive for April, July and September, zero for November and negative for all other months. The Sen's slope value for annual rainfall over entire Seonath river basin is also 2.832, showing a clear increase in annual rainfall for the study area in these 32 years.

5. Conclusions

Statistical analysis of annual rainfall data for Seonath river basin, Chhattisgarh, for 32 years (1981-2012) using Mann-Kendall and Sen's slope estimator test, has been presented in this article. For annual rainfall, the Z_{mk} value of Mann-Kendall Test was positive for 5 stations and negative for only 3 stations at 5% significance level for pre-whitened data, although most of the stations showed positive value of Mann-Kendall co-efficient. The Sen's slope values for most of the stations were also found to be positive. For overall Seonath river basin, an observable rising trend was there for the months of July and September and decreasing trend for February, May and October. For seasonal variation, the trend is clearly positive for monsoon season from Sen's slope estimator test.

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