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Aims and Scope

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Contents

An Alternate Efficient Sorting Algorithm Applicable for Classification of Versatile Data

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ABSTRACT

This paper plans a new alternate sorting technique, "Zigzag Sort" which has exclusively been developed to sort an odd number of elements. The initial pass (step) involves scanning elements in the left-to-right direction pair-wise and swapping elements in each pair as per the necessity to keep elements in each pair in the sorted order with keeping the last element unprocessed. The next pass involves the identical approach in the reverse direction. Thus in a zigzag way, one in the left-to-right direction, followed by the next in the right-to-left direction, a maximum of n passes are executed to sort n elements offering the complexity of O(n2). Under our numerical observations, discussions and conclusions, ithas been revealed that the "Zigzag Sort" technique is an efficient and stable sorting technique for a versatile data.

Keywords: Sorting, Swapping, Data Structure

1. Introduction

In data structures, a sorting algorithm is an algorithm that arrays input elements of a list in a certain order. There are many sorting techniques available like "Selection Sort", "Bubble Sort", "Marge Sort", "Quick Sort", "Heap Sort", "Bin Sort", "Radix Sort". Several researchers [1, 2…., 9] confined their attention in this connection and explored number ofdifferent sorting algorithms. There is a "Folk Theorem" to the effect that sorting n elements "requires n log n time". We observed that the statement of Folk Theorem is not always true; if the key type is such that "Bin Sort" or "Radix Sort" can be used to advantage, then O(n) time suffices. However, these sorting algorithms believe on keys being of a special type i.e. a type with a limited set of values. All the other general sorting algorithms we have studied believe only on the fact that we can test whether one key value is less than another. We should notice that in some of the sorting algorithms progress toward determining the proper order for the elements is made when we compare two keys, and then the flow for control in the algorithm goes one of only two ways. In contrast, an algorithm like “Bin Sort” causes one of n different things to happen in only one step, by storing a record with an integer key in one of n bins depending on the value of that integer. All programs use a capability of programming languages and machines that is much more powerful than a simple comparison of values, namely the ability to find in one step a location in an array, given the index of that location [1]. “Zigzag Sort” is exclusively developed for sorting an odd number of elements indirectly influenced by the existing Sorting Algorithms. The algorithm carries an overhead as the process of sorting an even number of elements using this technique can only be initiated by inserting the reasonably largest element of equivalent type into the list to make the number an odd one and finally to discard the same from the sorted list. The initial pass in the algorithm involves scanning elements in the left-to-right direction pair-wise and swapping elements in each pair as per the necessity to keep elements in each pair in the sorted order with keeping the last element in the sequence unprocessed. The following pass involves the identical approach in the reverse direction. Thus in a zigzag way, one in the left-to-right direction, followed by the next in the right-to-left direction, a maximum of N passes are executed to complete the process of sorting N elements. The time to execute the sorting a set of elements using this approach evidently increases with the number of elements. Section 2 presents the algorithmic approach. An application of the proposed algorithm is shown in section 3. Results and analysis based on these results are presented in section 4. Section 5 depicts conclusionson the entire approach

2. Algorithmic Approach

The algorithm [3] of the “Zigzag Sort” illustrated in Figure 2.1:

```
ZIGZAG_SWAP_SORT (ARRAY [N], N)
If N is even then N = N+1 and Array [N-1] = Highest Type Value
    1.  Start
    2.  Loop from i = 0 to N-1
    3.  If i is even
        Loop from K = 0 to K <= (N/2)-1
        If Array [2*K] > Array [2*K+1]
        Interchange Array [2*K] and Array [2*K+1]
        End of if
        End of Loop
        End of outer if
    4.  If i is odd
        Loop from K = 0 to K <= (N/2)-1
        If Array [(N-1) - 2*K] < Array [(N-1) - (2*K+1)]
        Interchange Array [(N-1) - 2*K] and Array [(N-1) - (2*K+1)]
        End of if
        End of Loop
        End of outer if
    5.  End of outer Loop  (2)
    6.  End
```

Figure: 2.1: Algorithm of the Proposed Sorting Technique

3. Application of Proposed Sorting Algorithm with Numerical Example

In Section 3, we discuss application aspect of the proposed sorting technique. To serve our purpose, we consider two cases of odd and even number of input elements. Case I illustrates an implementation for an odd number of elements, whereas Case 2 illustrates the same for an even number of elements.

Case I. Application for Odd Number of Input Elements

Suppose seven elements are taken, which are as follows:

20 50 100 10 2 500 400

For these above seven elements, position of the each element in the array after each pass will be as follows:

AFTER PASS 1:	20	50	10	100	2	500	400
AFTER PASS 2:	20	10	50	2	100	400	500
AFTER PASS 3:	10	20	2	50	100	400	500
AFTER PASS 4:	10	2	20	50	100	400	500
AFTER PASS 5:	2	10	20	50	100	400	500
AFTER PASS 6:	2	10	20	50	100	400	500
AFTER PASS 7:	2	10	20	50	100	400	500

In this example, sorting has been completed after pass 5 but loop has been repeated twice more (pass 6 and pass 7). This sorting technique is only applicable for odd number of elements, but in case of even number of elements this is only applicable through an approach stated separately.

Case II. Application for Even Number of Input Elements

Suppose eight elements has been considered, which are as follows:

10 1 15 800 100 400 300 5

As the number of elements inputted through keyboard is even, first we have to make the number of elements odd. So, an external element 2147483647 has been inserted at the end of the list of values.

After insertion of external element the list of unsorted values will be as follows:

10 1 15 800 100 400 300 5 2147483647

For the above list of values, the position of each element of the array after each pass will be as follows:

AFTER PASS 1:	1	10	15	800	100	400	5	300	2147483647
AFTER PASS 2:	1	10	15	100	800	5	400	300	2147483647
AFTER PASS 3:	1	10	15	100	5	800	300	400	2147483647
AFTER PASS 4:	1	10	15	5	100	300	800	400	2147483647
AFTER PASS 5:	1	10	5	15	100	300	400	800	2147483647
AFTER PASS 6:	1	5	10	15	100	300	400	800	2147483647
AFTER PASS 7:	1	5	10	15	100	300	400	800	2147483647
AFTER PASS 8:	1	5	10	15	100	300	400	800	2147483647
AFTER PASS 9:	1	5	10	15	100	300	400	800	2147483647

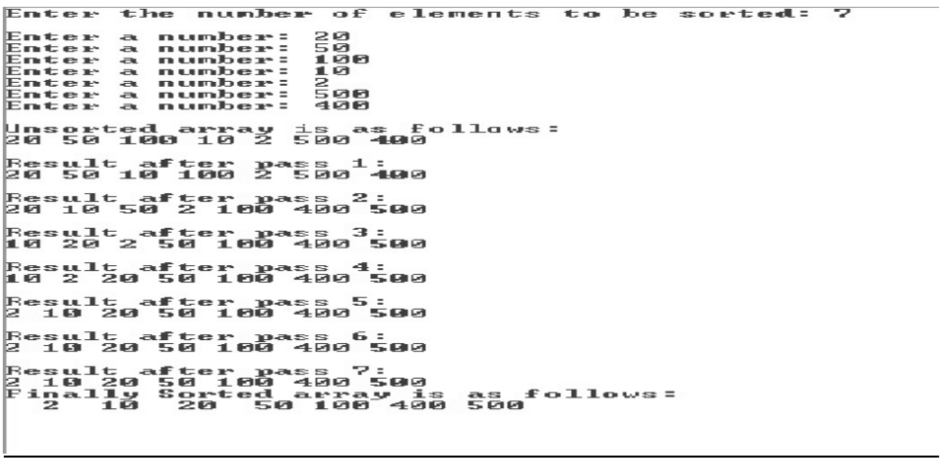
In the above passes, after pass 6 sorting has been completed, but loop has been repeated thrice more (pass 7, pass 8 and pass 9).

4. Result and Analysis

Section 4.1 illustrates execution for odd number of inputted elements and section 4.2 illustrates execution for even number of inputted elements. Section 4.3 is a presentation of analysis based on the observed result.

4.1 Results for Odd Number of Elements

Figure 4.1 is an extract from the screen showing the result of execution for odd number of elements.

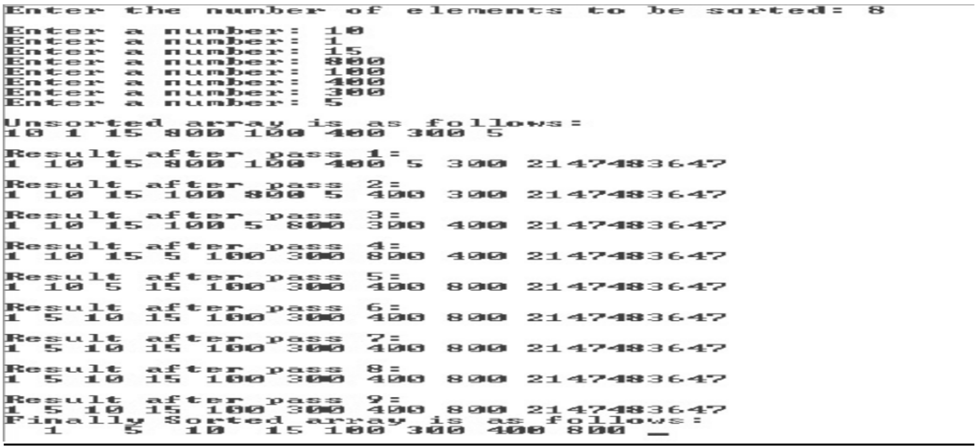


```
Enter the number of elements to be sorted: 7
Enter a number: 20
Enter a number: 50
Enter a number: 100
Enter a number: 10
Enter a number: 2
Enter a number: 500
Enter a number: 400
Unsorted array is as follows:
20 50 100 10 2 500 400
Result after pass 1:
20 50 10 100 2 500 400
Result after pass 2:
20 10 50 2 100 400 500
Result after pass 3:
10 20 2 50 100 400 500
Result after pass 4:
10 2 20 50 100 400 500
Result after pass 5:
2 10 20 50 100 400 500
Result after pass 6:
2 10 20 50 100 400 500
Result after pass 7:
2 10 20 50 100 400 500
Finally Sorted array is as follows:
  2  10  20  50 100 400 500
```

Figure: 4.1: Results for odd number of elements

4.2 Results for Even Number of Elements

Figure 4.2 is an extract from the screen showing the result of execution for even number of elements.



```
Enter the number of elements to be sorted: 8
Enter a number: 10
Enter a number: 1
Enter a number: 15
Enter a number: 800
Enter a number: 100
Enter a number: 400
Enter a number: 300
Enter a number: 5
Unsorted array is as follows:
10 1 15 800 100 400 300 5
Result after pass 1:
1 10 15 800 100 400 5 300 2147483647
Result after pass 2:
1 10 15 100 800 5 400 300 2147483647
Result after pass 3:
1 10 15 100 5 800 300 400 2147483647
Result after pass 4:
1 10 15 5 100 300 800 400 2147483647
Result after pass 5:
1 10 5 15 100 300 400 800 2147483647
Result after pass 6:
1 5 10 15 100 300 400 800 2147483647
Result after pass 7:
1 5 10 15 100 300 400 800 2147483647
Result after pass 8:
1 5 10 15 100 300 400 800 2147483647
Result after pass 9:
1 5 10 15 100 300 400 800 2147483647
Finally Sorted array is as follows:
   1    5   10   15 100 300 400 800 _
```

Figure: 4.2: Results for even number of elements

It is calculated that 2147483647 is the highest integer as per Bloodshed Dev-C++ compiler. We have used this number as an external element to make the number of elements odd.

4.3 Analysis based on the Observed Result

Table: 4.3.1: Data on number of inputs and respective execution times

No. of input(s)	Execution time
0	0.001
1	0.001
2	0.002
3	0.003
4	0.003
5	0.003
6	0.004
7	0.003
8	0.003

No. of input(s)	Execution time
9	0.004
10	0.003
11	0.004
15	0.005
20	0.01
25	0.012
30	0.014
35	0.014
40	0.02

No. of input(s)	Execution time
45	0.022
50	0.025
60	0.028
70	0.031
80	0.035
90	0.041
100	0.043
110	0.048
130	0.055

No. of input(s)	Execution time
150	0.062
170	0.075
200	0.081
205	0.085
210	0.086
220	0.089
230	0.091
250	0.099
270	0.106

No. of input(s)	Execution time
300	0.1
330	0.097
350	0.1
370	0.105
400	0.11

4.4 Graphical Representation of Number of Input(s) vs. Execution Time

The graphical representation of execution is illustrated in Figure 4.4.1.

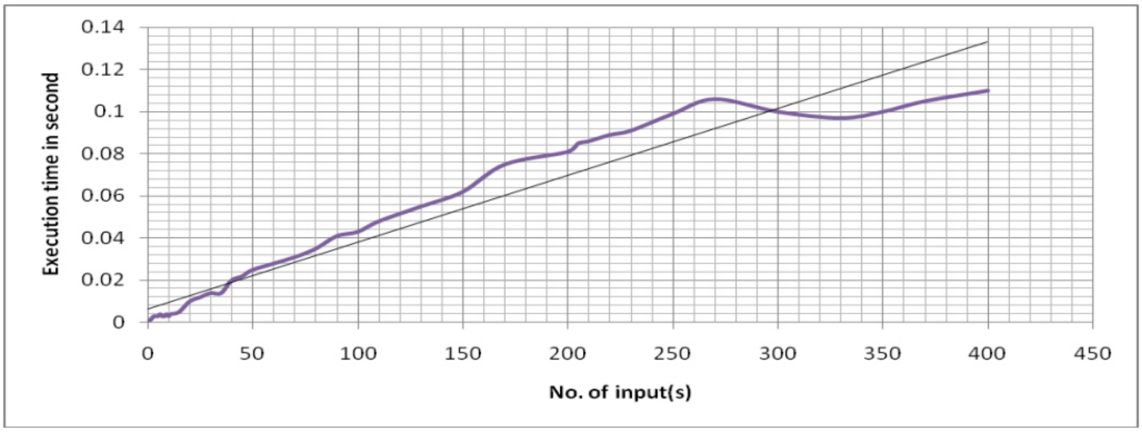


Figure: 4.4.1: Graphical representation of number of inputs and execution time

The execution is done at the platform with following configurations:

Processor:	1st generation Intel Dual Core @ 2.0 GHz
Main memory:	1 GB
Operating System:	Windows XP Service Pack 2
Input taking method:	The input(s) are taken randomly

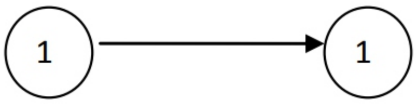
The graphical representation is derived from data given in Table: 4.3.1

4.5 Mathematical Induction of "<=n" Steps or Passes

Let, P(n) implies that, n number of elements will be sorted in <= n steps/passes.

In case of P(1), number of elements =1;

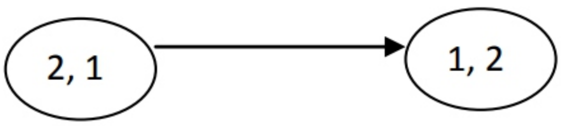
And 1 element will be sorted in one step that can be written directly:



Therefore n=1, then P(1) is true, as it takes n i.e. 1 step. That is number of step/pass taken <= 1.

Again, when n = 2; number of elements = 2;

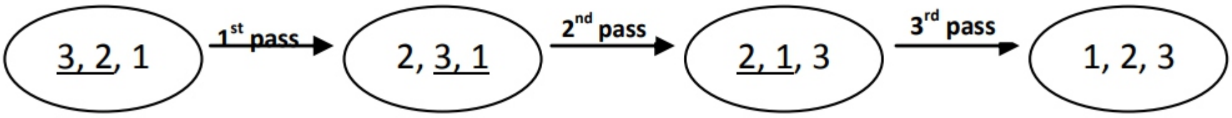
These two elements can be sorted in 1 step/pass:



Therefore when n=2, then P(2) is true, as it takes <= 2 steps/passes.

Again, when n = 3, number of elements = 3;

These three elements can be sorted in 3 steps/passes.



Therefore when n = 3.then P(3) is true as it takes <= 3 steps/passes.

Let we assume that P(m) is true;

That is at m(< n) it is true.

Therefore P(m): m number of elements sorted in <= m steps = k

In the case of (m+1);

Therefore for (m+1) elements:

m elements will be sorted by P(m) in <= m steps, and for the next element[(m+1)th element], it will take one more pass/step to arrange.

So, for sorting (m+1) elements we require <= (m+1) steps/passes.

Therefore P(m+1) is true as it takes <= (m+1) passes/steps.

Hence by mathematical induction it shows that to arrange n number of elements it takes maximum of n steps/passes.

4.6 Stability of Proposed Sorting Algorithm

Stable sorting algorithms maintain the relative order of records with equal keys. A key is that portion of the record which is the basis for the sort; it may or may not include all of the record. If all keys are different then this distinction is not necessary. But if there are equal keys, then a sorting algorithm is stable if whenever there are two records (let's say Rand S) with the same key, and R appears before S in the original list, then R will always appear before S in the sorted list. When equal elements are indistinguishable, such as with integers or more generally, any data where the entire element is the key, stability is not an issue.

The sorting of proposed algorithm is stable enough. Stability of this sorting technique can be examined from the following example:

5 elements have been taken to be sorted which are as follows:

	7	9(a)	5	1	9(b)
AFTER PASS 1:	7	9(a)	1	5	9(b)
AFTER PASS 2:	7	1	9(a)	5	9(b)
AFTER PASS 3:	1	7	5	9(a)	9(b)
AFTER PASS 4:	1	5	7	9(a)	9(b)
AFTER PASS 5:	1	5	7	9(a)	9(b)

4.7 Time Complexity

The complexity of the proposed sorting technique, "Zigzag Sort" algorithm if is used to sort n elements is observed as O(n2).

5. Conclusion and Policy Recommendation

In this paper, a sorting technique, "Zigzag Sort" has been proposed for a versatile data. Graphical representation of number of input(s) vs. execution time has been displayed in fig. 4.4.1 and Mathematical Induction of the suggestedsorting technique is described in sub-section 4.4. The developed sorting technique, "Zigzag Sort" is sufficiently stablewhich is utmost useful to reduce the execution time. From this point of view the suggested sorting technique is quite efficient with an overhead of adding a reasonably largest element in the case of even number of elements. Further process is on to eliminate the overhead.

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Acknowledgements

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Improved upper bound on the *L(2, 1)*-labeling of Cartesian sum of graphs1

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ABSTRACT

An L(2, 1)-labeling of a graph G is defined as a function f from the vertex set V (G)into the nonnegative integers such that for any two vertices x, y, $|f(x)-f(y)| \geq 2$ if $d(x, y) = 1$ and $|f(x) - f(y)| \geq 1$ if $d(x, y) = 2$, where d(x, y) is the distancebetween x and y in G. The L(2, 1)-labeling number $\lambda_{2,1}(G)$ of G is the smallestnumber k such that G has an L(2, 1)-labeling with $k = \max\{f(x)|x \in V(G)\}$. Inthis paper, we consider the graph formed by the Cartesian sum of two graphs andgive new upper bound of the L(2, 1)-labeling number, which improves the boundobtained by [Z.D.Shao, D.Zhang, The L(2, 1)-labeling on Cartesian sum of graphs,Applied Mathematics Letters 21(2008) 843-848.] significantly.

AMS subject classification: *05C15, 05C78.*

Keywords: L(2, 1)-labeling, Cartesian sum, frequency assignment.

1. Introduction

An L(2, 1)-labeling of a graph G is defined as a function f from the vertex set V (G) into the nonnegative integers such that for any two vertices x, y, $|f(x) - f(y)| \geq 2$ if $d(x, y) = 1$ and $|f(x) - f(y)| \geq 1$ if $d(x, y) = 2$, where d(x, y), the distance of x and y, is the length of a shortest path between x and y. A k-L(2, 1)-labeling is an L(2, 1)-labeling such that no label is greater than k. The L(2, 1)-labeling number $\lambda_{2,1}(G)$ of Gis the smallest number k such that G has a k-L(2, 1)-labeling.

The L(2, 1)-labeling problem comes from the frequency assignment problem. The task of the frequency assignment problem is to assign a frequency to each of the given transmitters such that the interference between nearby transmitters is avoided and the span of the assignment between the largest and smallest frequencies is minimized. Hale [1] first formulated the frequency assignment problem into a graph vertex coloring problem. Later Roberts [2] proposed a variation of the problem in which there are two levels of inference, “close” and “very close”, depending on the distance between the transmitters. In order to reduce the inference, any two “close” transmitters must receive different frequencies, and any two “very close” transmitters must receive frequencies by at least two apart. The transmitters are represented by the vertices of a graph. There is an edge between two vertices if the corresponding transmitters are “very close”. Two vertices are defined “close” if they are of distance two in the graph. Then in 1992, Griggs and Yeh [3] formulated the L(2, 1)-labeling of graphs.

The L(2, 1)-labeling of graphs has been extensively studied in the past decade [3- 5,8-18]. Most of these papers consider the values of $\lambda 2,1(G)$ on particular classes of graphs. However, Griggs and Yeh [3] proved an upper bound $2 + 2$ for any simple graph with maximum degree . Later, Chang and Kuo [4] improved the bound to $\lambda 2,1(G) \leq 2 +$. And Gonçalves [5] decreased it to $2 + \; - 2$. In general, Griggs and Yeh [3] suggested the following conjecture:

Conjecture 1.1. For any graph G with maximum degree $\Delta \geq 2$, $\lambda_{2,1}(G) \leq \Delta^2$.

We can claim that this is the most famous open problem in this area [6]. If G is a diameter 2 graph, then $\lambda_{2,1}(G) \leq \Delta^2$ [3]. Papers [8] and [9] proved that the conjecture is true for three classes of graphs that are the direct and strong product and the cartesian product of general non trivial graphs. Van den Heuvel and McGuinnes [10] proved that the conjecture holds for planar graphs with maximum degree $\Delta \geq 7$. Sakai [11] proved that chordal graphs satisfy the conjecture and more precisely that $\lambda_{2,1}(G) \leq \frac{1}{4}(\Delta + 3)^2$. See [6, 7] for more information.

Graph products play an important role in the graph labeling problems. The Cartesian sum of two graphs G and H is the graph $G \bigoplus H$ with vertex set $V(G) \times V(H)$, and edge set

$$E(G \bigoplus H) = \{((u_1, v_1), (u_2, v_2)) | u_1 u_2 \in E(G) \text{ or } v_1 v_2 \in E(H)]\}.$$

In this paper, we consider the graph formed by the Cartesian sum of two graphs and improve the upper bound of the L(2, 1)-labeling number obtained by Shao and Zhang [12] significantly.

2. A labeling algorithm

For any fixed positive integer k, a k-stable set of a graph G is a subset S of V (G) such that every two distinct vertices in S are of distance greater than k. A 1-stable set is an usual independent set. A maximal 2-stable subset S of a set F is a 2-stable subset of F such that S is not a proper subset of any 2-stable subset of F.

Chang and Kuo [4] proposed the following algorithm for obtaining an L(2, 1)- labeling and the maximum value of that labeling on a given graph.

Algorithm 2.1.

Input: A graph G = (V , E).

Output: The value l is the maximum label.

Idea: In each step, find a maximal 2-stable set from these unlabeled vertices that are distance at least 2 away from those vertices labeled in the previous step. Then label all vertices in that 2-stable set with the index i in current stage. The index i starts from 0 and the increases by 1 in each step. The maximum label l is the final value of I. Initialization: Set S−1 =   ; V = V (G); i = 0.

Iteration:

1. Determine F_i and S_i.
   $F_i = \{x \in V : x \text{ is unlabeled and } d(x, y) \geq 2 \text{ for all } y \in S_{i-1}\}$
   S_i is a maximal 2-stable subset of F_i.
   If $F_i = \emptyset$ then set $S_i = \emptyset$.
2. Label these vertices in S_i(if there is any) by i.
3. $V \leftarrow V \setminus S_i$.
4. $V \neq \emptyset$ then $i \leftarrow i + 1$; go to Step 1.
5. Record the current i as l (which is the maximal label). Stop.

Therefore l is an upper bound on $\lambda_{2,1}(G)$. We would like to find a bound in terms of the maximum degree $\Delta(G)$ of G.

Let x be a vertex with the largest label l obtained by Algorithm 2.1. Define
$I_1 = \{i : 0 \leq i \leq l - 1 \text{ and } d(x, y) = 1 \text{ for some } y \in S_i\}$,
$I_2 = \{i : 0 \leq i \leq l - 1 \text{ and } d(x, y) = 2 \text{ for some } y \in S_i\}$,
$\overline{I_2} = \{i : 0 \leq i \leq l - 1 \text{ and } d(x, y) \leq 2 \text{ for some } y \in S_i\}$,
$I_3 = \{i : 0 \leq i \leq l - 1 \text{ and } d(x, y) \geq 3 \text{ for all } y \in S_i\}$.

It is clear that $|\overline{I_2}| + |I_3| = l$. For any $i \in I_3$, $x \notin F_i$, for otherwise $S_i \cup \{x\}$ is a 2-stable of F_i, which contradicts the choice of S_i. That is, $d(x, y) = 1$ for some vertices y in S_{i-1}, i.e., $i - 1 \in I_1$. This implies $|I_3| \leq |I_1|$. Hence

$$l = |\overline{I_2}| + |I_3| \leq |\overline{I_2}| + |I_1| \leq 2|I_1| + |I_2|.$$

In order to find l, it suffices to estimate $2|I_1| + |I_2|$ in terms of $\Delta(G)$.

3. The Cartesian sum of graphs

By the definition of the Cartesian sum $G \bigoplus H$ of two graphs G and H, if $\Delta(G) = 0$ or $\Delta(H) = 0$, then $G \bigoplus H$ is disjoint copies of H or G. Therefore we assume $\Delta(G) \geq 1$ and $\Delta(H) \geq 1$.

In this section, we obtain an upper bound in terms of the maximum degree of $G \bigoplus H$ for any two graphs G and H.

Theorem 3.1. Let Δ, Δ_1, Δ_2 be the maximum degree of $G \bigoplus H$, G, H and v_1, v_2 be the number of vertices of G, H respectively. Then

$$\lambda_{2,1}(G \bigoplus H) \leq \Delta^2 + v_2\Delta_1 + v_1\Delta_2 + \Delta_1\Delta_2 - \Delta_1\Delta_2^2(v_1 + 2) - \Delta_1^2\Delta_2(v_2 + 2) - \Delta_1^2\Delta_2^2(v_1 + v_2 - 4).$$

Proof. Let $x = (u, v)$ be a vertex of $G \bigoplus H$ with the largest label l obtained by Algorithm 2.1. Denote

$$d = \deg_{G \bigoplus H}(x), d_1 = \deg_G(u), d_2 = \deg_H(v).$$

Then

$$d = v_2 d_1 + v_1 d_2 - d_1 d_2$$

and

$$\Delta = \Delta(G \bigoplus H) = v_2 \Delta_1 + v_1 \Delta_2 - \Delta_1 \Delta_2.$$

The goal of the proof is to evaluate $|I_1|$ and $|I_2|$. Obviously, $|I_1| = d \le \Delta$, $|I_2| \le d(\Delta - 1)$. Next, we will investigate the better upper bound of $|I_2|$ by the structure of the Cartesian sum $G \bigoplus H$.

Let the number of vertices in G with distance 2 from u be r, then $0 \le r \le d_1(\Delta_1 - 1)$. If $r > 0$, then for any vertex u'' in G with distance 2 from u, there must be a path $uu'u''$ of length 2 between u'' and u in G. Note that $\deg_H(v) = d_2$, i.e., v has d_2 adjacent vertices in H, and so by the definition of the Cartesian sum $G \bigoplus H$, there must be $d_2 v_1 + 1$ internally disjoint paths of length 2 between (u'', v) and (u, v) in $G \bigoplus H$. Hence for any vertex in G with distance 2 from u, there must be corresponding $d_2 v_1 + 1$ vertices with distance 2 from $x = (u, v)$ which are coincided in $G \bigoplus H$. And they can only be counted once, we have to deduct $d_2 v_1$ from the value $d(\Delta - 1)$. If $r = 0$, there do not exist such corresponding $d_2 v_1 + 1$ vertices with distance 2 from $x = (u, v)$ which are coincided in $G \bigoplus H$, we have to deduct $d_2 v_1 + 1$ from the value $d(\Delta - 1)$. The maximum number of vertices with distance 2 from $x = (u, v)$ in $G \bigoplus H$ occurs when $r = d_1(\Delta_1 - 1)$, and hence in this sense the number of vertices with distance 2 from $x = (u, v)$ in $G \bigoplus H$ will decrease $d_1(\Delta_1 - 1)d_2 v_1$ from the value $d(\Delta - 1)$.

For H, we can analyze similarly with above paragraph and know that the number of vertices with distance 2 from $x = (u, v)$ in $G \bigoplus H$ will still decrease $d_2(\Delta_2 - 1)d_1 v_2$ from the value $d(\Delta - 1)$.

Also in this sense, for any vertex u'' in G with distance 2 from u and any vertex v'' in H with distance 2 from v, (u'', v'') and (u, v) are distant 2 in $G \bigoplus H$. By the definition of the Cartesian sum $G \bigoplus H$, there must be $v_1 + v_2 - 1$ internally disjoint paths of length 2 between (u'', v'') and (u, v) in $G \bigoplus H$, and be corresponding $v_1 + v_2 - 1$ vertices with distance 2 from $x = (u, v)$ which are coincided in $G \bigoplus H$. Hence the number of vertices with distance 2 from $x = (u, v)$ in $G \bigoplus H$ will still decrease $d_1(\Delta_1 - 1)d_2(\Delta_2 - 1)(v_1 + v_2 - 2)$ from the value $d(\Delta - 1)$.

Let F be the subgraph induced by the neighbors of x. Whenever there is an edge in F, the number of vertices with distance 2 from x in $G \bigoplus H$ will decrease by 2. Denote

For any adjacent vertex u' of u in G and any adjacent vertex v' of v in H, (u', v) and (u_s, v') (where u_s is any vertex of G) are all adjacent to (u, v) in $G \bigoplus H$. By the definition of $G \bigoplus H$, there also must be an edge between (u', v) and (u_s, v'). And there are totally d_1 neighbors (u', v) of $x = (u, v)$ and totally $v_1 d_2$ neighbors (u_s, v') of $x = (u, v)$, hence the number of edges of the subgraph F induced by the neighbors of x is at least $v_1 d_2 d_1$. By a symmetric analysis and excluding the coincided edges between (u, v') and (u', v), the edges of F should again add at least $v_2 d_1 d_2 - d_1 d_2$.

For any vertex u'' in G with distance 2 from u, there must be a path $uu'u''$ of length 2 between u'' and u in G. Then (u'', v') (where v' is adjacent to v in H) and (u', v_t) (where v_t is any vertex of H except for v) are all adjacent to (u, v) in $G \bigoplus H$. And there also must be an edge between (u'', v') and (u', v_t). Note that there are totally $d_2 d_1(\Delta_1 - 1)$ vertices (u'', v') and totally $v_2 - 1$ vertices (u', v_t) (where v_t is any vertex of H except v), hence the edges of F should again add at least $d_2 d_1(\Delta_1 - 1)(v_2 - 1)$. By a symmetric analysis and excluding the coincided edges between (u'', v') and (u', v''), the edges of F should again add at least

$$d_1 d_2(\Delta_2 - 1)(v_1 - 1) - d_1(\Delta_1 - 1)d_2(\Delta_2 - 1).$$

Therefore, we have

$$\begin{aligned} e &\geq v_1 d_2 d_1 + v_2 d_1 d_2 - d_1 d_2 + d_2 d_1(\Delta_1 - 1)(v_2 - 1) \\ &\quad + d_1 d_2(\Delta_2 - 1)(v_1 - 1) - d_1(\Delta_1 - 1)d_2(\Delta_2 - 1) \\ &= d_1 d_2(\Delta_2 v_1 + \Delta_1 v_2 - \Delta_1 \Delta_2). \end{aligned}$$

Hence the number of vertices with distance 2 from $x = (u, v)$ in $G \bigoplus H$ will decrease

$$\begin{aligned} &d_1(\Delta_1 - 1)d_2 v_1 + d_2(\Delta_2 - 1)d_1 v_2 \\ &\quad + d_1(\Delta_1 - 1)d_2(\Delta_2 - 1)(v_1 + v_2 - 2) \\ &\quad + 2d_1 d_2(\Delta_2 v_1 + \Delta_1 v_2 - \Delta_1 \Delta_2) \end{aligned}$$

from the value $d(\Delta - 1)$ altogether.

Thus, we have $|I_1| = d \leq \Delta$,

$$\begin{aligned} |I_2| &\leq d(\Delta - 1) - d_1(\Delta_1 - 1)d_2 v_1 - d_2(\Delta_2 - 1)d_1 v_2 \\ &\quad - d_1(\Delta_1 - 1)d_2(\Delta_2 - 1)(v_1 + v_2 - 2) \\ &\quad - 2d_1 d_2(\Delta_2 v_1 + \Delta_1 v_2 - \Delta_1 \Delta_2). \end{aligned}$$

Then

$$\begin{aligned} l \leq 2|I_1| + |I_2| &\leq 2d + d(\Delta - 1) - d_1(\Delta_1 - 1)d_2 v_1 \\ &\quad - d_2(\Delta_2 - 1)d_1 v_2 - d_1(\Delta_1 - 1)d_2(\Delta_2 - 1)(v_1 + v_2 - 2) \end{aligned}$$

$$
\begin{aligned}
&- 2d_1d_2(\Delta_2 v_1 + \Delta_1 v_2 - \Delta_1\Delta_2)\\
&\quad = (v_2 d_1 + v_1 d_2 - d_1 d_2)(\Delta + 1) - d_1(\Delta_1 - 1)d_2 v_1\\
&- d_2(\Delta_2 - 1)d_1 v_2 - d_1(\Delta_1 - 1)d_2(\Delta_2 - 1)(v_1 + v_2 - 2)\\
&- 2d_1d_2(\Delta_2 v_1 + \Delta_1 v_2 - \Delta_1\Delta_2) \triangleq f(d_1, d_2).
\end{aligned}
$$

Then $f(d_1, d_2)$ has the absolute maximum when $d_1 = \Delta_1, d_2 = \Delta_2$ for $0 \leq d_1 \leq \Delta_1, 0 \leq d_2 \leq \Delta_2$. And

$$
\begin{aligned}
f(\Delta_1, \Delta_2) \;=\;& \Delta^2 + v_2\Delta_1 + v_1\Delta_2 - \Delta_1\Delta_2 - \Delta_1(\Delta_1 - 1)\Delta_2 v_1\\
&-\Delta_2(\Delta_2 - 1)\Delta_1 v_2 - \Delta_1(\Delta_1 - 1)\Delta_2(\Delta_2 - 1)(v_1 + v_2 - 2)\\
&-2\Delta_1\Delta_2(\Delta_2 v_1 + \Delta_1 v_2 - \Delta_1\Delta_2)\\
=\;& \Delta^2 + v_2\Delta_1 + v_1\Delta_2 - \Delta_1\Delta_2(\Delta_2 v_1 + \Delta_1 v_2 - 4\Delta_1\Delta_2\\
&+\Delta_1\Delta_2 v_1 + \Delta_1\Delta_2 v_2 + 2\Delta_1 + 2\Delta_2 - 1)\\
=\;& \Delta^2 + v_2\Delta_1 + v_1\Delta_2 + \Delta_1\Delta_2 - \Delta_1\Delta_2^2(v_1 + 2)\\
&-\Delta_1^2\Delta_2(v_2 + 2) - \Delta_1^2\Delta_2^2(v_1 + v_2 - 4).
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\lambda_{2,1}(G \bigoplus H) \leq l \leq\;& \Delta^2 + v_2\Delta_1 + v_1\Delta_2 + \Delta_1\Delta_2\\
&- \Delta_1\Delta_2^2(v_1 + 2) - \Delta_1^2\Delta_2(v_2 + 2)\\
&- \Delta_1^2\Delta_2^2(v_1 + v_2 - 4).
\end{aligned}
$$

In [12], it is proved that

$$
\begin{aligned}
\lambda_{2,1}(G \bigoplus H) \leq\;& \Delta^2 - v_1(\Delta_1 - 1)\Delta_2 - v_2(\Delta_2 - 1)\Delta_1\\
&- (\Delta_1 + \Delta_2)\Delta_1\Delta_2 - \Delta_1 - \Delta_2 + 1.
\end{aligned}
$$

Because

$$
\begin{aligned}
&\Delta^2 - v_1(\Delta_1 - 1)\Delta_2 - v_2(\Delta_2 - 1)\Delta_1\\
&\quad - (\Delta_1 + \Delta_2)\Delta_1\Delta_2 - \Delta_1 - \Delta_2 + 1\\
&\quad - (\Delta^2 + v_2\Delta_1 + v_1\Delta_2 + \Delta_1\Delta_2\\
&\quad - \Delta_1\Delta_2^2(v_1 + 2) - \Delta_1^2\Delta_2(v_2 + 2)\\
&\quad - \Delta_1^2\Delta_2^2(v_1 + v_2 - 4))\\
&\quad = (\Delta_2 - 1)(\Delta_1(\Delta_2 + 1) - 1)\\
&\quad + \Delta_1\Delta_2(\Delta_1 + \Delta_2 v_1 + \Delta_1 v_2 - 5)\\
&\quad + \Delta_1\Delta_2(\Delta_1\Delta_2 - 1)(v_1 + v_2 - 4) \geq 0,
\end{aligned}
$$

we reduce the bound significantly.

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On the Job Training: A Step Towards Job SatisfactionA Case Study of Public Sector Organization in Indian Scenario

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ABSTRACT

Present research paper deals with problems of public sector employee on the job training from the job satisfaction point of view in the present Indian scenario, where the public sector and the private sector are in a neck to neck competition. This paper examines the level of job satisfaction and its strategic implementation and implications towards the retention of the employees and the development human resources of the organization in particular and the society in general. In this paper, our central attention has been made to find scores on job satisfaction index (JSI) for employees of all cadres officers, supervisors and workers of a public sector organization. It is proposed here that public sector employee in officer's category has scored 53/100 on Job Satisfaction Index whereas the supervisors of public sector employees have scored 116/200. The workers of public sector employee have scored 465/700. The composite awareness index of public sector employees is 634/1000, which is 63.4 percent. This paper is based on the research conducted in the domain of selected public sector organizations. As far as the government/public sector employees are concerned, the constituent respondents are mainly from Indian Railways, Indian Defense Services, Gujarat State Electricity Board, Gujarat University and its Constituents College's teaching and non-teaching staff, banking sector employees etc. We have examined the satisfaction level of the employee of selected public sector organizations, 200 employees in the officers, supervisors and the worker's category were selected based on proportionate, purposive and random sampling technique. By following this technique a sample of 20 officers, 40 supervisors and 140 workers were collected, from various Public Sector Organizations of India.

Key words- Public sector employees, OJT, Job Satisfaction, Strategic Implementation, Human Resource Development

1. Objectives of Case Study

The main objectives of the present study are:

1. To find out the level of job satisfaction among the employees of public sector organizations, with respect to their on the job training facilities provided by their organization
2. To find out difference of job satisfaction among officers, supervisors and workers, with respect to pre service training facilities provided by their organization facilities

2. Definition of Job Satisfaction

The term job satisfaction is commonly referred in the context of employee's behaviour at work. Job satisfaction can be understood more clearly in the context of employee's extent of satisfaction in general in his total work/professional life situations.

Job satisfaction has been defined as a pleasurable emotional state resulting from the appraisal of one's job, an affective reaction to one's job; and an attitude towards one's job. Weiss (2002) has argued that job satisfaction is an attitudinal concept but points out that researchers should clearly distinguish the objects of cognitive evaluation which are affect (emotion), beliefs and behaviours. This definition suggests that we form attitudes towards our jobs by taking into account our feelings, our beliefs, and our behaviours.

3. Universe and Sampling

For the purpose of the present study, both Public and private sector organizations have been selected. As far as the government / public sector employees are concerned, the constituent respondents are mainly from Indian Railways, Indian Defence Services, Gujarat State Electricity Board, Gujarat University and its Constituents College's teaching and non-teaching staff, banking sector employees etc. It has been selected because of the proximity to the researcher's work place.

It was decided to examine the satisfaction level of the employee of selected Public and private sector organizations, 400 employees in the officers, supervisors and the worker's category were selected based on proportionate, purposive and random sampling technique. The respondents belong to 15 states and union territories of India. By following this technique a sample of 40 officers, 80 supervisors and 280 workers were collected, equally from Public and Private sector organizations.

4. Determinants of job satisfaction

The following are the main factors of job satisfaction:

Job satisfaction is closely related to the aptitude of the employees
Democratic leadership style enhances the job satisfaction
Workers with high moral will be loyal to the organization
Job perspective affects job satisfaction considerably
Interpersonal relationship affects job satisfaction proportionately
Facilities provided by the organization enhance the job satisfaction proportionately
Job satisfaction relates to good working condition also
Economic rewards play a significant role in influencing job satisfaction
Job satisfaction is directly and indirectly related to the organizational culture & climate

5. On-The-Job Training

5.1 Definition

0n the job training are the instructions which may theoretical or practical depending upon the job requirement, which takes place when an individual joins an organization or a new assignment before a person begins a job or task.

5.2 Benefit 0n the Job Training

Here are the benefits of on the job training.

(i) The trainees can take all the time they need to focus on their learning without having other responsibilities.

(ii) The work environment and situations are provided with which the individual employee is likely to work during his service career especially in that particular organization, where the individual is going on the job training.

(iii) 0n The Job Training moulds the employees in a required direction and fills the gap between the present skill level and the required skill level. The employer is spending a lot of financial and other resources on the job training of employees to achieve the organizational goals.

5.3 Drawbacks 0n the Job Training

Here are disadvantages of -0n The Job Training:

(i) The real-life situations differ for trainees from one organization to another.

(ii) The situation and the internal and external environment may differ at a given point of time, as a result of dynamism

A general question was posed to the respondents regarding on the job training facility provided to the employee by the employer, and their satisfaction thereof.

5.4 On-The-Job Training- A Step Towards Job Satisfaction

Expert OJT is a structured on-the-job training system based on high performance principles of expert workers co-workers. Expert OJT recognizes that workers can best be trained at the job site under real life conditions. Expert OJT is a proven and award-winning program for delivering low-cost and high-quality training.Expert OJT builds self-sufficiency and worker ownership that reduces dependency on outside training providers and materials. Put skill standards to work by applying performance outcomes to your process-specific applications and equipment. Train SMART, not hard!

Topics Covered

Plan a Structured On-the-Job Training (OJT) Project

Develop Training Lists & Performance Expectations

Prepare SMART Job Aids

Develop OJT Lesson Plans

Prepare Training & Curriculum Schedules

Conduct Structured On-the-Job Training Sessions

What You Will Learn

How to identify critical work activities to make sure OJT is directed at essential skills that make a difference in your productivity.

How to plan OJT by completing project planners and training schedules that capture the most effective and efficient learning sequence.

How to prepare SMART Job Aids that tell workers how to perform a task. Job aids are actually used for training on-the-job and can serve as Standard Operating Procedures (SOPs).

How and when to write OJT Lesson Plans. Lesson Plans educate the worker about the process and provide a structured learning experience to guarantee that training is done correctly the first time.

Techniques for presenting structured on-the-job training using state of the art, one-to-one instructional methods.

How to integrate OJT with classroom, video and CD-ROM learning to reinforce critical skills and ensure teamwork in the training design

To what extent they are satisfied with their on the job training facility, various responses made by the respondents in this regards are categorized below:-

(i) Not Available,

(ii) Available,

(iii) Reasonable,

(iv) Good,

(v) Excellent

Table 1: Satisfaction with respect to on the job training of public sector employees

Satisfaction	Officer	Supervisor	Worker	Total
1	2	3	11	16
2	8	10	26	44
3	6	18	32	56
4	3	6	49	58
5	1	3	22	26
Total	20	40	140	200

5.5 Satisfaction with Respect to On the Job Training of Public Sector Employees

There are only 2 officers out of 20 officers of public sector employees, which form 10 percent of their respective sample, who have given their satisfaction rating as the 1 out of 5. There are 8 officers out of 20 officers of public sector employees, which form 40 percent of their respective sample, who have given their satisfaction rating as the 2 out of 4. There are 6 officers out of 20 officers of public sector employees, which forms 30 percent of their respective sample, who have given their satisfaction rating as the 3 out of 5. There are 3 officers out of 20 officers of public sector employees, which constitute 15 percent of their respective sample, who have given their satisfaction rating as 4 out of 5.

There is only 1 officer out of 20 in the sample and which forms 5 percent of their sample, who is satisfied to the highest level and given the satisfaction grading of 5 out of 5, who is fully satisfied with, on the job training.

There are 3 supervisors out of 40 supervisors of public sector employees, which form 7. 5 percent of their respective sample, who have given their satisfaction rating as the lowest, which is 1 out of 5. There are 10 supervisors out of 40 supervisors of public sector employees, which form 25 percent of their respective sample, who have given their satisfaction rating as low as 2 out of 5. There are 18 supervisors out of 40 supervisors of public sector employees, which forms 45 percent of their respective sample, who have given their medium satisfaction rating as 3 out of 5. There are 6 supervisors out of 40 supervisors of public sector employees, which form 15 percent of their respective sample, who are satisfied to a higher level and given the satisfaction grading of 4 out of 5. There are 3 supervisors out of 40 supervisors of public sector employees in this sample, which forms 7.5 percent of their respective sample, who are satisfied to the highest level and given the satisfaction grading of 5 out of 5, this implies that 7.5 percent of supervisors of public sector employees are completely satisfied with their on the job training.

There are 11 workers out of 140 workers of public sector employees, which form 7.86 percent of their respective sample, who have given their satisfaction rating as the lowest, which is 1 out of 5. There are 26 workers out of 140 workers of public sector employees, which form 18.57 percent of their respective sample, who have given their satisfaction rating as low as 2 out of 5. There are 32 workers out of 140 workers of public sector employees, which form 22.86 percent of their respective sample, who have given their medium satisfaction rating as 3 out of 5. There are 49 workers out of 140 workers of public sector employees, which form 35 percent of their respective sample, who are satisfied to a higher level and given the satisfaction grading of 4 out of 5.

There are 22 workers out of 140 workers of public sector employees, which forms 15.71 percent of their respective sample, who are satisfied to the highest level and given the satisfaction grading of 5 out of 5, this implies that 15.71 percent of workers of public sector employees are completely satisfied with respect to their on the job training.

There are 16 respondents out of 200 respondents of public sector employees, which form 8 percent of their respective sample, who have given their satisfaction rating as the lowest, which is 1 out of 5. There are 44 respondents out of 200 workers of public sector employees, which form 22 percent of their respective sample, who have given their satisfaction rating as low as 2 out of 5. There are 56 respondents out of 200 respondents of public sector employees, which form 28 percent of their respective sample, who have given their medium satisfaction rating as 3 out of 5. There are 58 respondents out of 200 respondents of public sector employees, which form 29 percent of their respective sample, who are satisfied to a higher level and given the satisfaction grading of 4 out of 5.

There are 26 respondents out of 200 respondents of public sector employees, which form 13 percent of their respective sample, who are satisfied to the highest level and given the satisfaction grading of 5 out of 5, this implies that 13 percent of respondents of public sector employees are completely satisfied with the attitude with respect to their on the job training.

6. Comparative Analysis

The public sector employee in officer's category has scored 53/100 on Job Satisfaction Index which is 53 percent of their category, whereas the supervisors of public sector employees have scored 116/200 on Job Satisfaction Index which is 58 percent of their category. The workers of public sector employee have scored 465/700 on Job Satisfaction Index which is 65 percent of their category. The composite job satisfaction index of public sector employees is 634/1000, which is 63.4 percent.

7. Findings of the Present Case Study

There is only 1 officer out of 20 in the sample and which forms 5 percent of their sample, who is satisfied to the highest level and given the satisfaction grading of 5 out of 5, who is fully satisfied with, on the job training.

There are 3 supervisors out of 40 supervisors of public sector employees in this sample, which forms 7.5 percent of their respective sample, who are satisfied to the highest level and given the satisfaction grading of 5 out of 5, this implies that 7.5 percent of supervisors of public sector employees are completely satisfied with their on the job training.

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The composite job satisfaction index of public sector employees is 634/1000, which is 63.4 percent. The workers category of public sector employees is the most satisfied category with 65 percent satisfaction scored on JSI. Rightly said the proverb that those who have more, they desire more and who have less they manage with that. The supervisors among public sector employees are moderately satisfied as far as on the job training is concerned and the officers among the public sector employees are the least satisfied with training provided by their respective organizations.

8. Conclusion

In this research paper, we have obtained composite job satisfaction index of public sector employees. The composite job satisfaction index of public sector employees is 634/1000, which is 63.4 percent. India being a developing country, with the constrained financial resources, the satisfaction level is reasonably quite good.

Acknowledgements

Dr. V. N. Maurya; author of the present paper and former founder Director at Vision Institute of Technology, Aligarh (Uttar Pradesh Technical University, Lucknow (India), former Principal/Director at Shekhawati Engineering College (Rajasthan TechnicalUniversity, Kota) and former Professor & Dean Academics, Institute of Engineering & Technology, Sitapur, UP, India; is now the Professor & Head of Shekhawati Engineering College (Rajasthan Technical University, Kota). He is the Chief Editor of Editorial Board of American Journal of Modeling and Optimization; Science and Education Publishing, NewYork, USA and Statistics, Optimization and Information Computing; International Academic Press, Hong Kong and Advisory Editor of World Research Journal of Numerical Analysis

and Mathematical Modeling; Bioinfo Publications, Pune, India and Member of Editorial and Reviewer Board of over 50 Indian and Foreign International journals published by leading publishers of USA, Italy, Hong Kong, Austria, U.K., Algeria, Nigeria and other European and African countries.. He has been associated with leading Indian Universities-U. P. Technical University, Lucknow during 2005-06 and Chhatrapati Shahu Ji Maharaj University, Kanpur for three terms during 2000-2004 for significant contribution of his supervision as Head Examiner of Central Evaluation for Theory Examinations of UG (B.Tech./B.Pharm.) and PG (MA/M.Sc.) programmes.

Dr. Maurya was born on 15th July 1974 and he is having an outstanding academic record. He earned his M.Sc. and Ph.D. Degree in Mathematics & Statistics with specialization in Operations Research with First Division from Dr. Ram Manohar Lohia Avadh University, Faizabad, UP, India in the year 1996 and 2000 respectively and thereafter he accomplished another two years Master's Professional Degree-MBA with First Division (B+ Grade) with specialization in Computer Science from NU, California, USA in 2003. His Ph.D. Thesis titled as "A study of use of stochasticprocesses in some queueing models" submitted to Department of Mathematics & Statistics, Dr. R.M.L. Avadh University, Faizabad under supervision of Prof. (Dr.) S.N. Singh, Ph.D. (BHU); was offered to publish in Scholar's Press Publishing Co., Saarbrucken, Germany in view of his excellent research work. Since his primary education to higher education, he has been a meritorious scholar and recipient of meritorious scholarship. He started his teaching career as Lecturer in 1996 to teach post-graduate courses MBA, MCA and M.Sc. and later he was appointed as Professor & Head, Department of Applied Sciences and Engineering at Singhania University, Rajasthan in the year 2004. Since then, Prof. V. N. Maurya has rendered his services as Professor & Head/Dean as well as keen Researcher for PostDoctoral research and he has devoted his entire scientific and professional career in teaching at various premier technical institutions of the country such as at Haryana College of Technology & Management, Kaithal (Kuruchhetra University, Kuruchhetra); Institute of Engineering & Technology, Sitapur and United College of Engineering & Research, Allahabad. On the basis of significant research work carried out by him in the last 17 years of his professional career, Prof. V. N. Maurya has authored three textbooks and published more than 55 scientific and academic research papers including 25 research papers as Principal Author based on his Post-Doctoral work and D.Sc. Thesis in Indian and Foreign leading International Journals in the field of Mathematical and Management Sciences, Industrial Engineering & Technology. Some of his published research papers in India, USA, Algeria, Malaysia and other European and African countries are recognized as innovative contributions in the field of Mathematical and Physical Sciences, Engineering & Technology. Prof. V. N. Maurya is an approved Supervisor of UGC recognized various Indian Universities for Research Programs leading to M. Phil. & Ph.D. such as Shridhar University, Pilani (Rajasthan), Singhania University, Rajasthan and CMJ University, Sillong, Meghalaya and JJT University Jhunjhunu, Rajasthan and U.P. Technical University Lucknow etc. and since last 7 years, he is actively engaged as Research Supervisor of M. Phil. & Ph.D. Scholars in wide fields of Operations Research, Optimization Techniques, Statistical Inference, Applied Mathematics, Operations Management and Computer Science. He has guided as Principal Supervisor and Co-Supervisor to several Research Scholars of M. Phil. and Ph.D.

During his tenure as the Director, Vision Institute of Technology, Aligarh (Uttar Pradesh Technical University, Lucknow) and as the Principal, Shekhawati Engineering College (Rajasthan Technical University, Kota); massive expansion of infrastructure, research facilities, laboratories upgradation/augmentation and other relevant facilities and services for B.Tech./M.Tech./MBA academic programmes in different branches had taken place to accommodate and facilitate the campus students. His major contribution was to enhance the result of weaker students of their University Examination. He planned strategically and developed some tools and methods and then finally implemented for getting successfully considerable better result of campus students particularly in numerical papers.

Prof. Maurya is also on active role of Fellow/Senior/Life Member of various reputed National and International professional bodies of India and abroad including Operations Research Society of India, Kolkata; Indian Society for Technical Education, New Delhi; Indian Association for Productivity, Quality & Reliability, Kolkata; Indian Society for Congress Association, Kolkata; International Indian Statistical Association, Kolkata; All India Management Association, New Delhi; Rajasthan Ganita Parishad, Ajmer and International Association of Computer Science & Information Technology, Singapore etc.

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Game Information Dynamics and Its pplication to Congkak and Othello

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A B S T R A C T

This paper is concerned with uncertainty of game outcome in Congkak and Othello. Firstly, information dynamic model on uncertainty of game outcome is derived based on fluid mechanics. Secondly, data analyses on Congkak and Othello have been done. It is found that Congkak is a unique regional game at South-East Asia, while Othello is one of the best game in view of entertainment in the globe. It is suggested that Shannon's entropy provides a measure of uncertainty of game outcome, but not itself. The true uncertainty is given by the present proposed model.

Index terms: *Uncertainty of Game outcome, Information dynamic model, Congkak, Othello, Entropy, entertainment.*

I. Introduction

The fundamental problem of information communication is that of reproducing at one point either exactly or approximately an information selected at another point. Frequently the information has meaning; that is, it refers to or is correlated according to some system with certain physical or conceptual entities. The significant aspect is that actual information is one selected a set of possible information, In the present paper, the selected information is data such as evaluation function scores either in Congkak, or in Othello. Information of game outcome here represents the data which is the uncertainty of game outcome. We consider that information is produced as the motion of particles, for stationary particles provide only trivial information. In this regard, it has been inferred by Solso (1994) that motion of visualized fluid particles, for example, is detected by the eye almost instantaneously through light having enormous high speed, 3x1010 cm/s, and is mapped on the retina. It may be evident that during this process, motion of "fluid particles" is transformed into that of "information particles" by light carrying the images of fluid particles. The eye and brain may work together in collecting the light reflected from the visualized fluid particles and processing the information particles, which flow in our brain.

Shannon(1948) has introduced quantities of the form

$$H=-\sum pi \log pi,$$

which plays a central role in information theory as a measure of information, choice or uncertainty. The measure H is normally called the entropy of the set of probabilities p1, p2, …., pn. The quantity H has a number of interesting properties which further substantiate it as a reasonable measure of information. For example, (1) H=0 if and only if all the pi but one are zero, this one having the value of unity. Thus, only when we are certain of the game outcome, does H vanish. Otherwise, H is positive, and (2) for a given n, H is a maximum and equal to log n when all the pi are equal, i.e., 1/n. This is also intuitively the most uncertainty situation.

The concept on intelligence transmission velocity has been proposed by Nyquist (1924):The velocity at which intelligence can be transmitted over a telegraph current with a given line speed, i.e., a given rate of sending of signal elements is expressed approximately by the following formula.

$$W = K \log m,$$

where W is the intelligence transmission velocity, m the number of current values employed, and K a constant.

By the technical term, intelligence transmission velocity is here meant the number of character, representing different letters, figures, etc., which can be transmitted in a given length of time assuming that the circuit transmits a given number of signal elements per unit time. Iida & Nakagawa(2011) has inferred that when information velocity become equal to the speed of light time stops completely. Can we find what happens if the intelligence transmission velocity reaches at the speed of light ?

When we speak of the capacity of a system to transmit information, some sort of quantitative measure of information must be specified (Hartley 1928). In the first place, there must be a group of physical symbols, such as words, dots and dashes or the like, which convey certain meanings to the parties communicating. In any given communication, the sender mentally selects a particular symbol and by some bodily motion, as of his vocal mechanism, causes the attention of the receiver. By successive selections, a sequence of symbols is brought to the listener's attention. At each selection, all of other symbols may be eliminated. As the selections proceed, more and more possible symbol sequences are eliminated, and we say that the information becomes more precise. In this study, as the most precise information of game, the evaluation function scores are used(e.g. Tsuruoka et al 2002).

The main purpose of the present paper is twofold: (1) to derive the information dynamic model on uncertainty of game outcome, and (2) to dig out data of Congkak and Othello with aiming at their future improvement regarding entertainment.

II. Modeling

The modeling procedure of information dynamic model on uncertainty of game outcome is summarized as follows(Iida et al 2011a) :

(a) Assume a flow problem as the information dynamic model and solve it.
(b) Get the solutions, depending on the position and time.
(c) Examine whether any solution of the problem can correspond to game information.
(d) If so, visualize the assumed flow with some means. If not, return to the first step.
(e) Determine the correspondences between the flow solution and game information.
(f) Obtain the mathematical expression of the information dynamic model.

The modeling procedure of information dynamics based on fluid mechanics has been established by Iida et al (2011a). Another information dynamics model for a series of approximate solutions of the flow between two parallel flat walls, one of which is at rest, the other is suddenly accelerated from the rest to a constant velocity U0, Fig.1, will be constructed by following the above procedure step by step.
Flow near a flat plate which is suddenly accelerated from rest and moves in its own plane with a constant velocity is solved by Stokes(1851). For a brief sketch of the solution, see Schlichting (1968).

(a) Let us assume the flow between two parallel flat walls, one of which is at rest, the other is suddenly accelerated from the rest to a constant velocity U0 . Fig.1.

Note that the walls are two-dimensional, horizontal and infinitely long.

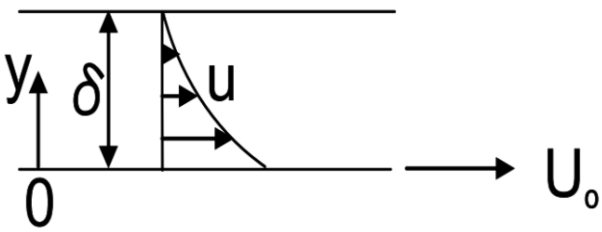

Figure 1: A definition sketch of flow between two parallel flat walls, one of which is at rest, the other is suddenly accelerated from the rest to a constant velocity U0 .

Since the system under consideration has no preferred length in the horizontal direction, it is reasonable to suppose that the velocity profile are independent of the horizontal x-direction, which means that the velocity profile u(y) for varying distance x can be made identical by selecting suitable scale factors for u and y. The scale factors for u and y appear quite naturally as the lower wall velocity U0 and gap between the two walls δ. Hence, the velocity profile after the time $t \succ 0$ can be written as the function in the following way.

$$u/U0 = f(y/\delta). \qquad (1)$$

(b) Get the solutions.

The velocity profile is here accounted for by assuming that the function f depends on y/δ only, and contains no additional free parameter. Since the fluid particles are fixed on the surface of two walls due to the viscous effect, the function must take the value of 1 on the lower wall(y=0) and the value of 0 on the upper wall($y=\delta$) . The boundary conditions are:

$t \leq 0$: $u/U0=0$ for $0 \leq y/\delta \leq 1$,
$t \succ 0$: $u/U0=1$ for $y/\delta=0$; $u/U0=0$ for $y/\delta=1$.

When writing down an approximate solution of the present flow, it is necessary to satisfy the above boundary conditions for u/ U0. It is evident that the following velocity profiles satisfy all of the boundary conditions.

$$u/U0 = (1 - y/\delta)q, \qquad (2)$$

in the range $0 \leq y/\delta \leq 1$, where q is positive real number parameter. Equation (2) is considered as the approximate solutions on the flow between two parallel flat walls, one of which is at rest, the other is suddenly accelerated from the rest to a constant velocity U0 , where each solution takes an unique value of q. The value of q must be determined by the boundary conditions and the Reynolds number Re= $U0 \cdot \delta/\nu$, where ν is the kinematic viscosity of the fluid.
It is known that the transition from laminar to turbulent flow in the boundary layer is governed by the Reynolds number Re= $U\backsim \cdot d/\nu$, where $U\backsim$ is the free stream velocity, d the boundary layer thickness. The critical Reynolds number Re crit., at which the transition is initiated, is of 2,800 approximately(e.g. Hansen 1928, Schlichting 1968).
In case of the present flow, as shown in Fig.1, at 1 atmospheric pressure and temperature at 20°C, water has the kinematic viscosity ν=1.004x10-2cm2/s. When water is chosen as the fluid, and the constant velocity U0 = 10 cm/s and the gap between the two walls δ=10 cm are set, we obtain the Reynolds number Re $\simeq$ 104. The result of this calculation clearly illustrates how the flow is liable to be turbulent under an ordinary situation.

The solution (2) is smooth analytical functions and thus this is only valid for laminar flow.

The fundamental equations for fluid mechanics are the Navier-Stokes equation. This inherently nonlinear set of partial differential equations has no general solution, only several exact solutions, which are trivial in practice, have been found(Wang 1991). All of these exact solutions are for laminar flows, and no turbulent flow solution is available yet. However, it is considered that each of the laminar solutions in (2) represents an approximate turbulent solution. In this regard, we consider that the solutions (2) are applicable for laminar flow as well as turbulent flow to some extent. However, it should be noted that the applicability of the present solutions to turbulent flow is severely limited.

(c)Let us examine whether this solution is game information or not.

The non-dimensional velocity u/U0 varies from 1 to 0 with increasing non-dimensional distance y/δ in many ways with changing the parameter q. It can be considered that u/U0 represents the uncertainty of game outcome. This is why uncertainty of game outcome takes the value of 1 at start, and it decreases with increasing the game length and becomes the value of 0 at the end of game.

(d) Visualize the assumed flow with some means.

Imagine that the assumed flow is visualized with neutral buoyant particles. Motion of the visualized particles is detected by the eye almost instantaneously through light and is mapped on our retina(Solso 1994), so that during these processes, motion of the “fluid particles” is transformed into that of the “information particles” by light carrying the images of fluid particles. This is why motion of the fluid particles is intact in the physical space, but only the reflected lights, or electromagnetic waves consisting of photons can reach the retina. Photons are then converted to electrochemical particles and are passed along the visual cortex for further processing in parts of the cerebral cortex(Solso 1994). Photons and /or electrochemical particles are considered to be information particles. It is, therefore, natural to expect that the flow in the physical world is faithfully transformed to that in the information world, or brain including eye, which is referred to “informatical world” here after. During this transformation, the flow solution in the physical world changes into the information in the informatical world.

(e) Proposed are correspondences between the flow and game information, which are listed in Table 1.

Table 1: Correspondences between flow and game information

Physical world (flow)	Informatical world (game)
u: flow velocity	I: current uncertainty of game outcome
U_0: plate velocity	I_0: initial uncertainty of game outcome
y: vertical distance	L: current game length
δ:gap between two walls	L_0: total game length

(f) Obtain the mathematical expression of the information dynamic model.

Considering the correspondences in Table 1, (2) can be rewritten as

$$I/ I0 =(1-L/ L0)q. \quad (3)$$

Introducing the following non-dimensional variables in (3),

ξ= I/ I0 and η= L/ L0,
we finally obtain the mathematical expression of the uncertainty of game outcome ξ as

$$\xi=(1-\eta)q \text{ for } 0 \leq \eta \leq 1, \quad (4)$$

where η is the non-dimensional current game length, and q the positive real number parameter. We expect that the greater the value of q is, the greater the strength difference between the two teams (or players) in a game is, and vice versa.

Fig.2 illustrates how the uncertainty of game outcome ξ due to (4) changes with increasing non-dimensional

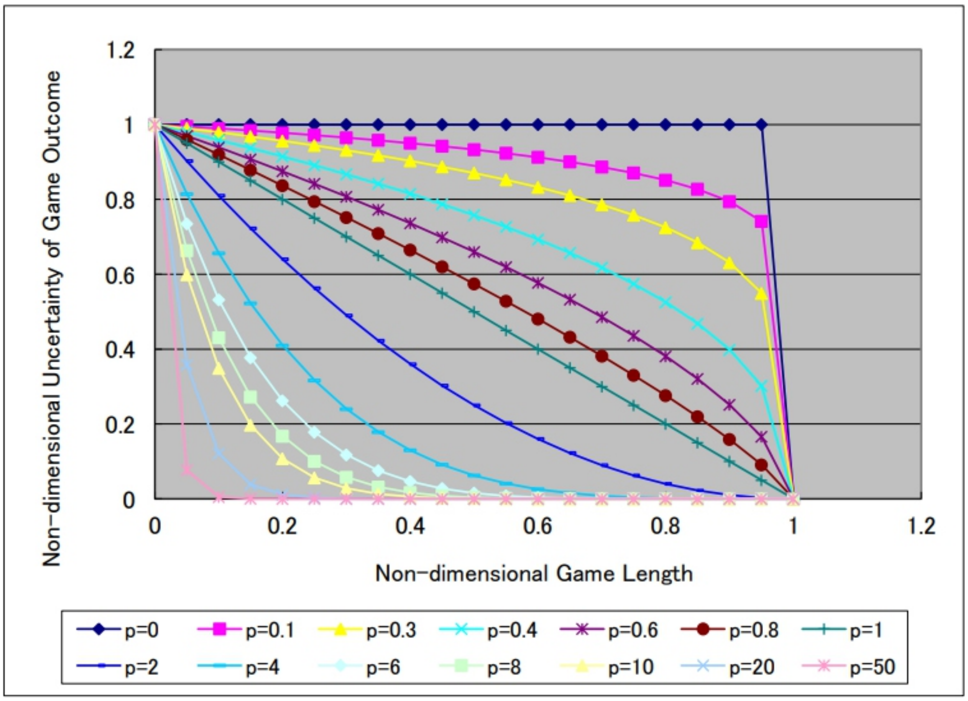

Figure 2: Uncertainty of game outcome ξ against non-dimensional game length η.

III. Verification of Model

A. Congkak

History of Congkak: Congkak is short for Main Congkak, which is Indonesian for “cowrie shell”, but some people believe that actually the name of the game originated from the word congak, which in old Malay language means mental calculation without writing it down. Congkak is a popular mancala game in Malaysia, Brunei, Singapore and Indonesia(Culin 1894, Hellier 1907, Overbeck 1915).

Many Indonesians believe that the game originated in Malacca Kingdom where it became very popular and spread to the South-East Asia region. This spread was due to the many travelers who visited the kingdom because it was a trading city. In the early days, Congkak was mostly played by the royal family and palace residents, however later it spread to the general population of the kingdom and today it is usually played by girls and women. As the Congkak board is often shaped like a boat it is believed that it is based on the legend of a fisherman unable to go to the sea during rainy season who lost his income during this time. To prevent boredom she or he created this game which is similar to her or his boat.

Today many Congkak tournaments are organized for children in Malaysia, e.g. in Kuala Lumpur, Kuala Terengganu, Pekan and Seremban. Several hotels in southern Borneo offer Congkak course to tourists. Since 2004, the Malaysian Embassy and the Malaysian Association in France sponsore each year a Congkak tournament to spread Malaysian culture in Europe. Another tournament is held in Wales during the Cardiff European Games, an annual meeting of Malaysians from all across Europe.

In Brunei, Congkak is also played during the night of royal ceremonials such as the Istiadat Malam Berjaga-jaga at the palace or nobility'sresidence

Congkak consists of: Congkak uses an oblong game board called papan congkak, which has two rows each one with five to ten playing pits. These pits are called lubang kampong("village") or lubang anak("child") in Malaysia. Most widespread are boards with 2x7 playing pits. In addition, there is at either end a larger hole to store the captured counters. The store is called lubang rumah("house") in Malaysia. Each player owns the store to her or his left.

Each of the small pits contains at the beginning of the game as many counters(usually cowrie shells or tamarind seeds called anak-anak buah in Malaysia) as each row counts small pits.

How to play Congkak: 2 players sit opposite each other. Each player owns the row of houses directly in front of her or his houses and the storehouse on her or his left.

(a) Players play simultaneously beginning with anyone of their hoses and dropping seeds clockwise into each house until each the player is finished with all the seeds in her or his hand. On her or his round, a seed is placed in a player's storehouse but not her or his opponent's.

(b) On ending her or his round, the player takes all the seeds of the house that she or he has dropped her or his last seed in and the process is repeated until the last seed is dropped into an empty house.

(c) If the last seed falls in a house that is part of a player's village, she or he can pick all the seeds from her or his opponent's house that lies opposite it and put them in her or his storehouse.

(d) If it drops in her or his storehouse, she or he can continue the game, picking a house of her or his choice from her or his side.

(e) When the last seed drops in an empty house, she or he is considered mati(“dead”) and ends her or his turn.
Her or his opponent continues until she or he similarly ends her or his turn.

Data analyses: Mardhiah plays Congkak against Husna under the rules mentioned in the above. The non-dimensional advantage $\alpha(\eta)$ is defined as

$$\alpha(\eta)=[SM(\eta)-SH(\eta)]/ST \text{ for } 0\leq\eta\leq 1,$$

where $SM(\eta)$ is Mardhiah’s current scores, $SH(\eta)$ Husna’s current scores, ST the total scores for the two players in the game, and η the non-dimensional game length. Sign of the non-dimensional advantage is defined to be positive when Mardhiah gets advantage, while it is negative when Husna advantage.
The uncertainty of game outcome ξ is derived by

$$\xi=1-|\alpha(\eta)| \text{ for } 0\leq\eta\prec 1,$$
$$0 \text{ for } \eta=1.$$

Fig.3 shows how the non-dimensional advantage $\alpha(\eta)$ and uncertainty of game outcome ξ depend on the non-dimensional game length η. Mardhiah leads the game until $\eta\simeq 0.369$, but after this point Husna gets advantage and keeps it until the end. However, uncertainty of game outcome ξ is kept within 0.8 and 1 until very end of the game, so this game is considered to be quite tight one. Furthermore, in this figure the best fit model curve $\xi=(1-\eta)0.15$ to the experimental data has been plotted concurrently.

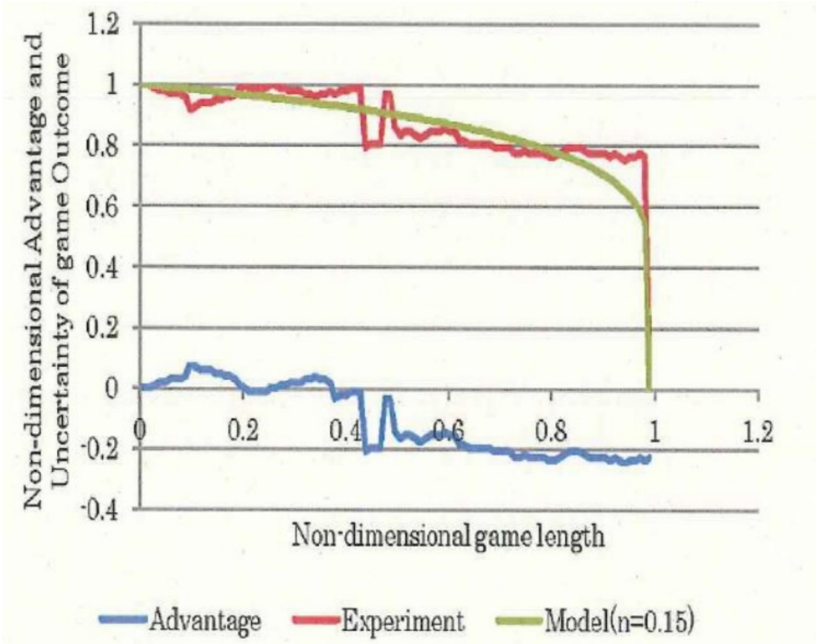

Figure 3: Non-dimensional advantage $\alpha(\eta)$ and uncertainty of game outcome ξ against Non-dimensional game length η for Congkak.

B. Othello

History of Othello: Othello is a board game involving abstract strategy and played by two players on a board with 8 rows and 8 columns and a set of distinct pieces for each side(Iwata & Kasai 1994, Victor 1994). Pieces typically are disks with a light and a dark face, each side belonging to one player. The player's goal is to have a majority of their pieces showing at the end of the game, turning over as many of their opponent's pieces possible.

The modern rule set used on the international tournamentstage originated in Mito,Japan.

How to play: Word, "outflank" means to place a disc on the board, so that your opponent's row(or rows) of disc(s) is bordered at each end by a disc of your color. A "row" may be made up of one or more discs. Othello rules are summarized as follows.

(a) Black always moves first.
(b) If on your turn you cannot outflank and flip at least one opposing disc, your turn is forfeited and your opponent moves again. However, if a move is available to you, you may not forfeit your turn.
(c) A disc may outflank any number of discs in one or more rows in any number of directions at the same time-horizontally, vertically or diagonally. A row is defined as one or more discs in a continuous straight line.

(d) You may not skip over your own color disc to outflank an opposing disc.
(e) Disc(s) may only be outflanked as a direct result of a move and must fall in the direct line of the disc placed down.
(f) All disc(s) outflanked in any one move must be flipped, even if it is to the player's advantage not to flip them all.
(g) A player who flips a disc which should not have been turned, may correct the mistake as long as the opponent has not made a subsequent move. If the opponent has already moved, it is too late for change and the disc(s) remain as is.
(h) Once a disc is placed on a square, it can never be moved to another square later in the game.
(i) If a player runs out of discs, but still has an opportunity to outflank an opposing disc on her or his turn, the opponent must give the player a disc to use. This can happen as many times as the player needs and can use a disc.
(j) When it is no longer possible for either player to move, the game is over. Discs are counted and the player with the majority of her or his color discs on the board is the winner. Note that it is possible for a game to end before all 64 squares are filled.

Othello has fast become one of the most popular and most often played games in our history, spawning contests, and tournaments on regional, national and even worldwide levels. And the rules of Othello explained as above, are very simple and the final destination is clear enough, but what exactly you are supposed to be trying to do in the early and middle stages of the game is unclear.

Data analyses: The present Othello game is played by Huy, who acts as both black and white players. The non-dimensional advantage$\alpha(\eta)$ is defined as follows,

$\alpha(\eta)=Ad(\eta)/ACT(1)$ for $0 \leq \eta \leq 1$,

where $Ad(\eta)$ is the advantage or evaluation function scores, ACT(1) the total advantage change at the end of game. $ACT(\eta)$ is expressed by
$ACT(\eta)=ACT(m./N)=$,

where m is current move, N the total moves at the end of game, and i the positive integer. And, $\eta = m/N$ the non-dimensional game length.

Uncertainty of game outcome $\xi$is expressed by

$s\xi = 1 - |\alpha(\eta)|$ for $0 \leq \eta \prec 1$,

0 for $\eta = 1$.

Fig.4 shows how non-dimensional advantage $\alpha(\eta)$ and uncertainty of game outcome ξdepend on the non-dimensional game length η.

It may be evident in Fig.4 that non-dimensional advantage $\alpha(\eta)$ is always positive, so that Black keeps advantage through the game, though it is smaller than 0.1. In this figure, the best fit model curve $\xi = (1-\eta)0.04$ to the experimental data has been plotted concurrently.

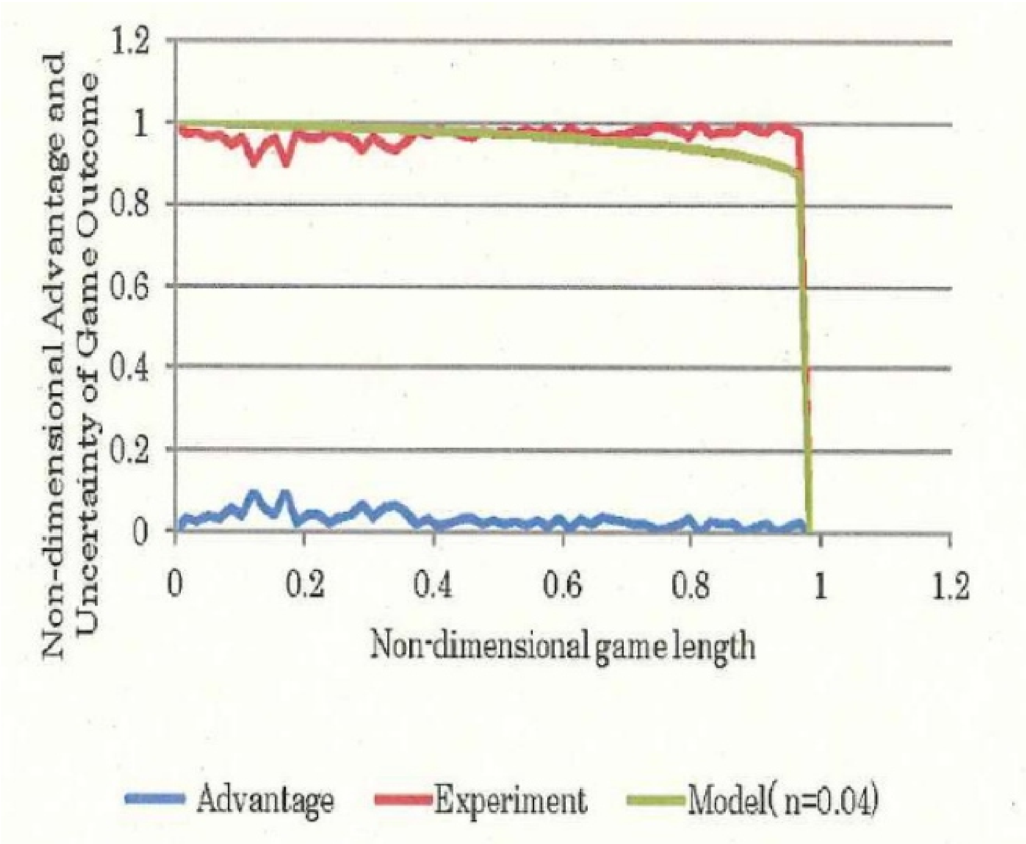

Figure 4: non-dimensional advantage $\alpha(\eta)$ and uncertainty of game outcome ξ against non-dimensional game length η for Othello.

IV. Discussion

This section describes how uncertainty of Soccer game outcome change with increasing the game length, where the goal scores of 2010 FIFA World Cup 3rd Place(Germany vs. Uruguay) are used for the illustration. Germany wins the game against Uruguay by the score 3 to 2: This game is full of thrill, with alternating changes from offense to defense, or from defense to offense many times. The game is balanced at the start, and then Germany gets the first goal. Uruguay makes the game balanced by taking the second goal, and then Germany is reversed by Uruguay due to the latter's third goal. The game is made balanced again by Germany's fourth goal. Finally, Germany gets the fifth goal near the end and keeps her lead until the end of game.

To begin with, the advantage $\alpha(\eta)$ is defined by

$$\alpha(\eta)=[S1(\eta)–S2(\eta)]/St \text{ for } 0\leq\eta\leq 1,$$

where $S1(\eta)$ is the current score sum for team 1, $S2(\eta)$ the current score sum for team 2, St the total score(s) for the game, and η the normalized game length. The sign of advantage is defined in such a way that it is positive when team 1 keeps advantage, while it is negative when team 2 takes advantage.

It may be worth noting the remarkable similarity between logarithmic uncertainty of game outcome

$\xi lu(\eta)$:

$$\xi lu(\eta)=-\sum_{i=1}^{2} pi(\eta)\log2\ pi(\eta) \text{ for } 0\leq\eta\prec 1,$$
$$=0 \text{ for } \eta=1, \qquad (1)$$

where $p1(\eta)$ and $p2(\eta)$ are winning rates for teams 1 and 2, respectively, and the entropy H(X) defined by Shannon(1948): Information theory has been used to study the properties of random variables. If a random variable X can assume the state x, and P(X=x) is the probability for X to assume the specific state x, we can define a measure H(X) called entropy as

$$H(X)=-\sum_{x}[P(X=x)]\log[P(X=x)].$$

This is often described as the uncertainty about the outcome of X gained if one is to observe the state of x, without having prior knowledge about X.

Note that in the expression of $\xi lu(\eta)$ when base of the logarithm is 2, the unit of $\xi lu(\eta)$ is «bit», when the base is Euler's number e, the unit is «nat», and when the nbase is 10, the unit is «digit».

We choose the value of 2 as the base, for it is unnecessary to normalize$\xi lu(\eta)$ in this case,
It may be evident in Fig. 5 that$\xi au(\eta)$:

$$\xi au(\eta)=1-|\alpha(\eta)| \text{ for } 0 \leq \eta \prec 1, =0 \text{ for } \eta=1, \quad (2)$$

is always smaller than the logarithmic uncertainty of game outcome $\xi lu(\eta)$. It may be evident that the logarithmic uncertainty of game outcome $\xi lu(\eta)$ obscures the uncertainty of game outcome by introducing the logarithmic value of winning rate $pi(\eta)$. Thus, it is here suggested the logarithmic uncertainty of game outcome $\xi lu(\eta)$ or Shannon's entropy provides only a measure of uncertainty of game outcome, but not itself. The uncertainty of game outcome is considered to be given by the present proposed advantageous uncertainty of game outcome $\xi au(\eta)$.

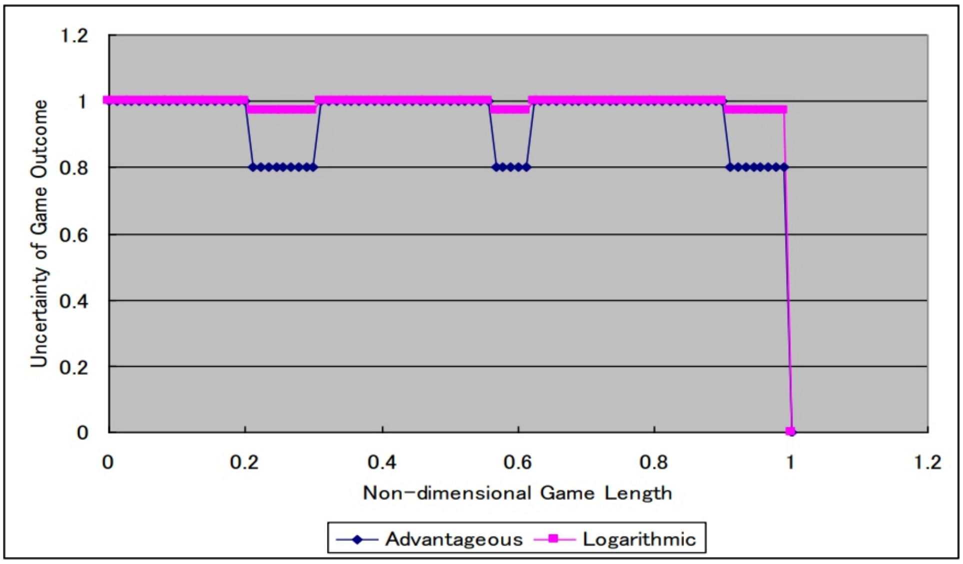

Figure 5: Uncertainty ξu of game outcome against normalized game length η for 2010 FIFA World Cup, 3rd Place.

V. Conclusion

New knowledge and insights obtained through the present study have been discussed and summarized as follows.

(a) Uncertainty of game outcome ξ for the present Congkak is approximated with the model curve:

$\xi=(1-\eta)0.15$,

while that for the present Othello with the model curve:

$\xi=(1-\eta)0.04$,

where η is the normalized game length. It is inferred that the Othello is more balanced than the Congkak, and thus it is considered that the former having the smaller value of n, is more exciting than the latter. However, this point of view is neither universal nor objective, because the results are highly depending on individual feeling or emotion of game players

According to the classification by Iida et al(2011b), the Othello can be classified as “one-sided game”, while the Congkak as “seesaw game”.

(b) It is inferred that the logarithmic uncertainty of game outcome or Shannon’s entropy(1948) provides only an order of uncertainty of game outcome, but not itself: It is considerd that the wanted value is given by the present proposed advantageous uncertainty of game outcome.

(c) An information dynamic model representing the uncertainty of game outcome has been derived based on the fluid mechanics. Its usefulness has been confirmed by comparing with the present actual game experiments due to congkak as well as Othello.

(d) Congkak has been here introduced into Japan for the first time, as far as the present authors are aware of, and analyzed in order to explore the game potential with aiming at the future improvement regarding its entertainment. As the result, it is realized that Congkak is a unique regional game at South-East Asia, having a high possibility to spread out widely.

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