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Aim & Scope

The Advances and Applications in Mathematical Sciences (ISSN 0974-6803) is a monthly journal. The AAMS's coverage extends across the whole of mathematical sciences and their applications in various disciplines, encompassing Pure and Applied Mathematics, Theoretical and Applied Statistics, Computer Science and Applications as well as new emerging applied areas. It publishes original research papers, review and survey articles in all areas of mathematical sciences and their applications within and outside the boundary.

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A SURVEY ON MACHINE LEARNING APPLICATIONS IN HEALTHCARE

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ABSTRACT

Now-a-days, the technology has become a part of our lives and we are living with the technology. Machine Learning technology has become a part to understand and know what is happening with the person's health and we need to analyze one's own health. Healthcare is the field where we can apply machine learning and big data applications on it. Prediction and detection of one's health has become a difficult for the health care practitioners. So, predicting heart disease, cancer, neuro and cardiac at the initial stages will be helpful to the people so that they can take the necessary steps/action before the situation gets severe. Over the years the machine learning algorithms are showing better observations in making decisions and prediction from a huge set of information provided by the healthcare industries. ML cannot be executed for all the problems in reality. A number of the supervised machine learning algorithms utilized in the predictions are Artificial Neural Networks, Decision Trees, Random Forest, SVM, Navie Bayes, K Nearest Neighbor algorithms.

I. Introduction

Healthcare is the largest domain which produces a great deal of information for the researchers to implement something new from data. In this article we will identify the machine learning methodology which helps the people to know the different things they have to follow to take care of a far better health mechanism. Here we will be following only the quality and traditional techniques to spot the human problems.

Machine learning as a part of AI, helps the computers to think like humans and take their own decisions without the intervention of the people. Moreover the algorithms give better prediction and performance on a specific problem. Machine learning is broadly divided into different types as shown in the following Figure 1:

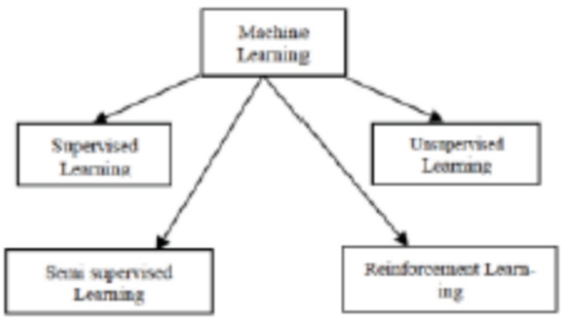

Figure 1. Different type of Machine learning.

Algorithms in Machine Learning:

Artificial Neural Networks (ANN): ANN is mostly implemented for computational purposes; the major principle of this model is to take care of the responsibility better than the conventional model. This is comparable to the shape of neurons in the brain. A single layer neural community is recognized as a perceptron which offers us a solitary output.

Support Vector Machine (SVM): It is a machine learning algorithm. In supervised support vector machine algorithms, if we consider any labeled data for training, which produce a classifier that divides the labeled information into various classes.

Decision tree (DT): It is one of the supervised algorithms which can be implemented for both regression and classification techniques. In this, the information will be split based on the parameters. A tree contains leaves and nodes. The nodes will split the data and at leaves we get the outcome. The decision trees can be implemented in two ways they are the classification and regression trees.

Random forest (RF): It is one of the supervised learning algorithms used for regression and classification. However, primarily it's for classification functions. The name itself indicates that it's a group of trees equally in an exceedingly random forest algorithmic rule square measure going to have trees and the trees are known as decision trees.

Navie Bayes (NB): It is one of the supervised machine learning classification algorithm. Initially it is used for classifying the text data. It handles the datasets with high spatiality.

The rest of the paper was organized as follows: In section 2, related works are discussed. Section 3 deals with data collection. Section 4 is about research challenges and section5 is about conclusion and future scope.

II. Related Work

Awais Nimat et al. [1] proposed a specialist framework supported by two support vector machines (SVM) to anticipate the heart condition efficiently. These two SVM's have their motivation; initially, one is utilized to get rid of the unnecessary features, and hence the other is utilized for prediction. Besides, they need the HGSA to optimize the two strategies. By utilizing this model, they accomplished around 3.3% preferably accurate than the traditional model.

Mehtaj Banu H [3] the researcher studied distinct techniques in machine learning like Supervised, unsupervised and reinforcement and also furthermore investigation on UCI dataset database and finalize that KNearest Neighbour and Support Vector Machine algorithms have proved execution and precision for the prediction of a specific disease.

Parthiban and Srivatsa [4] the author analyzed the condition of heart in diabetic patients by victimisation strategies of machine learning. Algorithms of Naive Bayes and Support Vector. Datasets of around 490 patients is utilized that are collected from Research Institute of Chennai. Patients that have the sickness are 141 and malady is absent in around 348 patients. By utilizing the Naive Bayes Algorithm 74% of precision is acquired. SVM provide the absolute accuracy of about 93.60.

Sarwar and Sharma [5] have recommended the work on Naive Bayes to anticipate diabetes Type-2. Diabetes is categorized into 3 types they are Type 1 diabetes, Type-2 diabetes and gestational diabetes. Type-2 diabetes originates from the extension of Insulin obstruction. The information comprises of 411 cases and for the motivation behind assortment; information are gathered from different areas of society in India. Around 94% of right forecasting is accomplished by Naive Bayes.

Fathima and Manimeglai [6] attempted to anticipate the disease Arbovirus-Dengue. Information processing is utilized by the analysts are SVM. Dataset for the examination is derived from King Institute of Preventive Medicine and studies of numerous emergency clinics and research centres in and around Chennai and Tirunelveli from India. It includes of 29 attributes and 5000 sample data and this was analyzed using the R version 2.12.2. Accuracy obtained using SVM is around 0.9041.

III. Data Collection

Machine learning is currently deployed widely across the various health sectors owing to its ability to form the real time predictions and draw the unnoticed insights from the given voluminous and unstructured datasets. Here are few repositories listed where we can get the data sets related to health care.

WHO (World Health Organization): Its open source data contains classes which include child nutrition, neglected diseases, risk factors pertaining to certain diseases among others.

OGD Platform India: This web site consists of all the information collected from the Indian health agencies and different entities.

Open fMRI: It is a project dedicated for sharing the free and open source datasets related to imaging.

Data.gov: This site consists of all the information collected from the various primary healthcare centres, community health centres of various district hospitals and mobile medical units from various states and union territories.

There are different other sites where we can get the information related to the particular research works being handled by various exploration research scholars. Depending on the area of work all the data that is available is the data collected from various sources.

IV. Research Challenges

The Indian healthcare scenario witnesses the rapid pace of change currently happening in the glorious tradition of the public health. There are many challenges being faced in the healthcare sector in India and various parts of the nation. We have many challenges but listing only few in this paper:

Awareness: How individuals know about the significant issues with respect to their own wellbeing? Studies on mindfulness are numerous and differing, yet satisfactory information and mindfulness seem to cut over the life expectancy in our nation.

Access or the lack of it: "The right or opportunity to use or benefit from healthcare" is the definition of access given by the Oxford dictionary in terms of healthcare. Physical reach is one of the fundamental determinants of access, [7].

Absence or the human power crisis in healthcare: According to the study India has approximately 20 health examiners for every 10,000 population, with medical aid comprising 31% of the manpower, medical attendants and 11% drug specialists. [8].

Affordability or the cost of healthcare: It is quite simple, how costly the medical services are in India. Practically 75% of the healthcare consumption comes from the households units. [9].

Conclusion and Future Scope

This article gives us the essential plan of the previously published paper of identification and assurance of different infections/diseases based on various learning algorithms. With this review and study it is clearly found and discovered that some machine learning algorithms like Decision tree, Random forest, Navie Bayes and ANN give the higher accuracy in detecting and predicting various diseases. And additionally the paper gives a review on various types of machine learning techniques utilized by totally different authors and every machine learning techniques has some smart and unhealthy outcome based upon the datasets and feature selection etc. With the review we have a tendency to know that the absoluteness and execution will be improved by victimisation of totally various combination or hybrid machine learning algorithms and in future we will additionally implement on a lot of variables that facilitate to urge higher execution than the current technique.

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TOWARDS MORE EFFICIENT 3NF DETERMINATION USING REDUCED FUNCTIONAL DEPENDENCY SETS

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ABSTRACT

3NF can reduce redundancies and anomalies from a relational database as far as possible while still observing lossless join and even dependency preservation properties. Obviously in spite of existence of higher order normal forms in theory, normalization into 3NF is widely viewed as necessary and sufficient for practical applications. Accordingly efficient determination of 3NF is a prerequisite for good database design. In this article the 3NF interpretation is strived to be reduced, yielding some more efficient 3NF determination techniques. One of the resulting interpretations is theoretically an optimal. Some prospective implications of the proposed interpretations are also discussed.

1. Introduction

Relational database theory has provided effective guidelines to the developers for using mathematical/logical models efficiently for ensuring good database design ([1]-[8]). These rules are particularly useful for investigating the keys, redundancies and anomalies with reference to given functional dependencies (FDs) and some other constraints. Normalization is an integral part of relational database design where the given relation schema ® is analysed with respect to keys and FDs, striving to reduce redundancies and anomalies. As per ordinary theory, for a given relation schema (R) defined on a set of FDs (F), expressed as (R, F),initially the normalization status is tested through some normal form (NF) in a particular order which is termed as determination of the normalization level. In case of non conformity with the NF, R is then either decomposed or synthesized into the NF. In theory, there are several NFs and in the ascending order of strength the NFs are 1NF, 2NF, 3NF, BCNF, 4NF, 5NF etc. However, in the standard literature normalization into 3NF is treated sufficient as it may reduce redundancies and anomalies as far as possible while still ensuring some other essential feature of database design, including dependency preservation ([1]-[8]). Clearly an efficient determination of 3NF may be a prerequisite for good database design and this article is focused on exploring more efficient 3NF determination schemes using some reduced interpretations of F.

The rest of the article is organized with four sections as follows. In the preliminaries section, the insight into the problem is intended to be provided with detailed background. In the proposed methods section 3NF is strived to be interpreted in terms of some reduced sets of FDs, immediately resulting in faster 3NF determination techniques and the proofs in favour of their correctness are also given. In the results and discussion section, the comparative merits of the proposed 3NF determination techniques are assessed. Finally the article ends in the conclusion section, outlining some future directions too.

2. Preliminaries

As per the original interpretation ®, F)is in 3NF iff for every non trivial FD $X \rightarrow Y$either X is a super key of R or Y is a prime (or key) attribute of R ([1]-[8]). In several classical texts 3NF is determined directly on the basis of the interpretation without referring any FD set explicitly ([1]-[4]).

[4]). In some other standard text books the 3NF testing addresses the closure of $F(F^+)$ i.e. (R, F) is in 3NF iff for every nontrivial $FD\ X \rightarrow Y \in F^+$ either X is a super key of R or Y is a prime attribute of R ([5]-[6]). The F^+-centric interpretation is also advocated in article [7] but implicitly, as it refers to the set of all FDs implied by F which is again F^+. In some other texts 3NF testing involves only the FDs in F i.e. (R, F) is in 3NF iff for every nontrivial $FD\ X \rightarrow Y \in F^+$ either X is a super key of R or Y is a prime attribute of R ([8]-[9]). In addition in a recent work 3NF is supposed to be determinable merely on the basis of an optimal canonical cover (G) of F i.e. (R, F) is in 3NF iff for every nontrivial $FD\ X \rightarrow Y \in G$ either X is a super key of R or Y is a prime attribute of R [10]. The interpretation of 3NF in terms of F^+ obviously holds true as $(F^+)^+ = F^+$. However, one problem regarding 3NF determination is that, in the literature it is not known whether the F-centric interpretation and G-centric interpretation are also correct or not. Even if both interpretations hold true, then the question of other interpretations, including possibly an optimal interpretation may arise.

It is needless to say that finding asymptotically more efficient algorithms for 3NF determination is constrained by the intractability of the problem, caused by its sub-problems. Speaking more elaborately, most likely there is no polynomial-time algorithm for candidate keys' determination, attributes' primality detection and obviously 3NF identification as all these problems belong to the NP-complete class ([11]-[13]). However, motivated by some early works ([12], [14]-[15]) which shows capability to determine the candidate key(s) quickly for the problems subject to typical characteristics of their attribute sets, some 3NF-determination algorithms have been proposed ([16]-[17]) which may often run in polynomial time.

Another interesting aspect of the problem is that although 3NF determination problem is NP-complete, there exists a 3NF synthesis algorithm [18] which guarantees lossless join and dependency preservation and acquires polynomial-time if considered in stand-alone mode. So one may directly consider applying 3NF synthesis algorithm without its testing so that 3NF status of R may be achieved as a whole in polynomial time. However, this idea may ultimately appear to be ineffective owing to two problems.Firstly a prerequisite input for applying 3NF synthesis algorithm is an optimal (or minimal) canonical cover of F(G).G is obtained by eliminating all superfluous attributes as well as all superfluous FDs from F in any order. As determination of G is known to be NP-complete ([19]-[21]), 3NF synthesis algorithm is NP-complete as a whole. Secondly as the stand-alone 3NF synthesis algorithm [18] does not involve normalization status checking at any point, it may further decompose R even after achieving the desired 3NF status [17].

3. The Proposed Methods

In coherent form the axial part of the proposed reduced interpretations of 3NF, Theorem 1, is as follows.

Theorem 1. *3NF status of (R, F) is equivalently determinable $\forall H$ where $H^+ = F^+$.*

Proof. Two declarative statements (propositions), say statement A and statement B, are treated equivalent (or $A \equiv B$) iff simultaneously both hold true or both hold false.

Introduce Statement 1 and Statement 2 where

Statement 1. For every nontrivial $FD\ X \to Y \in H$ either X is a super key of R or Y is a prime attribute of R.

Statement 2. For every nontrivial $FD\ X \to Y \in F^+$ either X is a super key of R or Y is a prime attribute of R.

If possible, suppose that regarding the 3NF status of a given relation schema R statement 1 and statement 2 do not simultaneously hold. It means either case 1 or case 2 might arise where.

Case 1. For R statement 1 does not hold true but statement 2 holds true.

Case 2. For R statement 1 holds true but statement 2 does not hold true.

Without any loss of generality hereafter every FD is supposed to be non trivial and canonical, unless stated otherwise. In case 1, non satisfying of statement 1 means that $\exists$ a $FD\ P \to Q \in H \mid$ neither P is a super key of R nor Q is a prime attribute of R. However, as $\forall FDs \in H$ are also necessarily $\in F^+$, $\exists$ a $FD\ P \to Q \in F^+ \mid$ neither P is a super key of R nor Q is a prime attribute of R. It contradicts statement 2. Therefore case 1 does never arise.

In case 2, non satisfying of statement 2 means that $\exists$ a $FD\ S \to T \in F^+ \mid$ neither S is a super key of R nor T is a prime attribute of R. Consider the inference of $S \to T \in F^+$ with reference to H. As $H^+ = F^+$, either originally $S \to T \in H$ or there is some $FD(s) \in H$, called source FD(s), from which the $FD\ S \to T$ is inferred by applying one or more instances of augmentation rule or transitive rule or their derivatives or compositions. Let the derivation path of the $FD\ S \to T$ is given by.

$S \to W_1 \to W_2 \dots \to W_i \to T$ where $W_1, W_2, \dots, W_i$ are formed over the attribute(s) of R. It is obvious that $\{S \to W_1, W1 \to W_2, \dots, W_i \to T\}$ is a subset of F^+ and $\exists V \to T \in H \mid V$ is a subset of W_i. If possible assume that V is a super key of R. It means W_i is a super key of R. Then the FD $W_{i-1} \to W_i$ implies that W_{i-1} is a super key of R. Proceeding in this manner W_1 also appears to be a super key of R. But it is given that S is not a super key of R. Then the $FD\ S \to W_1$ implies that a non-key of R functionally determines a super key of R, which is a contradiction. Therefore V is not a super key of R and case 1 does never arise. Hence the result follows.

Corollary 1. *The 3NF status of R is equivalently determinable in terms of* F^+, F *and* G.

Corollary 2. *The 3NF status of R is optimally determinable in terms of G.*

Corollary 1 immediately follows Theorem 1. In order to prove corollary 2, if possible, suppose that $\exists$ a subset L of $G \mid$ 3NF status of R is also determinable in terms of L. It means, although $L^+ \neq G^+$ (given) and $G^+ = F^+$ (known), $L^+ = F^+$ which is a contradiction. Hence corollary 2 immediately holds true.

Let K be the set of FDs obtained from F by removing the superfluous FD(s) only and K' be the set of FDs obtained from F by removing the superfluous attribute(s) only.

Corollary 3. *The 3NF status of R is equivalently determinable in terms of* K.

Corollary 4. *The 3NF status of R is equivalently determinable in terms of* K'.

Corollary 3 and corollary 4 immediately follow Theorem 1. Let SK and P denote the set of all super keys and prime attributes of R respectively. Then corollary 2 and corollary 3 may air new 3NF testing proposals, say, Method 1 and Method 2, respectively as follows.

Method 1. For all non trivial $FDs\ X \to Y \in G$ test if $X \in SK$ or $Y \in P$.

Method 2. For all non trivial $FDs\ X \rightarrow Y \in K$ test if $X \in SK$ $Y \in P$.

It is needless to recall that the Method 1 and Method 2 represent an optimal and minimum 3NF testing approaches respectively. It may also be noted that although computing G most probably needs exponential time, there exists a polynomial-time solution for K ([19]-[20]).

4. Results and Discussion

For the known 3NF determination methods the various stake holders are computing SK, P and checking the determinant and attribute of every FD. For a given relation, RX, the set of Fds, Fxand the associated attribute set,

A_X; without any loss of generality, the time complexities for computing SK, P and checking the determinants and attributes of all FDs may be expressed as $O(f(|A_X|, |F_X|))$, $O(g(|A_X|, |F_X|))$ and $O(h(|A_X|, |F_X|))$ respectively where $|S|$ denotes the number of elements in the set S. Let $O(\Psi(|A_X|, |F_X|)) = O(f(|A_X|, |F_X|)) + O(g(|A_X|, |F_X|)) + O(h(|A_X|, |F_X|))$. As f ( ), g ( ) and h ( ) are all increasing order functions, $\Psi(\ )$ is also an increasing order function in terms of $|A_X|$ and $|F_X|$. For a given R, let the attribute set corresponding to the FDs F^+, F, K and G are A_{F^+}, A_F, A_K and A_G respectively. As here $|A_G| \leq |A_K| \leq |A_F| \leq |A_{F^+}|$ and $|G| \leq |K| \leq |F| \leq |F^+|$ hold true, $O(\Psi(|A_G|, |G|)) \leq O(\Psi(|A_K|, |K|)) \leq O(\Psi(|A_F|, |F|)) \leq O(\Psi(|A_{F^+}|, |F^+|))$ must also hold true. It means that the proposed 3NF determination methods can outperform the textbook prescribed traditional methods as well as the recently introduced graphical methods for 3NF determination relying on F^+ or F ([16]-[17]). Another interesting point is that if the proposed G-centric or K-centric methods are employed for 3NF determination instead of the traditional or even the recent graphical methods ([16]-[17]), the follow-up queries can also run faster significantly along with less memory consumption, particularly where $|G|$ and $|F|$ are considerably smaller than $|F^+|$ and $|F|$.

Consider (R, F) where $F = \{A \rightarrow C, B \rightarrow C, AB \rightarrow C, AC \rightarrow D\}$. Here $\{A\}^+ = \{A, B, C, D\}$ and so A is a candidate key of R. However,

$B^+ = \{B, C\}$, $C^+ = \{C\}$, $D^+ = \{D\}$, $BC^+\{B, C\}$, $BD^+ = \{B, D\}$, $CD^+ = \{C, D\}$, $BCD^+ = \{B, C, D\}$. So any other key does not exist. It means A is the only key and B, C and D are the non prime attributes. Then the $FD\ B \to C \in F$ indicates that (R, F) is not in 3NF. Again as $\{A\}^+ = \{A, B, C, D\}$ the attribute B and C are superfluous in the $FD\ AB \to C$ and $AC \to D$ respectively. Removal of the attribute B and C from the respective FDs causes F to reduce to $\{A \to B, A \to C, B \to C, A \to D\}$. As the rest of the FDs do not have composite determinant, they obviously do not contain any superfluous attribute. So $K'\{A \to B, A \to C, B \to C, A \to D\}$. The $FD\ A \to C$ is superfluous in K' as $K' - \{A \to C\} \models \{A \to C\}$. None of the other FDs is superfluous. Therefore $G = \{A \to B, B \to C, A \to D\}$. (R, F) is also not in 3NF owing to the $FD\ B \to C$. This example shows that the 3NF status of (R, F) can be checked merely from the 3NF status of (R, F). In addition in this example, for determining the 3NF status of (R, F), only three FDs of G are sufficient to consider instead of five FDs of F.

5. Conclusion

In this paper, for any given R and associated F, 3NF has been strived to be determined merely using some parts of F^+ (or F). It has been shown that the F-centric, K-centric, K'-centric and G-centric interpretations of 3NF equivalently hold true. The significance of K'-centric interpretation of 3NF has been observed elsewhere [22]. However, the K-centric and G-centric interpretations and in particular, the optimality of G-centric interpretation had never explicitly appeared in the literature to the best of the authors' knowledge and belief. The comparative merits of the proposed 3NF determination methods over their potential contenders have been demonstrated too.

The optimal and minimum interpretations of 3NF proposed in this paper may be immediately extended for BCNF. In future along with better resolutions on generating candidate keys and checking primality of attributes, the proposed methods may continue to lead in determining 3NF with greater efficiency. A following-up of this study suggests reassessing the information theoretic implications of 3NF ([23]-[24]) in view of its reduced interpretations proposed and it is left as an open problem.

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TOPOLOGICAL ASPECTS ON prw-COMPACT AND prw-LINDELOF SPACES VIA GROUP ACTION

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ABSTRACT

The main aim of this paper is to introduce and investigate group acting on prw-compact and prw-Lindelof spaces in topological spaces. Also some of the properties have been investigated. We also obtain some new results of group acting on prw-compact and prw-Lindelof spaces.

1. Introduction

Compactness is one of the most important, useful and fundamental concepts in topology. The productivity and fruitfulness of these notions of compactness and connectedness motivated mathematicians to generalize these notions. In the course of these attempts many stronger and weaker forms of compactness and connectedness have been introduced and investigated. In 1981, Dorsett [4] introduced and studied the concept of semicompactness. In 1991, K. Balachandran, P. Sundaram and J. Maki, [2] introduced a class of compact spaces called GO-compact spaces. The notion of -compactness in generalized topological space was introduced by Jyothi Thomas and Sunil Jacob John [5]. In [3] Á. Császár has also introduced the concept of -compact in generalized topological space. The concept of topological groups is introduced by Mikhail Tkaccnko [7]. In this paper we defined and studied on group action on prw-compact and prw-Lindelof spaces in topological spaces. Throughout this paper, group (G,)(or simply G) always means a topological group on which no separation axioms are assumed unless explicitly stated. For a subset M of a group G, cl(M), pcl(M)and int(M)denote the closure of M, pre-closure of M, the interior of M respectively.

2. Preliminaries

We require the following definitions.

Definition 2.1. A topological group is a set G with two structures: (i) G is a group, and (ii) G is a topological space, such that the two structures are compatible, i.e., the multiplication map $\mu : G \times G \to G$ and the inversion map $\upsilon : G \to G$ are both continuous.

Definition 2.2. A group G acting on a subset M is called regular open (briefly r-open) [6] set if $M = \text{int}(cl(M))$ and regular closed (briefly r-closed) [6] set if $M = cl(\text{int}(M))$.

Definition 2.3. A group G acting on a subset M is called pre-open [9] set if $M \subseteq \text{int}(cl(M))$ and pre-closed [9] set if $cl(\text{int}(M)) \subseteq M$.

Definition 2.4. A topological group (G, τ) acting on a subset M is said to be pre-regular weakly closed (briefly *prw*-closed) [8] set if $pcl(M) \subseteq U$ whenever $M \subseteq U$ and U is *rs*-open in G.

Definition 2.5. A group acting on a subset M of G is said to be b-open [1] if $M \subseteq Int(cl(M)) \cup Cl(Int(M))$. The complement of b-open set is said to be b-closed. The family of all b-open sets (respectively b-closed sets) of (G, τ) is denoted by $bO(G, \tau)$ [respectively $bCl(G, \tau)$].

Definition 2.6. If M be a subset of G, then

(i) b-interior [1] of M is the union of all b-open sets contained in M.

(ii) b-closure [1] of M is the intersection of all b-closed sets containing M. The b-interior [respectively b-closure] of M is denoted by $b - Int(M)$ [respectively $b - Cl(M)$].

3. Group Action on *prw*-Compact and *prw*-Lindelof Spaces

Definition 3.1. A collection $\{M_i : i \in \wedge\}$ of *prw*-open sets in topological group (G, τ) is called *prw*-open cover of a subset N of G if $N \subseteq \bigcup\{M_i : i \in \wedge\}$ holds.

Definition 3.2. A topological group (G, τ) is called *prw*-compact if every *prw*-open cover of G has a finite subcover.

Theorem 3.3. *A topological group (G, τ) acting on every prw-closed subset of a prw-compact space is prw-compact relative to G.*

Proof. Let M be *prw*-closed subset of a *prw*-compact space (G, τ). Then $G - M$ is *prw*-open in (G, τ). Let $V = \bigcup\{A_i : i \in \wedge\}$ be a cover of M by *prw*-open sets. Therefore, $M \cup (G - M)$ is a *prw*-open cover of G. Since G is *prw*-compact, there exists a finite subset $\wedge_\circ$ of $\wedge$ such that $M \cup (G - M)$ $= \bigcup\{A_i : i \in \wedge_0\}$. Therefore $M \subseteq \bigcup\{A_i : i \in \wedge_\circ\}$. Hence M is *prw*-compact relative to G.

Definition 3.4. A group G is said to be prw-Lindelof space if every cover of G by prw-open sets contains a countable subcover.

Theorem 3.5. *A group acting on a function $g : G \to G'$ is prw-open and G' is prw-Lindelof space, then G is Lindelof space.*

Proof. Let $\{U_i\}$ be an open cover for G. Then $\{g(U_i)\}$ is a cover of G' by prw-open sets. Since G' is prw-Lindelof, $\{g(U_i)\}$ contains a countable subcover, namely $\{g(U_{ij})\}$. Then $\{(U_{ij})\}$ is a countable subcover for G. Thus G is Lindelof space.

Theorem 3.6. *A group G acting on a function $g : (G, \tau) \to (G', \tau')$ be prw-irresolute onto and G be prw-Lindelof, then G' is prw-Lindelof space.*

Proof. Let $g : G \to G'$ be prw-irresolute onto and G be prw-Lindelof. Let $\{U_i\}$ be an open cover for G'. Then $\{g^{-1}(U_i)\}$ is a cover of G by prw-open sets. Since G is prw-Lindelof, $\{g^{-1}(U_i)\}$ contains a countable subcover, namely $\{g^{-1}(U_{ij})\}$. Then $\{(U_{ij})\}$ is a countable subcover for G'. Thus G' is prw-Lindelof space.

4. Conclusion

In this paper, group action on *prw*-compact and *prw*-Lindelof spaces in topological spaces are introduced and some new results are analyzed.

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A NOTE ON t-DERIVATIONS OF BH-ALGEBRAS

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ABSTRACT

The notion of BCK-algebra was proposed by Imai and Iseki in 1966. In the same year Iseki introduced the notion of a BCI-algebras, which is generalization of a BCK-algebra. Y. B. Jun, E.H. Roh and H. S. Kim defined the notion of BH-algebra. Motivated by some results on derivations on rings and the generalizations of BCK and BCI-algebras. In 2019, P. Ganesan and N. Kandaraj introduced the notion of derivations on BH-algebras. In this paper, we study the notion of t-derivations on BH-algebras and investigate simple, interesting and elegant results.

1. Introduction

Imai Y. and Iseki K. [7, 8] introduced the on axiom system of propositional calculi and have been extensively investigated by many researchers. Iseki K. and Tanaka S. [9] introduced the theory of BCK algebra. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. Y.B. Jun, E.H. Roh and H.S. Kim [12] introduced the notion of BH-algebras. They investigated several relations between BH algebras and BCK-algebras. In 1957, Posner [18] introduced the notion of derivations in Prime rings theory. Also Lee P. H. and Lee T. K. [16] developed on derivations of prime rings.

The notion of derivation in ring theory is quite old and plays an important role in algebra. Al-Shehri N.O. and Bawazeer S. M. [4] introduced the notion of derivations of BCC algebras Many Research papers have appeared on the derivations of BCI-algebras in different ways. Zhan J and Liu Y. L. [19] developed the notion of f-derivations on BCI-algebras. Muhiuddin G and Abdullah Al-roqi M [17] introduced on t-derivations of BCIalgebras. Recently, in the year 2019 Ganesan p and Kandaraj N. defined and studied the notion of derivations, Compositions of derivations and fderivations of BH-algebras using the idea of regular derivations in BHalgebras and obtained some of its properties. The term algebra is used here to denote the algebraic structure defined on a non-empty set with a binary composition satisfying certain laws that resemble the algebra of logic but not the usual algebra.

The notion of the derivations is the same as that in ring theory and the usual algebraic theory. Motivated by a lot of Work done on derivations of BH-algebras and on derivations of other related abstract algebraic, structures such as BCI, TM, and d-algebras. In this paper we introduce the notion of t-derivations and show that if θ_t and θ'_t are t-derivations on U, then $(\theta_t \circ \theta'_t)$ is also a t-derivation on BH-algebra U and $(\theta_t \circ \theta'_t) = (\theta'_t \circ \theta_t)$. Finally we prove that $(\theta'_t * \theta_t) = (\theta_t * \theta'_t)$ where θ_t and θ'_t are t-derivations of BH-algebras.

2. Preliminaries

We review some basic definitions and properties that will be useful in our results.

Definition 2.1 [12]. Let X be a set X with a binary operation $*$ and a constant 0. Then $(X, *, 0)$ is called a BH-algebra, if it satisfies the following axioms

(1) $x * x = 0$

(2) $x * 0 = x$

(3) If $x * y = 0$ and $y * x = 0 \Rightarrow x = y$ for all $x, y \in X$.

Define a binary relation $\leq$ on X by taking $x \leq y$ if and only if $x * y = 0$. In this case $(X, \leq)$ is a partially ordered set [ ]

Let $(X, *, 0)$ be a BH-algebra and $x \in X$. Define $x * X = \{x * y \mid y \in X\}$.

Then X is said to be edge BH-algebra if for any $x \in X$, $x * X = \{x, 0\}$

Definition 2.2 [12]. Let S be a nonempty subset of a BH-algebra X. Then S is called Sub algebra of X, if $x * y \in S$ for all $x, y \in S$.

Definition 2.3 [12]. Let X be a BH-algebra and $I(\neq 0) \subseteq X$. Then I is called a BH-ideal of X if

(1) $0 \in I$

(2) $x * y \in I$ and $y \in I \Rightarrow x \in I$ for all $x, y \in I$

In BH-algebra X for all $x \cdot y, z \in x$, the following Property hold [ ]

1. $((x * y) * (x * z)) * (z * y) = 0$

2. $(x * y) * x = 0$

3. $(x * (x * y)) = y$

Theorem 2.4 *[11]. Every BH-algebra satisfying the condition (1) is a BCI algebra and satisfying the conditions (1) and (2) is a BCK-algebra*

3. Composition of t-Derivations on BH-algebras

Definition 3.1. Let U be a BH-algebra and θ_t, θ'_t be two self-maps of U. we define $\theta_t \circ \theta'_t : U \to U$ such that $(\theta_t \circ \theta'_t)(u) = \theta_t, (\theta'_t(u))$ for all $u \in U$.

Example 3.2. Let $U = \{0, a, b, c\}$ be a BH-algebra with the following Cayley table.

*	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Let $\theta_t : U \to U$ such that $\theta_t(x) = \begin{cases} c & \text{if } u = 0, a \\ 0 & \text{if } u = b, c \end{cases}$

Let $\theta'_t : U \to U$ such that $\theta'_t(x) = 0 \; \forall \, u = 0, a, b, c$

Then $(\theta_t \circ \theta'_t)(u) = \theta_t(\theta'_t(u)) \; \forall u \in U$.

Proposition 3.3. *Let U be a BH-algebra and* θ_t, θ'_t be (l, r)-*t-derivations on U. Then the composition of* θ_t *and* θ'_t *is a left-right t-derivations on U.*

Proof. Let U be a BH-algebra and θ_t, θ'_t be (l, r)-t-derivations on U.

Since θ_t, θ'_t are left-right-t-derivations on U.

$\theta_t(u * v) = (\theta_t(u) * v) \wedge (u * \theta_t(v)) \forall \, u, v \in U$

Therefore $\theta_t(u * v) = (\theta_t(u) * v)$

To claim: $(\theta_t \circ \theta'_t)$ is a left-right t-derivation on U.

(i.e.) $(\theta_t \circ \theta'_t)(u * v) = (\theta_t \circ \theta'_t)(u) * v$

$$(\theta_t \circ \theta'_t)(u * v) = (\theta_t(\theta'_t(u * v)))$$
$$= (\theta_t(\theta'_t(u) * v))$$
$$= (\theta_t(\theta'_t(u)) * v))$$
$$= (\theta_t(\theta'_t(u))) * v$$
$$= (\theta_t \circ \theta'_t)(u) * v. \text{ Hence the result}$$

Proposition 3.4. *Let U be a BH-algebra and θ_t, θ'_t be right-left t-derivations on U. Then the composition of θ_t and θ'_t is a right left t-derivation on U.*

Proof. Let θ_t, θ'_t be two right-left t-derivations on U.

$\theta_t(u * v) = (u * \theta_t(v)) \wedge (\theta_t(u) * v)\ \forall\, u, v \in U$

Therefore $\theta_t(u * v) = (u * \theta_t(v))$

$\theta'_t(u * v) = (u * \theta'_t(v)) \wedge (\theta'_t(u) * v)$ for all $u, v \in U$

Therefore $\theta'_t(u * v) = (u * \theta'_t(v))$

Claim: $(\theta_t \circ \theta'_t)$ is a right-left t-derivations on U.

(i.e.) $(\theta_t \circ \theta'_t)(u * v) = u * (\theta_t \circ \theta'_t)(v)$

$$
\begin{aligned}
(\theta_t \circ \theta'_t)(u * v) &= \theta_t(\theta'_t(u * v)) \\
&= \theta_t(u * \theta'_t(v)) \\
&= u * \theta_t(\theta'_t(v)) \\
&= u * (\theta_t \circ \theta'_t)(v)
\end{aligned}
$$

$(\theta_t \circ \theta'_t)$ is a right-left t-derivation on U.

From the above two propositions we get the following theorem.

Theorem 3.5. *Let U be a BH-algebra and θ_t, θ'_t be two t-derivations on U, then the composition of θ_t and θ'_t is also a t-derivation on U.*

Theorem 3.6. *Let U be a BH-algebra and θ_t be right-left t-derivation on U. Let θ'_t be a left -right t-derivation on U. Then $(\theta_t \circ \theta'_t) = (\theta_t \circ \theta'_t)$.*

Proof. Let θ'_t be left-right t-derivation on U.

$\theta'_t(u * v) = (\theta'_t(u) * v) \wedge (u * \theta'_t(v))$ for all $u, v \in U$

Therefore $\theta'_t(u * v) = (\theta'_t(u) * v)$

Let θ_t be right-left t-derivation on U.

$\theta_t(u * v) = (u * \theta_t(v)) \wedge (\theta_t(u) * v)$ for all $u, v \in U$

Therefore $\theta_t(u * v) = (u * \theta_t(v))$

Now $(\theta_t \circ \theta'_t)(u * v) = \theta_t(\theta'_t(u * v))$

$= \theta_t(\theta'_t(u) * v)$

$= \theta'_t(u) * (\theta_t(v)), \ldots 1$

Similarly $(\theta'_t \circ \theta_t)(u * v) = \theta'_t(\theta_t(u * v))$

$= \theta'_t(u * \theta_t(v))$

From 1 and 2 $(\theta_t \circ \theta'_t)(u * v) = (\theta'_t \circ \theta_t)(u * v)$

Hence $(\theta_t \circ \theta'_t) = (\theta'_t \circ \theta_t)$.

Definition 3.7. Let U be a BH-algebra and θ_t, θ'_t be two self-maps on U. We define $\theta_t * \theta'_t : U \to U$ such that $(\theta_t * \theta'_t)(u) = \theta_t(u) * \theta'_t(u)$ for all $u \in U$

Example 3.8. Let $U = \{0, a, b, c\}$ be a BH-algebra with the following Cayley table

*	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Let $\theta_t : U \to U$ such that $\theta_t(u) = c \;\forall\; u = 0, a, b, c$.

$\theta'_t(u) = 0 \;\forall\; u = 0, a, b, c$.

Define $(\theta_t * \theta'_t) : U \to U$ such that $(\theta_t * \theta'_t)(u) = c$ for all $u = 0, a, b, c$

Therefore $(\theta_t * \theta'_t)(u) = \theta_t(u) * \theta'_t(u)$ for all $u \in U$.

Theorem 3.9. *Let X be a BH-algebra and* θ_t, θ'_t *be t-derivations on U. Then* $(\theta_t * \theta'_t)(u) = \theta_t(u) * \theta'_t(u)$

Proof. Let θ'_t be left-right t-derivation on U.

Now $(\theta_t \circ \theta'_t), (u * v) = \theta_t(\theta'_t(u * v))$

$= \theta_t(\theta'_t(u) * v)$

$= \theta'_t(u) * \theta_t(v)$

There are $(\theta_t \circ \theta'_t)(u * v) = \theta'_t(u) * \theta_t(v)), \ldots 1$

Again $(\theta_t \circ \theta'_t)(u * v) = \theta_t(\theta'_t(u * v))$

$$= \theta_t(u * \theta'_t(v))$$

$$= \theta_t(u) * \theta'_t(v)$$

Therefore $(\theta_t \circ \theta'_t)(u * v) = \theta'_t(u) * \theta'_t(v), \ldots 2$

From equations 1 and 2, we have

$$\theta'_t(u) * \theta_t(v) = \theta_t(u) * \theta'_t(v)$$

Replacing v by u, we have

$$\theta'_t(u) * \theta_t(u) = \theta_t(u) * \theta'_t(u)$$

$$(\theta'_t * \theta'_t)(u) = (\theta_t * \theta'_t)(u) \text{ for all } u \in U.$$

Hence we have $(\theta'_t * \theta_t) = (\theta_t * \theta'_t)$.

Definition 3.10. Let $L_t\text{Der}(U)$ denote the set of all left-right t-derivations of a BH-algebra U. Define the binary operation $\wedge$ on $L_t\text{Der}(U)$ as given below.

For θ_t, θ'_t belongs to $L_t\text{Der}(U)$. We define $(\theta_t \wedge \theta'_t)(u) = \theta_t(u) \wedge \theta'_t(u)$ for all $u \in U$.

Lemma 3.11. *If θ_t, θ'_t be left-right t-derivations of U, then $(\theta_t \wedge \theta'_t)$ is also a left right t-derivations of U.*

Proof. Let U be a BH-algebra and θ_t, θ'_t be left-right t-derivations on U.

$\theta_t(u * v) = \theta_t(u) * v$. Since (l, r) t-derivations and $\theta'_t(u * v) = \theta'_t(u) * v$

Claim: $(\theta_t \wedge \theta'_t)(u * v) = ((\theta_t \wedge \theta'_t)(u)) * v$

Now $(\theta_t \wedge \theta'_t)(u * v) = \theta_t(u * v) \wedge \theta'_t(u * v)$

$$= (\theta_t(u) * v) \wedge (\theta'_t(u) * v)$$

$$= (\theta'_t(u) * v) * ((\theta'_t(u) * v) * (\theta_t(u) * v))$$

$$= (\theta'_t(u) * v)$$

$$(\theta_t \wedge \theta'_t)(u * v) = (\theta'_t(u) * v), \ldots 1$$

$$((\theta_t \wedge \theta'_t)(u) * v = (\theta_t(u) \wedge \theta'_t(u)) * v$$

$$= (\theta'_t(u) * (\theta'_t(u) * \theta_t(u))) * v$$

$$= \theta_t(u) * v$$

$$((\theta_t \wedge \theta'_t)(x)) * v = \theta_t(u) * v), \ldots 2$$

Using the equations 1 and 2 we have

$$(\theta_t \wedge \theta'_t)(u * v) = (\theta_t \wedge \theta'_t)(u) * v$$

Hence $(\theta_t \wedge \theta'_t)$ is a left-right t-derivation on U.

Lemma 3.12. *The binary composition* $\wedge$ *defined on* $L_t\mathrm{Der}(U)$ *is associative.*

Proof. θ_t, θ'_t, θ''_t be left-right t-derivations on BH-algebra U.

Claim: $(\theta_t \wedge \theta'_t) \wedge \theta''_t = \theta_t \wedge (\theta'_t \wedge \theta''_t)$

Now $((\theta_t \wedge \theta'_t) \wedge \theta''_t)(u * v) = (\theta_t \wedge \theta'_t)(u * v) \wedge \theta''_t(u * v)$

$$= (\theta_t(u) * v) \wedge (\theta''_t(u) * v)$$

$$= \theta_t(u) * v, \ldots 1$$

Also $\theta_t \wedge (\theta'_t \wedge \theta''_t))(u * v) = \theta_t(u * v) \wedge (\theta'_t \wedge \theta''_t)(u * v)$

$$= \theta_t(u) * v) \wedge (\theta'_t(u) * v)$$

$$= (\theta'_t(u) * v) * ((\theta'_t(u) * v) * (\theta_t(u) * v))$$

$$= \theta_t(u) * v, \ldots 2$$

From the results 1 and 2

$$((\theta_t \wedge \theta'_t) \wedge \theta''_t)(u * v) = (\theta_t \wedge (\theta'_t \wedge \theta''_t))(u * v)$$

put $v = 0$ in the above equation, we have

$$((\theta_t \wedge \theta'_t) \wedge \theta''_t)(u) = (\theta_t \wedge (\theta'_t \wedge \theta''_t))(u)$$

Equating the operator $(\theta_t \wedge \theta'_t) \wedge \theta''_t = \theta_t \wedge (\theta'_t \wedge \theta''_t)$

Hence the lemma.

From the above two lemma we get the following theorem

Theorem 3.13. *$L_t Der(u)$ is a semi group under the binary operation $\wedge$ which is defined by $(\theta_t \wedge \theta'_t)(u) = \theta_t(u) \wedge \theta'_t(u)\ \forall\ u, v \in U$ and θ_t, θ'_t belongs to $L_t Der(u)$.*

Definition 3.14. Let $R_t Der(u)$ denote the set of all right left t-derivations on BH-algebras U. Define the operation $\wedge$ on $R_t Der(u)$ as given below. For $\theta_t, \theta'_t \in R_t \mathrm{Der}(u)$, we define $(\theta_t \wedge \theta'_t)(u) = \theta_t(u) \wedge \theta'_t(u)\ \forall\ u \in U$.

Note:

Analogously we can prove the following result $R_t Der(u)$ is a semi group under the binary operation $\wedge$ which is defined by $(\theta_t \wedge \theta'_t)(u) = \theta_t(u) \wedge \theta'_t(u)\ \forall\ u \in U$. and θ_t, θ'_t belongs to $R_t Der(u)$.

From the above two results we have

Theorem 3.15. *If $tDer(u)$ denotes the set of all t-derivations on U, then it is a semigroup under the binary operation $\wedge$ defined by $(\theta_t \wedge \theta'_t)(u) = \theta_t(u) \wedge \theta'_t(u)\ \forall\ u \in U$ and $\theta_t, \theta'_t \in tDer(u)$.*

4. Conclusion

An algebraic structure that arises from the study of algebraic formulations of propositional logic. Taking different theorems or statements of propositional logic, different algebraic structures could be obtained. The BH - Algebra is one such algebra. The derivation concept is an important and Mathematics. The deep theory has been developed for derivations through various algebras. It plays an important role in algebra, algebraic geometry and linear differential equations.

We have considered the concept of t-derivations in BH-algebras. Finally, we investigated the notion of the composition of t-derivations in BH-algebras. In our opinion these definitions and main results may be similarly extended to some other algebras such as BCI-algebras [1, 2, 10, 13], d-algebras [5, 6,14, 15] and B-algebras [3] so forth. In future any researcher can study the notion of t-derivations in different algebraic structures which may have a lot of applications in various fields. This work is a foundation for the further study of the researcher on derivations of algebras.

The future study of derivations on BH-algebras may be the following topics should be covered.

(a) To find the generalized derivations on BH-algebras.
(b) To find the t-derivations of Q-algebras, d-algebras, B-algebras and so on so.
(c) To find more results and its applications in t-derivations on BHalgebras.
(d) To find to investigate how these concepts could be applied to the field of computers for processing information.

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EVALUATION OF PERFORMANCE MEASURES OF FUZZY QUEUES WITH PREEMPTIVE PRIORITY USING DIFFERENT FUZZY NUMBERS

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ABSTRACT

This paper investigates the performance measures in crisp values for preemptive priority fuzzy queues. This paper gives a procedure to convert fuzzy environment to crisp environment by Yager ranking Index in order to analyse the performance measures of fuzzy queues. Ranking the fuzzy numbers is an important characteristic in fuzzy environment. We take both the arrival time and service time as fuzzy numbers and derive the performances measure for triangular, trapezoidal and pentagonal fuzzy numbers. An example is given to derive the performance measures of 3-priority queues.

1. Introduction

Queueing models are applied in several fields such as transportation engineering, service industry, production, communication systems, health care and information processing systems. The waiting discipline where customers are served with respect to their order of arrival is frequently found in queuing models, but then again many real queueing systems follow priority discipline model. A priority mechanism is a useful method that allows different customer types of customers to receive different performance level. Priority queueing have many applications such as communication network, call centres and hospitals etc. Priority schemes are also known for their ease of implementation. Many queuing works is devoted to analyse priority queues. There are two possible segments in priority situation, the preemption and non-preemption. In the preemptive priority a customer with high priority is permitted to go into service immediately even if the lower priority is already in service. The preemptive priority queues are useful for performance evaluation of production, manufacturing, inventory controls and computer systems. The service of low customer is interrupted when on arrival of customer belonging to a higher class arrives, and will be restarted from the point of pause, when all the queues of higher priority have been emptied. In this situation the lower priority class customers are completely unseen and do not affect in any way the queues of the higher classes. Here the queuing model considered is one where few customers are given a priority service over routine. Given a model of this kind we require to find the performance measures.

The parameters in the preemptive priority may be fuzzy. The fuzzy queuing models are more realistic and practical than classical ones. Queuing models along with fuzzy increases their application. Uncertainty is determined by fuzzy set theory. Dissimilar to the classic model that considers arrivals to follow a Poisson process and exponentially distributed service times, the arrival rate in numerous real conditions is more possibility than probabilistic and are not denoted by exact terms. Bellman and Zadeh [1] presented the idea of fuzziness so that inexact information could be solved by decision making problems. The researchers like Li and Lee [10], Buckley [2], Negi and Lee [12], Kaufmann [9], Kao et al. [8], Chen [3, 4] has examined fuzzy queues by Zadeh's extension principle. Parametric linear programming approach to derive the membership functions of the system in fuzzy queues has been derived by Kao et al. [8]. Recent developments on fuzzy numbers by random variables can be used to analyse the queueing system. Zadeh, L. A [16] has presented the idea of fuzzy probabilities and the properties of fuzzy probability Markov chains were discussed. Buckley [2] studies multi-server queues with finite and infinite capacity queueing models with arrivals and departures follow possibilistic pattern. Chen [3] suggests a strategy of parametric programming in order to derive membership functions of the fuzzy queues.

Further, the conversion of fuzzy queues to crisp queues has also been widely discussed in works and many methods and approaches have been used. One of such methods is the robust ranking method [15]. Non-preemptive priority fuzzy queues have been explained by many authors where fuzzy problem are reduced to crisp problem. Most of the previous work have been done on non-preemptive queues. In this paper we study the preemptive priority queue and henceforth derive the performance measures of fuzzy queues using Yager ranking technique.

Definitions

Fuzzy set

A fuzzy set is characterized by a membership function mapping elements of a domain space, or universe of discourse X to the unit interval $[0, 1]$. (i. e.) $A = \{(x, \mu_A(x)); x \in X\}$. Here $\mu_A : X \to [0, 1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from $[0, 1]$.

α-cut of a fuzzy number

The α-cut of a fuzzy number $A(x)$ is defined as $A(\alpha) = \{x : \mu(x) \geq \alpha, \alpha \in [0, 1]\}$.

Addition of two Triangular fuzzy numbers can be performed as $(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$

Triangular fuzzy number

Fuzzy Triangular number can be represented by $\widetilde{A}(a, b, c)$ where $a < b < c$ with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise.} \end{cases}$$

Trapezoidal fuzzy number

Fuzzy Trapezoidal number can be represented by $\widetilde{A}(a, b, c, d)$ where $a < b < c < d$ with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & \text{otherwise.} \end{cases}$$

Pentagon fuzzy number

Fuzzy Trapezoidal number can be represented by $\widetilde{A}(a, b, c, d, e)$ where $a < b < c < d < e$ with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{x-b}{c-b}, & b \le x \le c \\ 1, & x = c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ \dfrac{e-x}{e-d}, & d \le x \le e \\ 0, & \text{otherwise.} \end{cases}$$

Definition. A fuzzy set $\widetilde{A}$ is convex subset of Z if and only if $\mu_{\widetilde{A}}(\lambda x_1 + (1 - \lambda x_2)) \ge \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2))$ for all $x_1, x_2 \in X$ and $\lambda \in [0, 1]$.

Yager ranking Index

Fuzzy problem is solved by reformulating into crisp problem by using Yager ranking Index. To solve the problem the fuzzy numbers are defuzzified into crisp ones by ranking method. Yager suggested the following index to order the fuzzy numbers. $Y(\widetilde{a}) = \frac{1}{2}\int_0^1 (a_\alpha^L + a_\alpha^U)\, d\alpha$, $Y(\widetilde{a})$ has the properties of linearity and additivity.

Mathematical formulation

A FM/FM/1 queuing system with a single server and 3-priority queues is studied. The inter arrival time $\widetilde{A}_\iota$, $i = 1, 2, 3$ of the first, second and third priority queues and service times are approximately known are given by the fuzzy sets

$$A_\iota = \{(x, \mu_{A_l}(X))/x \in X\}$$

$$\tilde{S} = \{(y, \mu_{\tilde{S}}(y))\ /y \in Y\}$$

Where X and Y are crisp universal sets of the inter arrival time and inter service time respectively and $\mu_{A_\iota}(x), i = 1, 2, 3$ and $\mu_{\tilde{S}}(y)$ are the respective membership functions. Using α-cut the inter arrival times and service time can be denoted by different levels of confidence intervals $[0, 1]$. Therefore a fuzzy preemptive priority queue can be reformulated to family of crisp queues with different α-cuts. Using the notion of α-cut the FM/FM/1 queue with 3-priority queues are reformulated to M/M/1 queue with 3-priority customers with equal service rates, i.e., $\mu_1 = \mu_2 = \mu_3 = \mu$. Further $\rho_1 = \frac{\lambda_1}{\mu_1}$, $\rho_2 = \frac{\lambda_2}{\mu_2}$, $\rho_3 = \frac{\lambda_3}{\mu_3}$

$$\rho = \rho_1 + \rho_2 + \rho_3, \rho = \frac{\lambda}{\mu}, \lambda = \lambda_1 + \lambda_2 + \lambda_3, \rho = \frac{\lambda_1 + \lambda_2 + \lambda_3}{\mu}, \sigma_k = \sum_{i=1}^{i=k} \rho_i, \sigma_0 = 0$$

Without loss of generality let us assume the performance measures for 3-priority queues.

From the traditional queuing theory, the waiting time in the queue is

$$W_{q,i} = \frac{\frac{1}{\tilde{\mu}}(1 - \sigma_{i-1}) + \frac{1}{\tilde{\mu}^2}\sum_{j=1}^{p}\tilde{\lambda}_j}{(1 - \sigma_{i-1})(1 - \sigma_i)} - \frac{1}{\tilde{\mu}}$$

The expected queue size is

$$L_q = \sum_{i=1}^{r} L_q^{(i)} = \sum_{i=1}^{r} \lambda_i(W_{q,\ i})$$

From which we can derive

$$W_q^i = \frac{\frac{1}{\tilde{\tilde{\mu}}}}{(1 - \sigma_{i-1})(1 - \sigma_i)} - \frac{1}{\tilde{\tilde{\mu}}}$$

$$W_q^{(1)} = \frac{1}{\mu}\left[\frac{\mu}{\mu-\lambda_1} - 1\right] = \frac{1}{\mu-\lambda_1} - \frac{1}{\mu}$$

$$W_q^{(2)} = \frac{\mu}{(\mu-\lambda_1)\,(\mu-(\lambda_1+\lambda_2))} - \frac{1}{\mu}$$

$$W_q^{(3)} = \frac{\mu}{(\mu-\lambda)\,(\mu-(\lambda_1+\lambda_2))} - \frac{1}{\mu}$$

$$L_q^{(1)} = \frac{\lambda_1}{\mu-\lambda_1} - \frac{\lambda_1}{\mu}$$

$$L_q^{(2)} = \frac{\lambda_2}{\mu\left(1-\frac{\lambda_1}{\mu}\right)\left(1-\frac{(\lambda_1+\lambda_2)}{\mu}\right)} - \frac{\lambda_2}{\mu}$$

$$L_q^{(3)} = \frac{\lambda_3}{\mu\left(1-\frac{\lambda}{\mu}\right)\left(1-\frac{(\lambda_1+\lambda_2)}{\mu}\right)} - \frac{\lambda_3}{\mu},$$

where $\lambda_1, \lambda_2, \lambda_3$ are the arrival rates of first, second and third priority units and μ is the service rate.

Numerical Example

Expected waiting time and expected number of customers in the queue for FM/FM/1 queue with 3-preemptive priority classes.

For Trapezoidal Fuzzy number

The rates of first, second and third priority with same service rates are trapezoidal fuzzy numbers represented by $A_1 = [3, 4, 6, 7]$, $A_2 = [5, 6, 8, 9]$, $A_3 = [7, 8, 10, 11]$, $\tilde{S} = [20, 21, 23, 24]$ per hour respectively. The α-cut of the membership functions $\mu_{A_1}(\alpha), \mu_{A_2}(\alpha), \mu_{A_3}(\alpha)$ and $\mu_{\tilde{S}}(\alpha)$ are $[3+\alpha,\ 7-\alpha]$, $[5+\alpha,\ 9-\alpha]$, $[7+\alpha,\ 11-\alpha]$ and $[20+\alpha,\ 24-\alpha]$ respectively.

We calculate $Y(3\ 4,\ 6,\ 7)$ by applying Yager ranking method. The membership function of the Trapezoidal fuzzy number $(3\ 4,\ 6,\ 7)$ is

$$\mu_{(x)} = \begin{cases} \dfrac{x-3}{1}, & 3 \le x \le 4 \\ 1, & 4 \le x \le 6 \\ \dfrac{7-x}{1}, & 6 \le x \le 7 \\ 0, & \text{otherwise} \end{cases}$$

The α cut of the fuzzy number (3, 4, 6, 7) is $(a_\alpha^L, a_\alpha^U) = (\alpha + 3, 7 - \alpha)$ and $Y(A_1) = R(3, 4, 6, 7) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)d\alpha = \int_0^1 0.5(10)d\alpha = 5.$

The Yager ranking indices for the fuzzy numbers A_2, A_3, S are given by $Y(A_2) = 7, Y(A_3) = 9, Y(\tilde{S}) = 22$ and hence $\lambda_1 = 5, \lambda_2 = 7, \lambda_3 = 9,$ $\mu = 22, \lambda = 21.$

Average waiting time of first priority customer in the queue is

$$W_q^{(1)} = \frac{1}{\mu - \lambda_1} - \frac{1}{\mu} = 0.01336.$$

Average waiting time of second priority customer in the queue is

$$W_q^{(2)} = \frac{1}{(\mu - \lambda_1)(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 0.08395.$$

Average waiting time of third priority customer in the queue is

$$W_q^{(3)} = \frac{\mu}{(\mu - \lambda)(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 2.1545.$$

Average queue length of first priority is

$$L_q^{(1)} = \frac{\lambda_1}{\mu - \lambda_1} - \frac{\lambda_1}{\mu} = 0.0668.$$

Average queue length of second priority is

$$L_q^{(2)} = \frac{\lambda_2}{\mu\left(1 - \frac{\lambda_1}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_2}{\mu} = 0.58765.$$

Average queue length of third priority is

$$L_q^{(3)} = \frac{\lambda_3}{\mu\left(1-\frac{\lambda}{\mu}\right)\left(1-\frac{(\lambda_1+\lambda_2)}{\mu}\right)} - \frac{\lambda_3}{\mu} = 19.3905.$$

For Triangular Fuzzy number

Let the rates of first, second and third priority with the same service rates are triangular fuzzy numbers denoted by $A_1 = [3, 6, 8], A_2 = [4, 7, 9], A_3 = [5, 8, 10], \tilde{S} = [20, 23, 25]$ per hour respectively. The α-cut of the membership functions $\mu_{A_1}(\alpha), \mu_{A_2}(\alpha), \mu_{A_3}(\alpha)$ and $\mu_{\tilde{S}}(\alpha)$ are $[3+3\alpha,\ 8-2\alpha], [4+3\alpha,\ 9-2\alpha], [5+3\alpha,\ 10-2\alpha],$ and $[20+3\alpha,\ 25-2\alpha]$ respectively.

We calculate $Y(3,\ 6,\ 8)$ by Yager ranking method.

The membership function of the triangular fuzzy number $(3,\ 6,\ 8)$ is

$$\mu_{(x)} = \begin{cases} \frac{x-3}{3}, & 3 \le x \le 6 \\ 1, & x = 6 \\ \frac{8-x}{2}, & 6 \le x \le 8 \\ 0, & \text{otherwise.} \end{cases}$$

The α cut of the fuzzy number $(3,\ 6,\ 8)$ is $(a_\alpha^L,\ a_\alpha^U) = (3+3\alpha,\ 8-2\alpha)$ for which $Y(A_1) = R(3, 6, 8) = \int_0^1 0.5(a_\alpha^L + a_\alpha^U)d\alpha = \int_0^1 0.5(3+3\alpha+8-2\alpha)$ $d\alpha = 5.75.$

Similarly the Yager ranking indices for the fuzzy numbers A_2, A_3, S are calculated as $Y(A_2) = 6.75, Y(A_3) = 7.75, Y(\tilde{S}) = 22.75$ and hence $\lambda_1 = 5.75,\ \lambda_2 = 6.75,\ \lambda_3 = 7.75,\ \mu = 22.75,\ \lambda = 20.25.$

Average waiting time of first priority customer in the queue is

$$W_q^{(1)} = \frac{1}{\mu-\lambda_1} - \frac{1}{\mu} = 0.01486.$$

Average waiting time of second priority customer in the queue is

$$W_q^{(2)} = \frac{\mu}{(\mu - \lambda_1)\,(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 0.08660.$$

Average waiting time of third priority customer in the queue is

$$W_q^{(3)} = \frac{\mu}{(\mu - \lambda)\,(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 0.8438.$$

Average queue length of first priority is

$$L_q^{(1)} = \frac{\lambda_1}{\mu - \lambda_1} - \frac{\lambda_1}{\mu} = 0.0854.$$

Average queue length of second priority is

$$L_q^{(2)} = \frac{\lambda_2}{\mu\left(1 - \frac{\lambda_1}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_2}{\mu} = 0.5846.$$

Average queue length of third priority is

$$L_q^{(3)} = \frac{\lambda_3}{\mu\left(1 - \frac{\lambda}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_3}{\mu} = 6.5398.$$

For Pentagon Fuzzy number

Let the rates of first, second and third priority with the same service rates are pentagon fuzzy numbers denoted by $A_1 = [1, 2, 3, 4, 5]$, $A_2 = [3, 4, 5, 6, 7]$, $A_3 = [9, 10, 11, 12, 13]$, $\tilde{S} = [20, 23, 25, 27, 30]$ per hour respectively. The α-cut of the membership functions $\mu_{A_1}(\alpha), \mu_{A_2}(\alpha), \mu_{A_3}(\alpha)$ and $\mu_{\tilde{S}}(\alpha)$ are $[1 + \alpha, 5 - \alpha], [3 + \alpha, 7 - \alpha], [9 + \alpha, 13 - \alpha]$, and $[20 + 3\alpha,\ 30 - 3\alpha]$ respectively.

Average waiting time of first priority customer in the queue is

$$W_q^{(1)} = \frac{1}{\mu - \lambda_1} - \frac{1}{\mu} = 0.00545.$$

Average waiting time of second priority customer in the queue is

$$W_q^{(2)} = \frac{\mu}{(\mu - \lambda_1)(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 0.0268.$$

Average waiting time of third priority customer in the queue is

$$W_q^{(3)} = \frac{\mu}{(\mu - \lambda)(\mu - (\lambda_1 + \lambda_2))} - \frac{1}{\mu} = 0.20509.$$

Average queue length of first priority is

$$L_q^{(1)} = \frac{\lambda_1}{\mu - \lambda_1} - \frac{\lambda_1}{\mu} = 0.01636.$$

Average queue length of second priority is

$$L_q^{(2)} = \frac{\lambda_2}{\mu\left(1 - \frac{\lambda_1}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_2}{\mu} = 0.13422.$$

Average queue length of third priority is

$$L_q^{(3)} = \frac{\lambda_3}{\mu\left(1 - \frac{\lambda}{\mu}\right)\left(1 - \frac{(\lambda_1 + \lambda_2)}{\mu}\right)} - \frac{\lambda_3}{\mu} = 2.25607.$$

Conclusion

The fuzzy preemptive priority queues are represented more exactly and the results are derived. Numerical examples for triangular, trapezoidal and pentagonal numbers are explained effectively to determine the validity of the suggested model. The ranking approach used here in the paper is more effective while transforming fuzzy queues into crisp queues and − cut is used to reduce the fuzzy queues to crisp queues. The expected queue length and expected waiting time are derived. The future work can be done in examining the efficiency of this technique to other queueing models and applying different methods to find the performance measures in fuzzy preemptive queues.

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