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Aim & Scope

The Advances and Applications in Mathematical Sciences (ISSN 0974-6803) is a monthly journal. The AAMS's coverage extends across the whole of mathematical sciences and their applications in various disciplines, encompassing Pure and Applied Mathematics, Theoretical and Applied Statistics, Computer Science and Applications as well as new emerging applied areas. It publishes original research papers, review and survey articles in all areas of mathematical sciences and their applications within and outside the boundary.

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ABSOLUTELY HARMONIOUS LABELING OF SOME SPECIAL GRAPHS

M. SEENIVASAN, P. ARUNA RUKMANI and A. LOURDUSAMY

ABSTRACT

Absolutely harmonious labeling f is an injection from the vertex set of a graph G with q edges to the set {0, 1 , 2 , q-1} if each edge uv is assigned f (v) + f (u) then the resulting edge labels can be arranged as .$a_0, a_1, a_2, \ldots, a_{q-1}$ where $a_i = q - i$ or $q + i, 0 \le i \le q-1$. *however, when G is a tree one of the vertex labels may be assigned to exactly two vertices. A graph which admits absolutely harmonious labeling is called absolutely harmonious graph. In this paper, we study absolutely harmonious labeling of some special graphs.*

Keywords: star graph, globe graph, triangular ladder, bistar graph, shadow graph, W-graph.

1. Introduction

In this paper, we consider finite, simple and undirected graphs. M. Seenivasan and A. Lourdusamy [3] introduced another variation of harmonious labeling, namely, absolutely harmonious labeling of graphs. In this paper we investigate the absolutely harmonious labeling of some special graphs such as triangular ladder, globe graph, shadow graph, Sudivision, splitting and duplication of star graph.

Theorem 1. *The graph* $K_2 + mK_1$ *is absolutely harmonious.*

Proof. Let $G = K_2 + mK_1$ graph.

The vertex set $V(G) = \{x, y, w_1, w_2, \ldots, w_m\}$ and the edge set $E(G) = \{xy, xw_i, yw_i : 1 \le i \le m\}$ here, G is of order $m + 2$ and size $2m + 1$.

Now, define $f : V(G) \to \{0, 1, 2, 3, \ldots, q-1\}$ as follows

$f(x) = 0$

$f(y) = m + 1$

$f(w_i) = i, 1 \le i \le m$

The induced edge labels are as follows

$f^*(xy) = a_m$

$f^*(xw_i) = a_{q-j}; 1 \le i \le m$ and $1 \le j \le m$.

$f^*(yw_i) = a_k; 0 \le k \le m-1$

From the above, $a_0, a_1, a_2, \ldots, a_{q-1}$ where $a_i = q - i$ (or) $q + i : 0 \le i \le q-1$ are the arranged edge labels. Therefore f admits absolutely harmonious labeling.

and hence $K_2 + mK_1$ is an absolutely harmonious graph. □

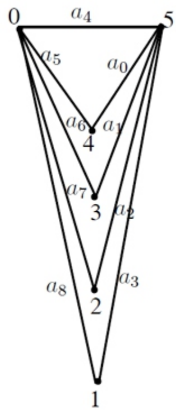

Figure 1. $K_2 + 4K_1$.

Definition 1. Let G be a (p, q) graph. The subdivision of each edge of a graph G with a vertex is called the subdivision graph and it is denoted by $S(G)$.

Theorem 2. *$S(K_{1,n})$ is absolutely harmonious for all* $n \geq 1$.

Proof. Let $G = S(K_{1,n})$

Let $V(G) = \{u, v_i, u_i : 1 \leq i \leq n\}$ and $E(G) = \{uv_i, v_iu_i : 1 \leq i \leq n\}$

Here G is of order $2n+1$ and size $2n$. Now, Define $f : V(G) \to \{0, 1, 2, \ldots, q-1\}$ as follows

$f(u) = 1$

$f(v_1) = 0$

$f(v_i) = 2i - 1;\ 2 \leq i \leq n$

$f(u_i) = 2i;\ 1 \leq i \leq n-1$

$f(v_n) = 2n - 2$

Then the induced edge labels are as follows

$f^*(uv_1) = a_{q-1}$

$f^*(uv_k) = a_{[q-2i]};\ 2 \leq k \leq n,\ 2 \leq i \leq n$

$f^*(v_1u_1) = a_{q-2}$ and $f^*(v_nu_n) = a_{2n-3}$

$f^*(v_ku_k) = a[q-(4i-1)];\ 2 \leq k \leq n-1;\ 2 \leq i \leq n-1$

From the above, $a_0, a_1, a_2, \ldots, a_{q-1}$ where $a_i = q - i$ (or) $q + i; 0 \leq i \leq q-1$ are the arranged edge labels. Therefore f admits absolutely harmonious labeling of $S(K_{1,n})$ and hence $S(K_{1,n})$ is an absolutely harmonious graph. □

Definition 2. Let the graphs G_1 and G_2 have disjoint vertex sets V_1 and V_2 the edge sets E_1 and E_2 respectively. Then their union $G = G_1 \cup G_2$ is a graph with vertex set $V = V_1 \cup V_2$ and the edge set $E = E_1 \cup E_2$. Clearly, $G_1 \cup G_2$ has $p_1 + p_2$ vertices and $q_1 + q_2$ edges.

Theorem 3. $S(K_{1,n}) \cup K_{1,m},\ n, m \succ 1$ *is not absolutely harmonious.*

Proof. Let $G = S(K_{1,n}) \cup K_{1,m}$ be a graph with p vertices and q edges.

Let $\{u, u_i, v_i, w, w_j; 1 \leq i \leq n, 1 \leq j \leq m\}$ be the vertices of G and $\{uu_i, u_iv_i, ww_j; 1 \leq i \leq n, 1 \leq j \leq m\}$ be the edges of G.

Here, $p = 3n + 2$ and $q = 3n$ since $p \succ q$, we cannot give the distinct labels from $\{0, 1, 2, \ldots, q-1\}$ to the vertices of G. Hence G is not an absolutely harmonious graph. □

Definition 3. For a graph G, the splitting graph $S'(K_{1,n})$ of G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Theorem 4. *The splitting graph* $S'(K_{1,n})$ *is absolutely harmonious.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the pendant vertices and v be the apex vertex of $K_{1,n}$ and $u, u_1, u_2, \ldots, u_n$ be added vertices corresponding to $v, v_1, v_2, \ldots v_n$ to obtain $S'(K_{1,n})$.

Let G be the splitting graph $S'(K_{1,n})$. Then G is of order $2n + 2$ and size $3n$. Now, Define $f : V(G) \to \{0, 1, 2, \ldots, q-1\}$ as follows

$f(u) = 3n - 1$

$f(v) = 0$

$f(v_i) = i; 1 \leq i \leq n$

$f(u_i) = f(v_n) + i; 1 \leq i \leq n$

Then the induced edge labels are as follows

$f(uv_i) = a_{i-1}; 1 \leq i \leq n$

$f(vv_i) = a_{q-1}; 1 \leq i \leq n$

$f(vu_i) = a_{2n-i}; 1 \leq i \leq n$

From the above, $a_0, a_1, a_2, \ldots, a_{q-1}$ where $a_i = q-1$ (or) $q+i; 0 \leq i \leq q-1$ are the arranged edge labels. Therefore f admits absolutely harmonious labeling of $S'(K_{1,n})$ and hence $S'(K_{1,n})$ is an absolutely harmonious graph. □

Definition 4. The Bistar graph $B_{n,n}$ is obtained by joining the center (apex) vertices of two copies of $K_{1,n}$ by an edge.

Theorem 5. *The Bistar graph* $B_{n,n}$ *is absolutely harmonious.*

Proof. Let $G = B_{n,n}$. Let $V(G) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$, $E(G) = \{uv, vv_i, uu_i : 1 \leq i \leq n\}$. Then G is of order $2n+2$ and size $2n+1$. Now, define $f : V(G) \to \{0, 1, 2, \ldots, q-1\}$ as follows

$f(u) = 0$

$f(v) = q-1$

$f(v_i) = 2i-1; 1 \leq i \leq n$

$f(u_1) = 1 \; f(u_k) = 2i; 1 \leq i \leq n, 2 \leq k \leq n$

Then the induced edge labels are as follows

$f^*(uv) = a_1$

$f^*(vv_1) = a_0$

$f^*(vv_k) = a_{2i}; 1 \leq i \leq n, 2 \leq k \leq n$

$f^*(uu_1) = a_{q-1}$

$f^*(uu_k) = a_{2i-1}; n \leq i \leq 2 \leq k \leq n$

From the above, $a_0, a_1, a_2, \ldots, a_{q-1}$ where $a_i = q - 1$ (or) $q + i; 0 \le i \le q - 1$ are the arranged edge labels.

Therefore f is an absolutely harmonious labeling of Bistar graph $B_{n,n}$ and hence the Bistar graph $B_{n,n}$ is an absolutely harmonious graph. □

Theorem 6. *$S(K_{1,n}) \cup B_{r,s}$ is not Absolutely harmonious for all $n, r, s \succ 1$.*

Proof. Let $G = S(K_{1,n}) \cup B_{r,s}$ since the number of vertices of G is greater than the number of edges of G. We cannot give the distinct labels from $\{0, 1, 2, \ldots, q-1\}$ to be the vertices of G.

Hence G is not an absolutely harmonious graph. □

Definition 5. The triangular ladder TL_n is graph obtained from L_n by adding edges $u_i v_{i+1}, 1 \le i \le n$, where u_i and $v_i, 1 \le i \le n$ are the vertices of L_n such that $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ are the two paths of the length n in the graph L_n.

Theorem 7. *The Triangular ladder TL_n is an absolutely harmonious graph.*

Proof. Let G be a TL_n graph. Let $V(G) = \{u_i v_i : 1 \le i \le n\}$ and $E(G) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1} : 1 \le i \le n-1\} \cup \{u_i v_i : 1 \le i \le n\}$.

Then G is of order $2n$ and size $4n - 3$. Now, Define

$f : V(G) \to \{0, 1, 2, \ldots, q-1\}$ as follows

$f(u_1) = 0$

$f(v_i) = 2i - 1; 1 \le i \le n - 1$

$f(u_k) = 2i; 1 \le i \le n - 1, 2 \le k \le n$

Then the induced edge labels are as follows

$f^*(u_1 v_1) = a_{q-1}$

$f^*(u_i v_{i+1}) = a_{q-2k}, 1 \le i \le n - 1; 1 \le k \le n + 2$ and k is odd

$f^*(v_i v_{i+1}) = a_{q-4i}, 1 \le i \le n - 1$

$f^*(u_k v_k) = a_{q-(4i+1)}, 1 \le i \le n - 1, 2 \le k \le n$

$f^*(u_k v_{k+1}) = a_{q-(4i+3)}; 0 \le i \le n - 1, 1 \le k \le n - 1$

From the above, $a_0, a_1, a_2, \ldots, a_{q-1}$ where $a_i = q - i$ (or) $q + i, 0 \le i \le q - 1$ are the arranged edge labels.

Therefore f is an absolutely harmonious labeling of triangular ladder n TL and hence the triangular ladder n TL is an absolutely harmonious graph.

Definition 6. Duplication of a vertex k v of a graph G produces a new graph G by adding a vertex k v with ( ) ( ). k k v N v N =

Theorem 8. The graph obtained by duplication of apex vertex by an edge in n K, 1 is absolutely harmonious.

Proof. Let 0 v be the apex vertex of star n K, 1 and n v v v , , , 2 1 are pendant vertices of . , 1n K
Let G denote the graph obtained by duplication of apex vertex 0 v by an edge . 0 0v v Then G is of order 3 + n and size 3 + n Now, define ( ) 1 , , 2 , 1 , 0 : − → q G V f as follows

$f(v_0) = 0$

$f(v_0') = 1$

$f(v_0'') = n + 2$

$f(v_k) = i + 1; 1 \leq k \leq n, 1 \leq i \leq n$

Then the induced edge labels are as follows

$f^*(v_0 v_0'') = a_1$

$f^*(v_0' v_0'') = a_0$

$f^*(v_0 v_0') = a_{q-1}$

$f^*(v_0 v_k) = a[q - (i + 1)]; 1 \leq k \leq n, 1 \leq i \leq n$

From the above, $a_0, a_1, a_2, \ldots, a_{q-1}$ where $a_i = q - i$ (or) $q + i; 0 \leq i \leq q - 1$ are the arranged edge labels.

Therefore f is an absolutely harmonious labeling of the graph obtained by duplication of apex vertex by an edge in n K, 1 and hence the graph obtained by duplication of apex vertex by an edge in n K, 1 is an absolutely harmonious graph.

Definition 7. Globe graph is defined as the two isolated vertex are joined by n paths of length 2. It is denoted by ( ). n Gl

Theorem 9. Globe ( ) 3 , n n Gl is an absolutely harmonious graph.

Proof. Let ( ). n Gl G=

Let $V(G) = \{u, v, w_i : 1 \le i \le n\}$ and $E(G) = \{[uw_i] \cup [vw_i] : 1 \le i \le n\}$.

Here, $|V(G)| = n + 1$ and $|E(G)| = 2n$

Define $f : V(G) \to \{0, 1, 2, \ldots, q-1\}$ as follows

$f(u) = q - 1$

$f(v) = 0$

$f(w_i) = i; 1 \le i \le q - n$

Then the induced edge labels are as follows

$f^*(uw_i) = a_{i-1}, 1 \le i \le q - n$

$f^*(w_i v) = a_{q-i}; 1 \le i \le q - n$

From the above, $a_0, a_1, a_2, \ldots, a_{q-1}$ where $a_i = q - i$ (or) $q + i$, $0 \le i \le q-1$ are the arranged edge labels.

Therefore f is an absolutely harmonious labeling of the globe ( ) 3 , n n Gl and hence the globe ( ) 3 , n n Gl is an absolutely harmonious graph.

□ **Definition 8.** The shadow graph ( ) G D2 of a connected graph G is constructed by taking two copies of G say G and . G Join each vertex u in G to the neighbours of the corresponding vertex v in . G

Theorem 10. *The graph* $D_2(K_{1,n})$, $n \ge 2$ *is absolutely harmonious.*

Proof. Let $\{v, v_i, 1 \le i \le n\}$ be the vertices and $\{e_i, 1 \le i \le 4n\}$ be the edges.

Define $f : V(G) \to \{0, 1, 2, \ldots, q-1\}$ as follows

$f(u) = q - 1, f(v) = 0$

$f(u_i) = i; 1 \le i \le n$

$f(v_i) = n + i; 1 \le i \le n$

Then the induced edge labels are as follows

$f^*(uu_i) = a_{i-1}; 1 \leq i \leq n$

$f^*(vu_i) = a_{q-i}; 1 \leq i \leq n$

$f^*(uv_i) = a_{n-i}; 0 \leq i \leq n-1$

$f^*(uu_i) = a[q-(n+i)]; 1 \leq i \leq n$

From the above, $a_0, a_1, a_2, \ldots, a_{q-1}$ where $a_i = q-1$ (or) $q+i; 0 \leq i \leq q-1$ are the arranged edge labels.

Therefore f is an absolutely harmonious labeling of the graph $D_2(K_{1,n}), n \geq 2$ and hence the graph $D_2(K_{1,n}), n \geq 2$ is an absolutely harmonious graph. □

Definition 9. Let G be a graph with set of vertices and edges as $V(G) = \{(c_1, c_2, b, w, d) \cup (x^1, x^2, x^3, \ldots, x^n) \cup (y^1, y^2, y^3, \ldots, y^n)\}$ $E(G) = \{(c_1x^1, c_1x^2, c_1x^3, \ldots, c_1x^n) \cup (c_2y^1, c_2y^2, c_2y^3, \ldots, c_2y^n) \cup (c_1b, c_1w) \cup (c_2w, c_2d)\}$

We shall call it W-graph and it shall be denoted by $W(n)$

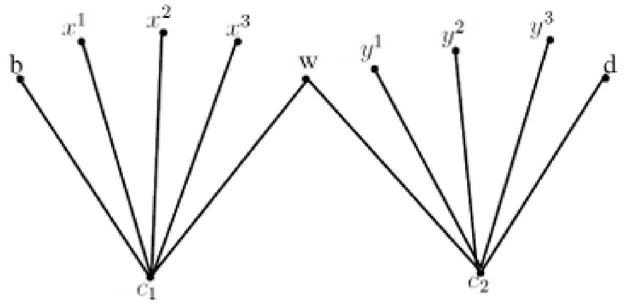

Figure 2. $w(3)$

Theorem 11. *The W-graph $W(n), n \geq 1$ admits absolutely harmonious labeling.*

Proof. Let $G = W(n)$ with $V = |V(G)|$ and $E = |E(G)|$. Here, $V = 2n+5$ and $E = 2n+4$.

The vertex set and the edge set of G are as follows

$$V(G) = \{(c_1, c_2, b, w, d) \cup (x^1, x^2, x^3, \ldots, x^n) \cup (y^1, y^2, y^3, \ldots, y^n)\}$$

$$E(G) = \{(c_1x^1, c_1x^2, c_1x^3, \ldots, c_1x^n) \cup (c_2y^1, c_2y^2, c_2y^3, \ldots, c_2y^n) \cup (c_1b, c_1w) \cup (c_2w, c_2d)\}$$

Now, we define the labeling $f : V(G) \to \{0, 1, 2, \ldots, q-1\}$ as follows

$f(b) = 1$

$f(w) = n + 2$

$f(d) = n + 3$

$f(c_1) = 0$

$f(c_2) = 1$

$f(x^i) = i + 1; 1 \leq i \leq n$

$f(y^j) = q - j; 1 \leq j \leq n$

From the above, $a_0, a_1, a_2, \ldots, a_{q-1}$ where $a_i = q - i$ (or) $q + i; 0 \leq i \leq q-1$ are the arranged edge labels.

Therefore f is an absolutely harmonious labeling of the W-graph.

and hence the W-graph is an absolutely harmonious graph. □

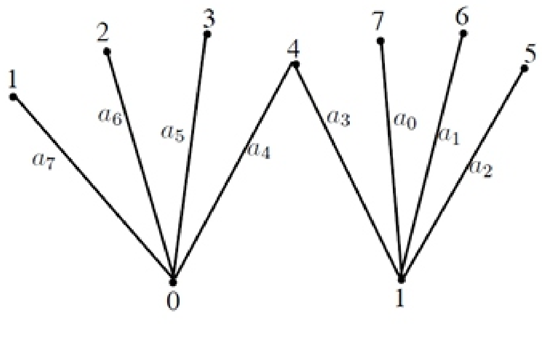

Figure 3. $W(2)$

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ROLE OF SUSPENDED PARTICLES IN COOLING A STRETCHING FILM AT A DESIRED RATE

S. MANJUNATH1, D. SREELAKSHMI2 and YASHAS JAYAPRAKASH3
1,2Department of Mathematics B.N.M. Institute of Technology Bengaluru-560070, India
3MS Mechanical Engineering San Jose State University San Jose, CA, USA

ABSTRACT

The main goal of this research is to look into velocity and heat transfer in an electrically conducting Newtonian fluid flowing over a quadratically extending sheet with Navier slip affecting velocity at the boundary. The flow events are described by a nonlinear partial differential equation which are translated to an ordinary differential equation using well established similarity transformations. We have adopted differential transformation method with Pade approximations to get numerical convergent series solution. The graphical and tabular results for velocity and heat transfer with slip effects are displayed for different dimensionless parameters for a range of values.

Introduction

Extrusion, melt-spinning, hot rolling, wire drawing, glass-fiber production, plastic and rubber sheet manufacturing and cooling of a huge metallic plate are all examples of engineering processes where flow across a stretched surface is a critical issue. Thin-film production is a difficult procedure that requires cooling a stretched film at a specific rate to offer unidirectional orientation to the finished product. Because the whole system is contained in cooling liquids, selecting a coolant and stretching approach that involves stretching thin films by equal and opposing pressures is critical. The thin film's properties are determined by the pace of cooling and the type of the coolant.

The concept of a boundary layer over a continuous solid surface moving at a same speed was initiated by Sakiadis [1]. Crane [2] assumed that the sheet's velocity varied linearly with axial distance. They focused on the flow field with the no-slip at the edge of the flow. In some cases, this boundary requirement should be substituted by the Navier slip boundary condition. Anderson [3] looked at the effects of a slip boundary condition on Newtonian fluid flow past a stretched sheet. Bidin et al. [4] investigated the flow over a stretching sheet with a convective boundary condition and slip effect. Nandeppanavar et al. [5] debriefed heat transfer over a stretching sheet with nonlinear navier second order slip flow boundary condition. In the presence of a transverse magnetic field and viscous dissipation, M.R. Krishnamurthy et al. [6] resolved the problem of steady, boundary layer flow and heat transfer of a nanofluid with fluid-particle suspension across an exponentially extending surface. The convective heat transfer properties of an incompressible viscous dusty fluid across an exponentially stretched surface with an exponential temperature distribution were investigated by Siti Nur HaseelaIzani et al. [7]. Najeeb Alam Khan et al. [8] explained the and heat transfer at the edge of flow considering fourth-grade fluid over an exponentially stretching sheet and Fazle Mabood et al. T. Gangaiah et al. [9] described the effects of thermal radiation and heat source/sink parameters on the mixed convective MHD flow of a Casson nanofluid with zero normal flux of nanoparticles over an exponentially stretching sheet. Basant K. Jha and Dauda Gambo [10] investigated nature of flow wrt applied magnetic field on unsteady MHD Coutte flow of dusty fluid in an annulus.

Mathematical relationship of the investigation:

Taking into account a two-dimensional uninterrupted flow of an incompressible, electrically conducting Boussinesq-Stokes suspension fluid (couple stress fluid) across an exponentially extending sheet. Using two equal and opposing pressures along the x-axis, the boundary sheet is made to move axially with an exponential velocity. The y-axis is transversal to it. The guiding equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0,\ x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\vartheta\frac{\partial^2 u}{\partial x^2}-\frac{\mu_m^2\sigma H_0^2}{\rho}u-\vartheta'\frac{\partial^4 u}{\partial x^4},$$

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2$$

$$\text{With } U_w(x)=U_0\exp\left(\frac{x}{l}\right)+\chi\gamma\frac{\partial u}{\partial y},\ \frac{\partial^2 u}{\partial x^2}=0,\ \upsilon=0 \text{ at } y=0$$

$$T=T_w=T_\infty+(T_w-T_\infty)\exp\left(\frac{x}{l}\right) \text{ in } PEST,\ -\kappa\frac{\partial T}{\partial y}=T_1e^{\frac{3x}{2l}} \text{ in } PEHF \text{ at}$$

$y=0$

With the concept of nondimensionalisation and stream function

$$(X,Y)=\left(\frac{x}{l},\frac{y}{l}\right),\ (U,V)=\left(\frac{u}{\sqrt{U_0\vartheta}},\frac{\upsilon}{\sqrt{U_0\vartheta}}\right),\ U=\frac{\partial\varphi}{\partial Y},\ V=-\frac{\partial\varphi}{\partial X}$$

$$C\frac{\partial^5\varphi}{\partial Y^5}-\frac{\partial^5\varphi}{\partial Y^3}+\frac{\partial\varphi}{\partial Y}\frac{\partial^2\varphi}{\partial X\partial Y}-\frac{\partial\varphi}{\partial X}\frac{\partial^2\varphi}{\partial Y^2}+Q\frac{\partial\varphi}{\partial Y}=0,$$

$$\frac{\partial\varphi}{\partial Y}\frac{\partial T}{\partial X}-\frac{\partial\varphi}{\partial X}\frac{\partial T}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2}+\frac{\mu c}{\rho C_p l^2}\left(\frac{\partial\varphi}{\partial Y}\right)^2$$

$$\text{with } \frac{\partial\varphi}{\partial Y}=\sqrt{\frac{U_0}{\gamma}}e^X+\chi\gamma\frac{\partial^2\varphi}{\partial Y^2},\ \frac{\partial\varphi}{\partial X}=0,\ \frac{\partial^3\varphi}{\partial Y^3}=0 \text{ at } y=0$$

$$T=T_w=T_\infty+(T_w-T_\infty)e^X \text{ in } PEST,\ -\kappa\frac{\partial T}{\partial y}=T_1e^{\frac{3x}{2l}} \text{ in } PEHF \text{ at}$$

$y=0$

$$\frac{\partial\varphi}{\partial Y}\to 0,\ \frac{\partial^3\varphi}{\partial Y^3}\to 0,\ T\to T_\infty \text{ at } Y\to\infty.$$

Using following similarity transformation:

$$\gamma(X,Y)=\sqrt{2\,\mathrm{Re}}f(\eta)e^{\frac{X}{2}} \text{ where } \eta=Y\sqrt{\frac{\mathrm{Re}}{2}}e^{\frac{X}{2}} \text{ is the similarity variable.}$$

Using above list of equations, momentum equation becomes

$$Cf_{\eta\eta\eta\eta\eta} - f_{\eta\eta\eta} + 2ff_{\eta\eta} + 2Qf_{\eta} = 0 \text{ with}$$

$$f = 0,\ f_{\eta} - 1 = Kf_{\eta\eta},\ f_{\eta\eta\eta} = 0 \text{ at } \eta = 0$$

$$f_{\eta} \to 0,\ f_{\eta\eta} \to 0 \text{ as } \eta \to \infty \text{ where } K = \frac{\chi U_0 \sqrt{\mathrm{Re}}}{\sqrt{2}}$$

Temperature distribution analysis:

PEST: $\theta(\eta) = \dfrac{T - T_{\infty}}{T_w - T_{\infty}}$

$$\theta_{\eta\eta} + \Pr f\theta_{\eta} - 2\Pr f_{\eta}\theta + \Pr Ef^2{}_{\eta\eta} = 0 \text{ with}$$

$$\theta = 1 \text{ at } \eta = 0,\ \theta \to 0 \text{ as } \eta \to \infty$$

PEHF: $\phi(\eta) = \dfrac{T - T_{\infty}}{T_w - T_{\infty}}$ where $T - T_{\infty} = \dfrac{T_1}{k}\sqrt{\dfrac{2}{\mathrm{Re}}}e^{\frac{3X}{2}}\phi(\eta)$ and $T_w - T_{\infty}$ $= \dfrac{T_1 l}{k}\sqrt{\dfrac{2}{\mathrm{Re}}}e^{\frac{3X}{2}}$.

Using the listed transformations, we get

$$\varphi_{\eta\eta} + \Pr f\varphi_{\eta} - 2\Pr f_{\eta} + \Pr Ef_{\eta\eta}^2 = 0 \text{ with}$$

$$\varphi_{\eta} = -1 \text{ at } \eta = 0,\ \varphi \to 0 \text{ as } \eta \to \infty.$$

Research Methodology:
The shooting technique is used to convert transformed nonlinear ordinary differential equations with defined boundary conditions into an initial value problem. The Runge-Kutta-Fehlberg 45 technique is then used to solve IVP.

Momentum equation:

$$\frac{dF_1}{d\eta} = F_2 \quad F_1(0) = 0, \quad \frac{dF_2}{d\eta} = F_3 \quad F_2(0) = 1 + KF_3$$

$$\frac{dF_3}{d\eta} = F_4 \quad F_3(0) = \alpha_4 \quad \frac{dF_4}{d\eta} = F_5 \quad F_4(0) = 0$$

$$\frac{dF_5}{d\eta} = \frac{1}{C}[F_4 - 2F_2^2 + F_1F_3 - 2QF_2] \qquad F_5(0) = \beta_4$$

PEST:

$$\frac{dF_6}{d\eta} = F_7 \qquad F_6(0) = 1,$$

$$\frac{dF_7}{d\eta} = -\Pr F_1F_7 + 2\Pr F_2F_6 - \Pr EF_3^2 \;\; F_7(0) = \gamma_4$$

PEHF:

$$\frac{dF_8}{d\eta} = F_9 \qquad F_8(0) = \lambda_4$$

$$\frac{dF_7}{d\eta} = -\Pr F_1F_7 + 2\Pr F_2F_6 - \Pr E_sF_3^2 \;\; F_9(0) = -1$$

Momentum Equation:

$$C(k+1)(k+2)(k+3)(k+4)(k+5)F[k+5] - (k+1)(k+2)(k+3)F[k+3]$$

$$-2\sum_{r=0}^{k}(r+1)F[r+1](k+1-r)F[k+1-r]$$

$$-\sum_{r=0}^{k}(r+1)(r+2)F[r+2]F[k-r] + 2Q(k+1)F[k+1] = 0$$

$$F[0] = 0,\; F[1] = 1 + k\alpha_4,\; F[2] = \frac{\alpha_4}{2},\; F[3] = 0,\; F[4] = \frac{\beta_4}{24}$$

PEST:

$$(k+1)(k+2)G[k+2]$$

$$+\Pr\sum_{r=0}^{k}(r+1)G[r+1]F[k-r]$$

$$-2\sum_{r=0}^{k}(r+1)F[r+1]G[k-r]$$

$$+ E\,\Pr\sum_{r=0}^{k}(r+1)(r+2)F[r+2](k-r+1)(k-r+2)$$

$$G[0]=1,\ G[1]=\gamma_4$$

PEHF:

$$(k+1)(k+2)H[k+2]$$

$$+\Pr\sum_{r=0}^{k}(r+1)H[r+1]F[k-r]$$

$$-2\sum_{r=0}^{k}(r+1)F[r+1]H[k-r]$$

$$+ E\,\Pr\sum_{r=0}^{k}(r+1)(r+2)F[r+2](k-r+1)(k-r+2)$$

$$H[0]=\lambda_4,\ H[1]=-1$$

Then, using the inverse differential transform, we arrive at

$$f(\eta)=F[0]+F[1]\eta+F[2]\eta^2+F[3]\eta^3+\ldots$$

$$f'(\eta)=F[1]+2F[2]\eta+3F[3]\eta^2+\ldots$$

$$g(\eta)=g[0]+g[1]\eta+g[2]\eta^2+g[3]\eta^3+\ldots$$

$$H(\eta)=H[0]+H[1]\eta+H[2]\eta^2+H[3]\eta^3+\ldots$$

In MATHEMATICA 7.0, with the help of command "PadeApproximant", we can obtain Pade approximation of ( ) x f around the point . 0

Findings and Analysis

1. Increasing the Chandrasekhar number (Q) causes the flow to be resisted, resulting in a drop in the momentum boundary layer.
2. Increasing the value of the couple stress ( ), C widens the momentum boundary layer and reduces the thermal boundary layer.
3. As the Prandtl number (Pr) is raised, the thermal boundary layer thickness reduces.
4. The PEHF boundary condition is appropriate for the adequate cooling of the stretching sheet.

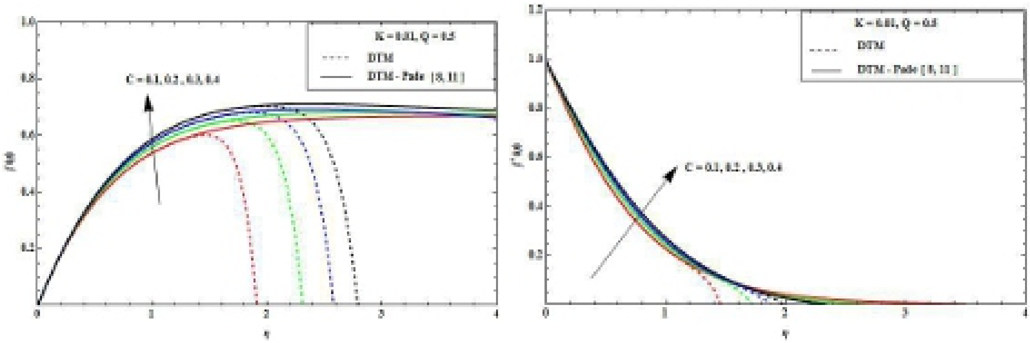

Figure 1.1. Illustration of $f(\eta)$ verses η for range of C values.

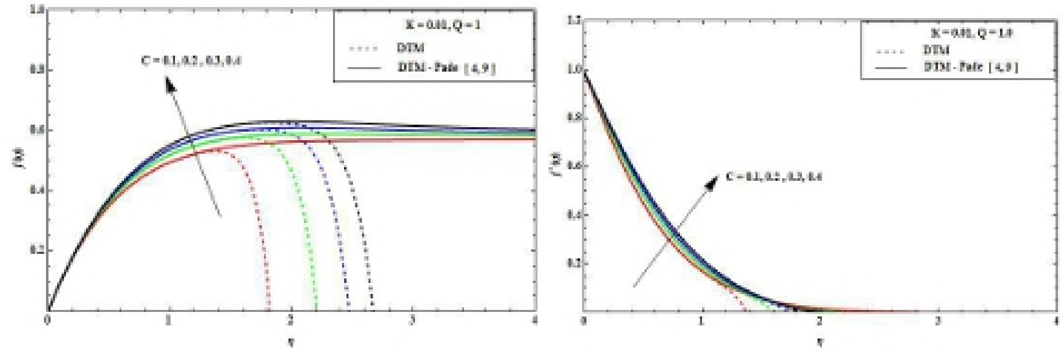

Figure 1.2. Illustration of $f(\eta)$ verses η for range C values.

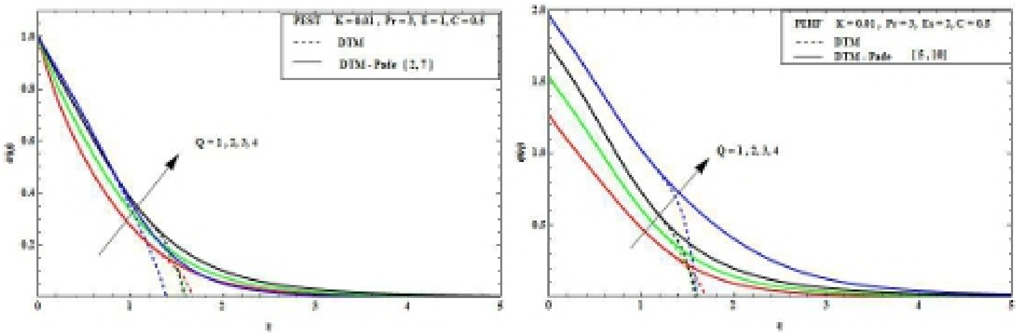

Figure 1.3. Illustration of heat transfer for range of Q values.

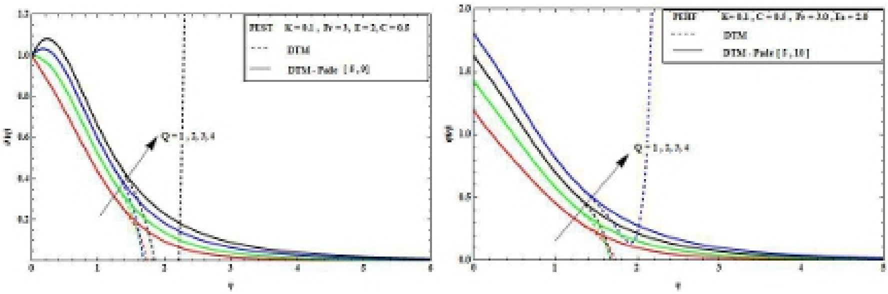

Figure 1.4. Illustration of temperature profiles for range of Q values.

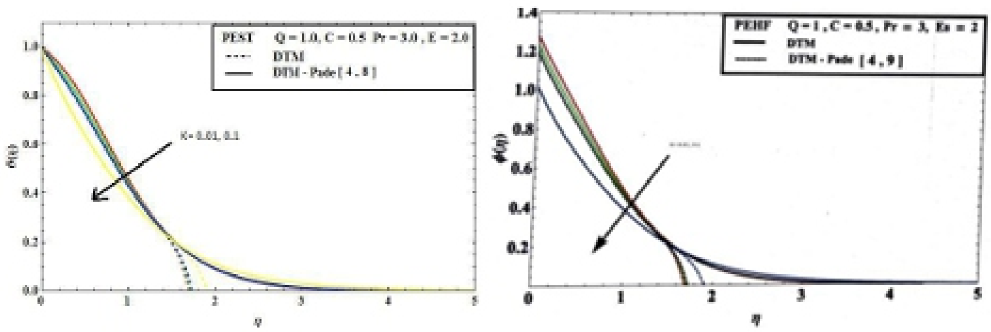

Figure 1.5. Illustration of heat transfer profiles for range of slip parameter K values.

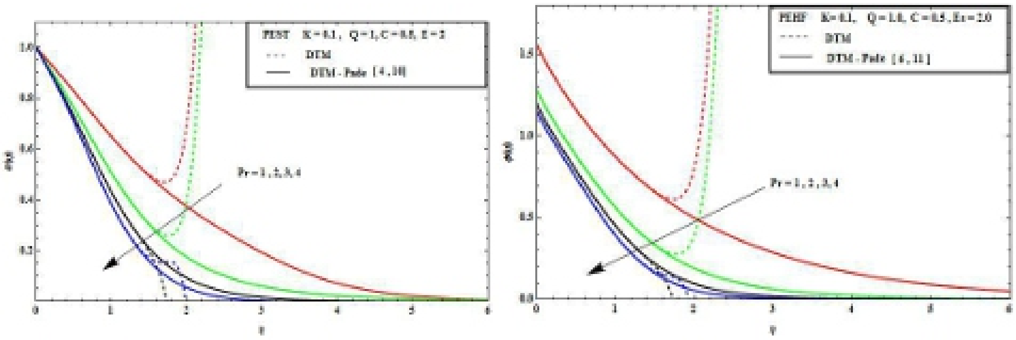

Figure 1.6. Illustration of heat transfer profiles for range of Pr values.

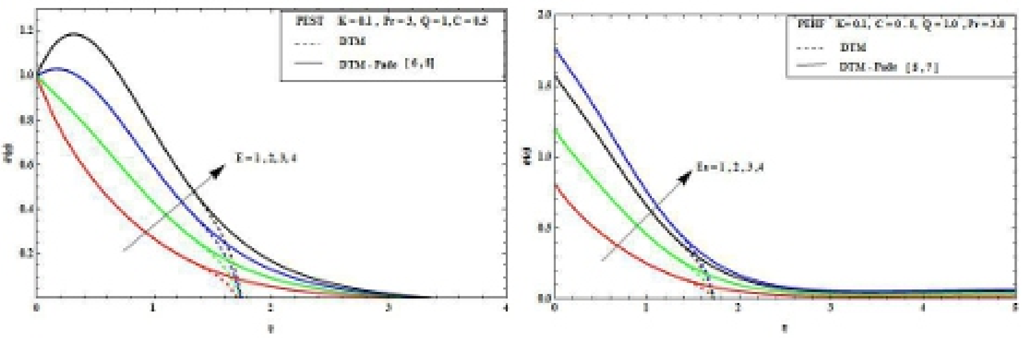

Figure 1.7. Illustration of heat transfer profiles for range of $E(Es)$ values.

Table 1. α_4 and β_4 for range of Q and C values.

Slip parameter K	Q	C	$C_f = \lvert f''(0) \rvert = \alpha_4$	$f'''(0) = \beta_4$
0.01	0.5	0.1	1.119926772	6.143065751
		0.2	1.023715885	3.927076649
		0.3	0.965503505	3.003995085
		0.4	0.923816576	2.477376318
0.01	0.5	0.1	1.263030327	7.956885243
		0.2	1.145729666	5.065095666
		0.3	1.075567284	3.865437398
		0.4	1.025698446	3.198751307
0.1	0	0.1	0.831284836	3.662315682
		0.2	0.775725168	2.378860715
		0.3	0.741562103	1.836375927
		0.4	0.717017904	1.523771535
0.1	0.5	0.1	0.988038328	5.311977266
		0.2	0.991451536	3.439556844
		0.3	0.874258662	2.651374678
		0.4	0.840072338	2.198575688
0.1	1.0	0.1	1.117388830	6.864520525
		0.2	1.025011109	4.430282239
		0.3	0.968670329	3.408740201
		0.4	0.928133292	2.823023705

Table 2. γ_4 and λ_4 for range of Q, C, Pr and $E(E_s)$ values.

Slip parameter	Q	C	Pr	$E(E_s)$	PEST $\theta'(0) = \lambda_4$	PEHF $\emptyset(0) = \lambda_4$
0.01	1	0.5	3	1	1.392967714	0.839263194
	2				1.052319124	0.978025785
	3				0.777138528	1.095719644
	4				0.542350085	1.200516246
0.01	1	0.5	1	1	0.743338084	0.839263194
			2		1.125953138	1.269481207
			3		1.392967714	1.699702309
			4		1.606361427	1.914813078
0.01	1	0.5	3	1	1.439787864	0.811513126
				2	0.546346308	1.194431102
				3	0.347097324	1.577351632
				4	0.240540874	1.960272652
0.01	1	0.1	3	2	0.517693860	1.645976811
		0.2			0.173479284	1.491501481
		0.3			0.048646915	1.394383539
		0.4			0.021226779	1.241094410

Table 3. γ_4 in PEST and λ_4 in PEHF for range of Q, C, Pr and $E(Es)$ values.

	Q	C	Pr	$E(Es)$	PEST $\theta'(0) = \lambda_4$	PEHF $\phi(0) = \lambda_4$
0.1	1	0.5	3	1	1.439787864	0.811513126
	2				1.138989365	0.938252582
	3				0.902484651	1.044646141
	4				0.704902578	1.138752686
0.1	1	0.5	1	1	0.743380841	1.211424418
			2		1.212370191	0.914746683
			3		1.385918477	0.811513126
			4		1.597262835	0.757265002
0.1	1	0.5	3	1	1.385918477	0.811513126
				2	0.313617540	1.194431021
				3	0.758684679	1.577351632
				4	0.790876161	1.960272652
0.1	1	0.1	3	2	0.517693865	1.645976810
		0.2			0.173479281	1.491501482
		0.3			0.048646910	1.394383539
		0.4			0.013585510	1.324109441

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SOME NEW RESULTS ON PALEY GRAPHS

A. HARITHA and J. CHITHRA
Department of Mathematics Lady Doak College Madurai, Tamil Nadu, India

ABSTRACT

Pq , A B) Abstract graph is a Paley graph with vertices and b B are edges between elements , x y A a A if and only if are the elements of finite field q y x − F q , where q 1 (mod 4) = p n (p F is a non-zero square in is a prime number, n is any positive integer) is a prime power.
This paper aims to prove some new results on Paley graph. The main new results are closure, planarity and the edge pebbling number of a Paley graph.

1. INTRODUCTION

Paley graph is a very good example which shows how graph theory and algebra interplace with each other. Paley graphs are named after Raymond Paley. They are closely related to the Paley construction for constructing Hadamard matrices from quadratic residues. They were introduced as graphs independently by Sachs and Erdos and Renyi. Sachs was interested in them for their self-complementarity properties, while Erdos and Renyi studied their symmetries. Paley graphs are dense undirected graphs constructed from the members of a suitable finite field by connecting pairs of elements that differ by a quadratic residue. The Paley graphs form an infinite family of conference graphs, which yield an infinite family of symmetric conference matrices. Anyone who seriously studies algebraic graph theory will, sooner or later come across the Paley graphs.

2. Preliminaries

In this section, for convenience of the reader and also for later use, we recall some known supporting results.

2.1 Theorem [2]. *For every prime 'p' and $n \in N$ there is a field with p^n elements.*

2.2 Theorem [2]. *The number of elements of a finite field F is equal to p^n, where p is prime and $n \in N$.*

2.3 Definition. Paley Graph [2]. Let p be a prime number and n be a positive integer such that $q = p^n \equiv 1 (\text{mod } 4)$. The graph $P = P_q = (V, E)$ with $v(P) = F_{p^n}$ and $E(P) = \{\{(x, y)\} : x, y \in F_{p^n}, x - y \in (F^*_{p^n})^2\}$ is called the Paley graph of order p^n. Here $F^*_{p^n} = F_{p^n} - \{0\}$ and $(F^*_{p^n})^2 = \{a^2 : a \in F^*_{p^n}\}$.

2.4 Illustration. Paley graphs exist for orders 5, 9, 13, 17, 25, 29, 37, 41, 49, 53, 61, 73, ... (1) Let $q = 5$, $Z_5 = \{0, 1, 2, 3, 4\}$ can be taken as F_q, $(F_q^*)^2 = \{1, 4\}$ $\therefore E(q) = \{(0, 1), (1, 2), (2, 3), (3, 4), (4, 0)\}$

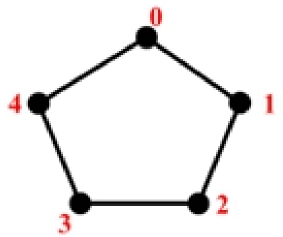

Figure 2.1. Paley graph of P_5.

Here we have illustrated the Paley graph of order 5, successive Paley graphs are illustrated in [2].

3. Closure of a Paley Graph

In this section, the closure of a Paley graph is determined.

3.1 Definition. Closure [5]. The closure of a graph G with n vertices, denoted by ( ), G C is the graph obtained from G by repeatedly adding edges between pair of non-adjacent vertices whose degree sum is at least n, until this can no longer be done.

3.2 Theorem. The closure of a Paley graph is itself.

Proof. Since Paley graphs are regular. The degree of each vertex is $\frac{q-1}{2}$, where 'q' is the number of vertices. Now, consider any two non-adjacent vertices, say v_i and v_j having a degree $\frac{q-1}{2}$. By adding the degree of vertices, we get

$$\frac{q-1}{2} + \frac{q-1}{2} = \frac{2q-2}{2} = \frac{2(q-1)}{2} = q-1 < q$$

Since i v and j v are arbitrary, it is true for every pair of non-adjacent vertices. So, by the definition of closure, no more edges can be added. Therefore, the graph q P is its own closure

Hence the proof.

4. Planarity of a Paley Graph

4.1 Definition[1]. A graph G is called “planar graph” if G can be drawn in the plane so that no two of its edges cross each other. Therefore, a graph that is not planar is called “non-planar”.

4.2 Theorem[1]. If G is a planar graph of order 3 n and size m, then . 6 3 − n m Here = n number of vertices and = m number of edges.

4.3 Theorem. Paley graphs are non-planar whenever . 5 q Proof. Paley graphs exist for order 5, 9, 13, 25, 17, 29,…. Claim. Paley graphs are non-planar except for 5 = q WKT, if G is a planar graph of order 3 n and size m, then

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When $q = 5 = n$, number of edges, $m = \frac{q(q-1)}{4} = \frac{5(4)}{4} = 5$

Substituting in (1), we get $5 \leq 3(5) - 6 \Rightarrow 5 \leq 15 - 6 \Rightarrow 5 \leq 9$

$\therefore$ This inequality holds

Also, it is obvious from the diagram of P_5, that no two edges intersect each other

$\therefore$ P_5 is a planar graph.

Let $q = 9 = n$.

We cannot draw a graph P_9 without the edge crossing $\exists$ at least any two edges say e_i and e_j cross with each other

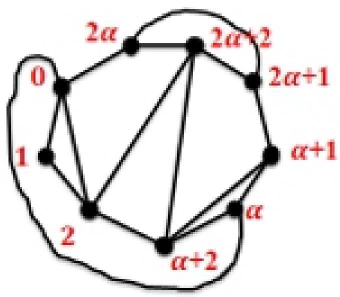

Figure 4.1. $P_9 - \{(2^{\alpha}, \alpha), (2^{\alpha} + 1, 1), (1, \alpha + 1)\}$.

In figure 4.1, if we draw any one of the edges in $\{(2^{\alpha}, \alpha), (2^{\alpha} + 1, 1), (1, \alpha + 1)\}$, then these edges will definitely intersect. P_9 is non-planar.

Now,

$$\frac{q(q-1)}{4} \leq 3q - 6 \qquad (2)$$

Suppose, $q \geq 13$ then

$$\frac{q(q-1)}{4} = \frac{q(13-1)}{4} = 3q$$

$\therefore$ (2) becomes, $3q \leq 3q - 6$ which doesn't hold

$\therefore$ q violates the inequality when $q \geq 13$

5. Edge Pebbling Number of a Paley Graph

5.1 Definition. Edge pebbling [3]. An edge pebbling move on a graph G is defined to be the removal of two pebbles from one edge and the addition of one pebble to an adjacent edge.

Edge pebbling number [3]. An edge pebbling number $P_E(G)$ is defined to be least number of pebbles such that any distribution of $P_E(G)$ pebbles on the edges of G allows one pebble to be any specific, but arbitrary edge.

5.2 Theorem. *The edge pebbling number of a Paley graph is* $\frac{q(q-1)}{4}$.

Proof.

Case (i). The pebbles are distributed in a way that all edges with exactly one pebble except the target edge, then the edge pebble move is not possible, so we should place one more pebble in any of the $\left\{\frac{q(q-1)}{4}\right\} - 1$ edges. Then the target edge can be reached. In this case, we need $\frac{q(q-1)}{4}$ pebbles.

Case (ii). All the pebbles are placed on a single edge, say e_1.

Subcase (i). If this target edge is adjacent to e_1, then only 2 pebbles are enough to reach the target edge because of adjacency.

Subcase (ii). If the target edge is not adjacent to e_1 [i.e. Immediate non-adjacent to e_1], then we need 4 pebbles to attain the target edge.

Subcase (iii). Consider another way of distributing the pebbles on any edge then we need 8 pebbles to reach the target edge. Here the target edge is arbitrary non-adjacent to . 1 e This is true for all possible distributions. But the maximum number of pebbles needed to reach the target edge is the edge pebbling number, so the edge pebbling number for a Paley graph is ( ) . 4 1 − q q

6. Conclusion

In this paper, we have determined the closure, planarity and the edge pebbling number of the Paley graph. There are many more interesting results pertaining to Paley graphs which have wider applications in other fields.

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A NOTE ON RANK OF TRAPEZOIDAL FUZZY NUMBER MATRICES

U. HARI NARAYANAN1 and K. KUMARESH2
1,2PG and Research Department of Mathematics A. V. C. College (Autonomous) (Affiliated to Annamalai University, Chidambaram) Mannampanadal-609305, India

ABSTRACT

In this article, some methods are described to find these types of ranks viz, row rank, column rank, fuzzy rank and fuzzy full rank using Trapezoidal Fuzzy Matrices (TrFMs). Also we investigated the relationship between them under the algorithm for Row Reduced Echelon Form (RREF). We studied the cross vector and Schein rank under the relationship is illustrated with suitable example.

Throughout this paper we deal with fuzzy number matrices that is matrices over fuzzy algebra . For A mn , A , A , ( ) ( ) ( ) ( ) A r f ( ) A A , c and denote the row space, column space, row rank, column rank, and fuzzy rank under trapezoidal fuzzy matrix respectively. Ismail and Morsi [1] established that fuzzy rank of fuzzy matrix in the product of two fuzzy matrices cannot exceed the fuzzy rank either fuzzy matrix. Also, Latha et al. [2] studied the rank of type 2 triangular fuzzy matrix.

The concept of fuzzy matrix (FM) [3] is one of the recent topics to develop for dealing with the rank of fuzzy number matrices fuzzy matrices defined first time by Thomson [6] and discussed about the convergence of the powers of a fuzzy matrix. In FMs, rows and columns are taken as uncertain. He also investigated different properties of these types of matrices along with application. Shyamal and Pal [4] introduced the concept of triangular fuzzy matrix. Stephen Dinagar and Harinarayanan [5] are studied the rank of fuzzy matrices.

1. INTRODUCTION

Paley graph is a very good example which shows how graph theory and algebra interplace with each other. Paley graphs are named after Raymond Paley. They are closely related to the Paley construction for constructing Hadamard matrices from quadratic residues. They were introduced as graphs independently by Sachs and Erdos and Renyi. Sachs was interested in them for their self-complementarity properties, while Erdos and Renyi studied their symmetries. Paley graphs are dense undirected graphs constructed from the members of a suitable finite field by connecting pairs of elements that differ by a quadratic residue. The Paley graphs form an infinite family of conference graphs, which yield an infinite family of symmetric conference matrices. Anyone who seriously studies algebraic graph theory will, sooner or later come across the Paley graphs.

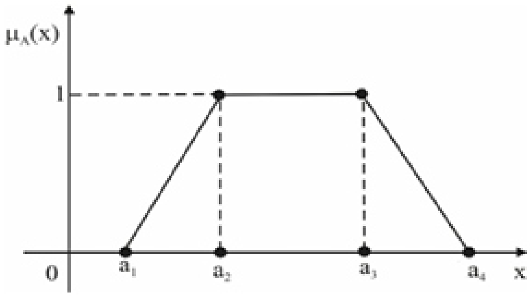
2. Preliminaries

Definition 2.1 (Fuzzy Set). A fuzzy set is defined as ( ) , : , ~ X x x x A A = with a membership function ( ) , 1 , 0 : → X x A where ( ) x A denotes the degree of membership of the element x to the set A.

Definition 2.2 (Fuzzy Number). A fuzzy set , ~ A defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics A ~ is normal A ~ is convex set. The support of A ~ is closed and bounded then A ~ is called fuzzy number.

Definition 2.3 (Trapezoidal Fuzzy Number). A fuzzy number on T A ~ is a trapezoidal fuzzy number denoted by ( ) 4 3 2 1 , , , ~ a a a a AT = are the membership function of this fuzzy number will be interpreted as

$$\mu_{\tilde{A}_T}(x)\begin{cases} 0, & x < a_1 \\ \left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \left(\dfrac{a_4-x}{a_4}\right), & a_4 \le x \le a_5 \\ 0, & x > a_5 \end{cases}$$



Trapezoidal Fuzzy Number

2.4. Arithmetic Operations on Trapezoidal Fuzzy Numbers (TrFNs).

Let us consider ( ) 4 3 2 1 , , , ~ a a a a AT = and ( ) 4 3 2 1 , , , ~ b b b b BT = be two trapezoidal fuzzy numbers then,

(i) Addition: $\tilde{A}_T(+)\tilde{B}_T = (a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3,\ a_4 + b_4)$.

(ii) Subtraction: $\tilde{A}_T(-)\tilde{B}_T = (a_1 - b_1,\ a_2 - b_2,\ a_3 - b_3,\ a_4 - b_4)$.

(iii) Multiplication:

$\tilde{A}_T(\times)\tilde{B}_T = \{(a_1 R(\tilde{B}_T), (a_2 R(\tilde{B}_T), (a_3 R(\tilde{B}_T), (a_4 R(\tilde{B}_T)\}$.

Where $R(\tilde{B}_T) = \dfrac{b_1 + b_2 + b_3 + b_4}{4}$

(iv) Division: $\tilde{A}_T(\div)\tilde{B}_T = \left(\dfrac{a_1}{R(\tilde{B}_T)}, \dfrac{a_2}{R(\tilde{B}_T)}, \dfrac{a_3}{R(\tilde{B}_T)}, \dfrac{a_4}{R(\tilde{B}_T)}\right)$.

Where $R(\tilde{B}_T) = \dfrac{b_1 + b_2 + b_3 + b_4}{4}$.

Definition 2.5 Ranking Function.

We define a ranking function ( ) R R F R → : which maps each fuzzy numbers to real line ( ) R F represent the set of all trapezoidal fuzzy numbers. If R be any linear ranking functions.

$$R(\tilde{A}_T) = \left(\frac{a_1 + a_2 + a_3 + a_4}{4}\right).$$

Definition 2.6 Inverse Trapezoidal Fuzzy Number. If $\tilde{a}_T$ is trapezoidal fuzzy number and $\tilde{a}_T \neq \tilde{0}_T$, then we define $\tilde{a}_T^{-1} = \frac{\tilde{1}_T}{\tilde{a}_T}$.

3. Trapezoidal Fuzzy Matrices (TrFMs)

Definition 3.1 (Trapezoidal Fuzzy Matrix). A trapezoidal fuzzy matrix of order $m \times n$ is defined as $\hat{A}_T = (\tilde{a}_{ij})_{m \times n}$ where $\tilde{a}_{ij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4})$ is ij^{th} element of $\hat{A}_T$.

3.2. Operations on Trapezoidal Fuzzy Matrices (TrFMs). Let $\hat{A}_T = (\tilde{a}_{ij})_{m \times n}$ and $\hat{B}_T = (\tilde{b}_{ij})_{m \times n}$ be two TrFMs of same order. Then we have the following

$$\hat{A}_T + \hat{B}_T = (\tilde{a}_{ij} + \tilde{b}_{ij})$$

$$\hat{A}_T - \hat{B}_T = (\tilde{a}_{ij} - \tilde{b}_{ij})$$

For $\hat{A}_T = (\tilde{a}_{ij})_{m \times n}$ and $\hat{B}_T = (\tilde{b}_{hij})_{m \times k}$ then $\hat{A}_T \hat{B}_T = (\tilde{c}_{ij})_{m \times k}$ where $\tilde{c}_{ij} = \sum_{p=1}^{n} \tilde{a}_{ip} \tilde{b}_{pj}$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, k$.

$\hat{A}_T^T$ or $\hat{A}_T' = (\tilde{a}_{ij})$

$k\hat{A}_T = (k\tilde{a}_{ij})$, where k is scalar.

Definition 3.3 (Zero Trapezoidal Fuzzy Matrix). A trapezoidal fuzzy matrix (TrFM) is said to be a zero TrFM and all its entries are 0 and it is denoted by $\hat{O}_T$.

Definition 3.4 (Unit Trapezoidal Fuzzy Matrix). The square TrFM is said to be a unit TrFM if the diagonal elements are 1 and the rest of elements are 0. It is denoted by $\hat{I}_T$.

4. Rank of Trapezoidal Fuzzy Matrices (TrFMs)

Definition 4.1 (Trapezoidal Fuzzy Vector). A trapezoidal fuzzy vector is an n-tuple of elements from a fuzzy algebra. That is, a trapezoidal fuzzy vector is of the form ( ) n x x x , , , 2 1 where each element , I x . , , 2 , 1 n i =

Definition 4.2 (Row Rank). The row space ( ) T A ˆ of an n m trapezoidal fuzzy matrix T A ˆ is the subspace of n generated by the rows of . ˆT A The row rank ( ) T r A z of T A ˆ is minimum possible size of a spanning set of ( ). ˆ T A

Definition 4.3 (Column Rank). The column space ( ) T A ˆ of an n m trapezoidal fuzzy matrix T A ˆ is the subspace of m generated by the rows of A The row rank ( ) T r A ˆ of T A ˆ is the smallest possible size of a spanning set of ( ). ˆ T A

Definition 4.4 (Fuzzy Rank). A trapezoidal fuzzy matrix T A ˆ is said to be a fuzzy rank of rank r, if ( ) ( ) r A A T c T r = = ˆ ˆ and it is denoted by ( ). ˆT f A

4.1 Elementary Transformation. The elementary transformation of a TrFM ( ) ij T a A ~ ˆ = of the following transformation,

1. Interchange of two rows or columns.
2. Multiplication of a row (or column) by an arbitrary trapezoidal fuzzy number not equal to . ~ T O
3. Addition of multiple of one row (or column) by trapezoidal fuzzy number not equal to T O ~ to another row (column).

4.2 Row Reduced Echelon form of TrFM.

Let ( ) n m ij T a A = ~ ˆ be the trapezoidal fuzzy matrix then the following steps are,

1. If T O a ~ ~11= then an interchange of rows and columns will change element in the position , ~11 a so that . ~ ~11 T O a
2. Convert the element 11 ~ a to T 1 ~ by multiplying the first row by . ~ 1 11 a
3. Subtract from the ith row, , 1 i the first row multiplied by , ~11 a whenever T O a ~ ~11 then the element ( ) 1 ~1 i ai will be replaced by . ~ T O
4. Subtract from the jth column, , 1 j the first column multiplied by , ~j a whenever , ~ ~ T j O a then the element ( ) 1 ~ j aj will be replaced by . ~ T O
5. Performing the same manipulations (step 1 to step 4) with the submatrix that remains in the lower right corner and so on. We finally after a finite number of manipulations arrive at a diagonal-equivalent TrFM with the same rank as the original TrFM . ~ T.

Example 4.5. Consider the TrFM,

$$\hat{A}_T = \begin{bmatrix} (-1, 0, 1, 4)(-1, 1, 2, 6)(-1, 0, 1, 4)(-1, 1, 4, 8) \\ (-1, 1, 2, 6)(-1, 2, 5, 10)(-1, 0, 1, 4)(-6, -2, 11) \\ (-1, 1, 4, 8)(-1, 6, 8, 11)(-1, 1, 4, 8)(-12, -10, -8, 2) \end{bmatrix}$$

into RREF.

$$\sim \begin{bmatrix} \left(\frac{-25}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{31}{2}\right) & \left(\frac{-41}{2}, \frac{-8}{2}, \frac{12}{2}, \frac{53}{2}\right) & \left(\frac{-25}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{27}{2}\right) & \left(\frac{-115}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{31}{2}\right) \\ \left(\frac{-23}{2}, \frac{-5}{2}, \frac{5}{2}, \frac{23}{2}\right) & \left(\frac{-41}{2}, \frac{-8}{2}, \frac{10}{2}, \frac{39}{2}\right) & \left(\frac{-19}{2}, \frac{-3}{2}, \frac{7}{2}, \frac{23}{2}\right) & \left(\frac{-44}{2}, \frac{-14}{2}, \frac{5}{2}, \frac{53}{2}\right) \\ \left(\frac{-9}{16}, \frac{-3}{16}, \frac{3}{16}, \frac{9}{16}\right) & \left(\frac{-13}{16}, \frac{-6}{16}, \frac{2}{16}, \frac{17}{16}\right) & \left(\frac{-9}{16}, \frac{-3}{16}, \frac{3}{16}, \frac{9}{16}\right) & \left(\frac{-5}{16}, \frac{11}{16}, \frac{22}{16}, \frac{36}{16}\right) \end{bmatrix}$$

$$c = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbb{C}(\hat{A}_T) = \{c_1, c_2, c_3, c_4\} \text{ and } \mathbb{R}(\hat{A}_T) = \{r_1, r_2, r_3\}$$

Clearly c_1, c_3 and c_4 are linearly independent,

$$\text{The basis of } \mathbb{C}(\hat{A}_T) = \left\{ \begin{bmatrix} (-1, 0, 1, 4) \\ (-1, 1, 2, 6) \\ (-1, 1, 4, 8) \end{bmatrix} \begin{bmatrix} (-1, 0, 1, 4) \\ (-1, 0, 1, 4) \\ (-1, 1, 4, 8) \end{bmatrix} \begin{bmatrix} (-1, 1, 4, 8) \\ (-6, -2, -1, 1) \\ (-12, -10, -8, 2) \end{bmatrix} \right\}$$

The basis of $\mathbb{R}(\hat{A}_T) =$

$$\sim \begin{bmatrix} \left(\frac{-25}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{31}{2}\right) & \left(\frac{-41}{2}, \frac{-8}{2}, \frac{12}{2}, \frac{53}{2}\right) & \left(\frac{-25}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{27}{2}\right) & \left(\frac{-115}{2}, \frac{-5}{2}, \frac{7}{2}, \frac{31}{2}\right) \\ \left(\frac{-23}{2}, \frac{-5}{2}, \frac{5}{2}, \frac{23}{2}\right) & \left(\frac{-41}{2}, \frac{-8}{2}, \frac{10}{2}, \frac{39}{2}\right) & \left(\frac{-19}{2}, \frac{-3}{2}, \frac{7}{2}, \frac{23}{2}\right) & \left(\frac{-44}{2}, \frac{-14}{2}, \frac{5}{2}, \frac{53}{2}\right) \\ \left(\frac{-9}{16}, \frac{-3}{16}, \frac{3}{16}, \frac{9}{16}\right) & \left(\frac{-13}{16}, \frac{-6}{16}, \frac{2}{16}, \frac{17}{16}\right) & \left(\frac{-9}{16}, \frac{-3}{16}, \frac{3}{16}, \frac{9}{16}\right) & \left(\frac{-5}{16}, \frac{11}{16}, \frac{22}{16}, \frac{36}{16}\right) \end{bmatrix}$$

$$\dim(\mathbb{C}(\hat{A}_T)) = \rho_c(\hat{A}_T) = 3, \ \dim(\mathbb{R}(\hat{A}_T)) = \rho_r(\hat{A}_T) = 3$$

The common value of row rank and column rank is fuzzy rank,

(i.e.), $\rho_r(\hat{A}_T) = \rho_c(\hat{A}_T) = \rho_f(\hat{A}_T) = 3$.

Hence the rank of trapezoidal fuzzy matrix $\hat{A}_T$ is 3.

Property 4.6. Let $\hat{A}_T \in \mathbb{F}_{mn}$ with $\rho_c(\hat{A}_T) = r$. Then there exist matrices $\hat{B}_T \in \mathbb{F}_{mr}$ and $\hat{C}_T \in \mathbb{F}_{rn}$ such that $\rho_r(\hat{A}_T) = \rho_r(\hat{C}_T) = r$ and $\hat{A}_T = \hat{B}_T\hat{C}_T$.

Property 4.7. Let $\hat{A}_T \in \mathbb{F}_{mn}$ with $\rho_c(\hat{A}_T) = s$. Then there exist matrices $\hat{B}_T \in \mathbb{F}_{ms}$ and $\hat{C}_T \in \mathbb{F}_{sn}$ such that $\rho_c(\hat{A}_T) = \rho_c(\hat{A}_T) = s$ and $\hat{A}_T = \hat{B}_T\hat{C}_T$.

Definition 4.8. Let $\hat{A}_T$ and $\hat{B}_T$ be the two TrFMs such that $\hat{A}_T\hat{B}_T$ is defined by $\rho(\hat{A}_T\hat{B}_T) \le \min\{\rho(\hat{A}_T), \rho(\hat{B}_T)\}$.

Definition 4.9. Let $\hat{A}_T = (\tilde{a}_{ij})_{m\times n} \in \mathbb{F}_{mn}$, then $\hat{A}_T$ is called row full rank if $\rho_r(\hat{A}_T) = m$ and $\hat{A}_T$ is called column full rank if $\rho_c(\hat{A}_T) = n$ and let $\hat{A}_T \in \mathbb{F}_{mn}$ then $\hat{A}_T$ is said to be fuzzy full rank if $\rho_f(\hat{A}_T) = \rho_r(\hat{A}_T) = \rho_c(\hat{A}_T) = n$.

Example 4.10. Consider a Trapezoidal fuzzy matrix,

$$\hat{A}_T = \begin{pmatrix} (-2, 0, 2, 4) & (-1, 3, 4, 6) & (-1, 6, 7, 8) \\ (-1, 0, 1, 8) & (4, 6, 8, 10) & (6, 10, 11, 13) \\ (-2, 0, 2, 4) & (-1, 3, 4, 6) & (4, 5, 7, 8) \end{pmatrix} \text{ into RREF is}$$

$$\sim \begin{pmatrix} (-2, 0, 2, 4) & (-7, -1, 1, 7) & (-9, -1, 1, 9) \\ (-9, -1, 1, 9) & (-20, 3, 8, 13) & (-34, 5, 11, 18) \\ (-6, -2, 2, 6) & (-13, -3, 4, 12) & (-16, -5, 7, 18) \end{pmatrix}$$

$$c = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbb{C}(\hat{A}_T) = \{c_1, c_2, c_3\} \text{ and } \mathbb{R}(\hat{A}_T) = \{r_1, r_2, r_3\}$$

Clearly c_1, c_2 and c_3 are linearly independent.

The basis of $\mathbb{C}(\hat{A}_T) = \begin{Bmatrix} (-2, 0, 2, 4) & (-1, 3, 4, 6) & (-1, 6, 7, 8) \\ (-1, 0, 1, 8) & (4, 6, 8, 10) & (6, 10, 11, 13) \\ (-2, 0, 2, 4) & (-1, 3, 4, 6) & (4, 5, 7, 8) \end{Bmatrix}$

The basis of

$$\mathbb{R}(\hat{A}_T) = \begin{Bmatrix} (-2, 0, 2, 4) & (-7, -1, 1, 7) & (-9, -1, 1, 9) \\ (-9, -1, 1, 9) & (-20, 3, 8, 10) & (-34, 5, 11, 18) \\ (-6, -2, 2, 6) & (-13, -3, 4, 12) & (-16, -5, 7, 18) \end{Bmatrix}$$

$$\dim(\mathbb{C}(\hat{A}_T)) = \rho_c(\hat{A}_T) = 3, \ \dim(\mathbb{R}(\hat{A}_T)) = \rho_r(\hat{A}_T) = 3$$

The common value of row rank and column rank order 3 is same for the fuzzy full rank is also 3

(i.e.), $\rho_r(\hat{A}_T) = \rho_c(\hat{A}_T) = \rho_f(\hat{A}_T) = 3.$

5. Cross Vector and Schein Rank

Cross vector play an important role in fuzzy as well as trapezoidal fuzzy matrix theory to evaluate the rank of matrices.

Definition 5.1. For vectors v u, the cross vector ( ) v u, is the TrFM ( ) v u a A t ij T = = ~ ˆ such that , ~ i i ij v u a = where i i ij v u a , , ~ in fuzzy algebra.

Example 5.2. We consider two vectors ( ) ( ), 6 , 3 , 0 , 1 , 4 , 1 , 0 , 1 − − = u ( ) 8 , 6 , 0 , 2 − and ( ) ( ) ( ) , 8 , 6 , 2 , 0 , 9 , 7 , 6 , 2 , 8 , 6 , 0 , 2 − = v then the cross vector of these two vectors u and v is

$$u^t v = \begin{pmatrix} (-1, 0, 1, 4) \\ (-1, 0, 3, 6) \\ (-2, 0, 6, 8) \end{pmatrix} ((-2, 0, 6, 8), (2, 6, 7, 9), (0, 2, 6, 8))$$

$$= \begin{pmatrix} (-3, 0, 3, 12) & (-6, 0, 6, 24) & (-4, 0, 4, 16) \\ (-3, 0, 9, 18) & (-6, 0, 18, 36) & (-4, 0, 12, 24) \\ (-6, 0, 18, 24) & (-12, 0, 24, 32) & (-8, 0, 24, 32) \end{pmatrix}$$

Here we see that the row vectors of the cross vectors are linear combination of the vector ( ) ( ) ( ) 8 , 6 , 2 , 0 , 9 , 7 , 6 , 2 , 8 , 6 , 0 , 2 − and the column vectors are the linear combination of the vector ( ) ( ) ( ) , 8 , 6 , 0 , 2 , 6 , 3 , 0 , 1 , 4 , 1 , 0 , 1 t − − − which shows that the row vectors and column vectors of the cross vector are linearly dependent and row rank = column rank = fuzzy rank = 1.

Example 5.3. Let us consider $u_1 = \{(-2, 0, 2, 4), (-1, 0, 1, 8)\} \in \mathbb{V}_2$, $u_2 = [(-1, 3, 4, 6), (-1, 1, 5, 11)] \in \mathbb{V}_2$,

$v_1 = [(-1, 3, 4, 6), (4, 5, 7, 8), (-2, 0, 2, 4)] \in \mathbb{V}_3$ and

$v_2 = [(-1, 1, 5, 11), (4, 6, 8, 10), (-1, 0, 1, 8)] \in \mathbb{V}_3$. Now the cross vectors

$$\hat{A}_{1T} = u_1^t v_1 = \begin{pmatrix} (-6, 0, 6, 12) & (-12, 0, 12, 24) & (-2, 0, 2, 4) \\ (-3, 0, 3, 24) & (-6, 0, 6, 32) & (-1, 0, 1, 8) \end{pmatrix}$$

$$\hat{A}_{2T} = u_2^t v_2 = \begin{pmatrix} (-4, 12, 16, 24) & (-7, 21, 28, 42) & (-2, 6, 8, 12) \\ (-4, 4, 20, 44) & (-7, 7, 35, 77) & (-2, 2, 10, 22) \end{pmatrix}$$

$$\hat{A}_T = \begin{pmatrix} (-10, 12, 22, 36) & (-19, 21, 40, 66) & (-4, 6, 10, 16) \\ (-7, 4, 23, 68) & (-13, 7, 41, 109) & (-3, 2, 11, 30) \end{pmatrix}$$

$$\hat{U}_T = \begin{pmatrix} (-2, 0, 2, 4) & (-1, 3, 4, 6) \\ (-1, 0, 1, 8) & (-1, 1, 5, 11) \end{pmatrix},$$

$$\hat{V}_T = \begin{pmatrix} (-1, 3, 4, 6) & (-4, 5, 7, 8) & (-2, 0, 2, 4) \\ (-1, 1, 5, 11) & (-4, 6, 8, 10) & (-1, 0, 1, 8) \end{pmatrix}$$

$$\hat{U}_T\hat{V}_T = \begin{pmatrix} (-10, 12, 22, 36) & (-19, 21, 40, 66) & (-4, 6, 10, 16) \\ (-7, 4, 23, 68) & (-13, 7, 41, 109) & (-3, 2, 11, 30) \end{pmatrix} = \hat{A}_{Tr}$$

$$\therefore \hat{U}_T\hat{V}_T = \hat{A}_T.$$

Definition 5.4 (Schein Rank). The Schein rank $\rho_s(\hat{A}_T)$ of a trapezoidal fuzzy matrix $\hat{A}_T$ is the least number of matrices of rank 1 whose sum is $\hat{A}_{Tr}$.

Example 5.5. The Schein ranks of the trapezoidal fuzzy matrix $\hat{A}_T = \begin{bmatrix} (-2, 0, 2, 4) & (0, 0, 0, 0) & (-2, 0, 2, 4) \\ (-2, 0, 2, 4) & (-2, 0, 2, 4) & (-1, 0, 1, 8) \\ (0, 0, 0, 0) & (-2, 0, 2, 4) & (-2, 0, 2, 4) \end{bmatrix}$ are 3, because $\hat{A}_T = \hat{A}_{1T} + \hat{A}_{2T} + \hat{A}_{3T}$.

Where $\hat{A}_{1T} = \begin{pmatrix} (-2, 0, 2, 4) & (0, 0, 0, 0) & (0, 0, 0, 0) \\ (-2, 0, 2, 4) & (0, 0, 0, 0) & (0, 0, 0, 0) \\ (0, 0, 0, 0) & (0, 0, 0, 0) & (0, 0, 0, 0) \end{pmatrix},$

$$\hat{A}_{2T} = \begin{pmatrix} (0, 0, 0, 0) & (0, 0, 0, 0) & (0, 0, 0, 0) \\ (0, 0, 0, 0) & (-2, 0, 2, 4) & (0, 0, 0, 0) \\ (0, 0, 0, 0) & (-2, 0, 2, 4) & (0, 0, 0, 0) \end{pmatrix},$$

$$\hat{A}_{3T} = \begin{pmatrix} (0, 0, 0, 0) & (0, 0, 0, 0) & (-2, 0, 2, 4) \\ (0, 0, 0, 0) & (0, 0, 0, 0) & (-1, 0, 1, 8) \\ (0, 0, 0, 0) & (0, 0, 0, 0) & (-2, 0, 2, 8) \end{pmatrix}$$

With all $\hat{A}_i$'s are of rank 1.

6. Conclusion

In this paper, we find out the different types of ranks using trapezoidal fuzzy matrices such as fuzzy rank, fuzzy full rank, cross vector and Schein rank. The evaluation of rank in topics of trapezoidal fuzzy matrices in fuzzy algebra have been attempted. Our future investigation of study may be in the domain of fuzzy linear space of fuzzy linear system under fuzzy rank method.

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AN APPLICATION OF TRIANGULAR FUZZY NUMBER MATRICES WITH TRIPLET OPERATOR IN MEDICAL DIAGNOSIS

A. NAGOOR GANI1, M. AFFROSE BEGUM1 and P. MURUGANANTHAM1
1PG and Research Department of Mathematics Jamal Mohamed College (Autonomous) Affiliated to Bharathidasan University Trichy-20, India.

ABSTRACT

Fuzzy set theory and fuzzy logic play an essential role in medical field. Fuzzy logic deals a monotony models in complicated structure of medical diagnosis model. Fuzzy logic systems are excellent in handling ambiguous and imprecise information prevalent in medical diagnosis. A new method of medical diagnostic model using triplet operator is presented in this paper.

1. INTRODUCTION

Nowadays, an artificial intelligence has been developed by using fuzzy set theory and fuzzy logic to deal many complicated problems. In some medical skills structure, fuzzy set theory has been applied. Professor Zadeh established fuzzy set theory in 1965 and accomplishes a qualitative computational approach which describes uncertainty. Fuzzy matrix theory was developed by Meenakshi who indicating the concepts of fuzzy set and the essential notion of complicated in number of fields. An interval valued fuzzy matrix was developed by Sanchez's study covered medical diagnosis in Meenakshi and Kaliraja [2] as well as S. Elizebeth and L. Sujatha [1]. The origins of the arithmetic mean matrix can also be traced back to an interval value.

Sanchez's work for medical diagnostics is represented by a fuzzy matrix is applied.

In this paper, we have given some preliminaries of fuzzy set theory and introduced a triplet-operator on triangular fuzzy number matrix in section 2. Then the procedure is carried out under the fuzzy medical diagnostic model in section 3. In section 4, the exemplification is comprised by giving the proof of proposal. Finally we approach at the end of process, which concludes the paper.

2. Basic Definitions

Definition 2.1 (Triangular Fuzzy Number). A triplet 3 2 1 , , m m m is known as triangular fuzzy number where " 1 m" represents smallest likely value, " 2 m" the most probable value, and " 3 m" the largest possible value of any fuzzy event.

Definition 2.2 (Triangular Fuzzy Number Matrix). A triangular fuzzy number matrix of order n m is defined as , n m ij a A where ij ij ij ij m a , , is the th ij element of ij m A, is the mean value of ij a and ij ij , are the left and right spreads of ij a respectively.

Definition 2.3 (Triplet operator on Triangular Fuzzy Number Matrix). A triangular fuzzy number matrix of order n m is defined as , n m ij a A where ij ij ij ij m a , , is the th ij element of ij m A, is the mean value of ij a and ij ij , are the left and right spreads of ij a respectively and the triplet.

3. Implementation of Medical Analysis on Triangular Fuzzy Number Matrix

Let the set of patients be 3 2 1 , , P P P and the set of symptoms be 4 3 2 1 , , , S S S S and the set of diseases be . , 2 1 D D Suppose that the parameter 2 1,D D of the family be 2 1 , D F D F over all diseases are included together under this term. The triangular fuzzy number matrix elements are defined as follows , n m ij a A where ij ij ij ij m a , , is the th ij elements of A and . 10 0 ij ij ij m The implementation involves the following five steps.

3.1. Procedure for fuzzy medical diagnostic model

Step (I). Let us define 1 A be the symptom-disease triangular fuzzy number matrix with mapping . : 1 S F D F

Step (ii). Let us define 2 A be the patient-symptom triangular fuzzy number matrix with mapping . : 2 P F S F

Step (iii). The triangular fuzzy membership matrices 1 A and 2 A are constructed from the matrices which is denoted by 1 A mem and 2 A mem using the definition of conversion.

Step (iv). Compute the following relation matrices

(i) c A mem A mem B 2 1

(ii) 2 1 A mem A mem C c

(iii) C BT

(iv) C B MAX T

Step (v). Finally we conclude that the minimum of the corresponding patient i P has affected strong confirmation of the disease Di.

4. Example

Purulent, cyanos, hemoptysis, and chest pain are symptoms that three patients are experiencing at the hospital. Bronchiectasis and pneumonia are two probable diseases associated with the above complaints. Let 3 2 1 , , P P P P denote the set of patients. Let 4 3 2 1 , , , S S S S S denote the set of symptoms and 2 1,D D D denote the set of diseases.

Step (I). Let us define 1 A be the symptom-disease triangular fuzzy number matrix with mapping . : 1 S F D F The triangular fuzzy number matrix with imprecise value of medical information of the two diseases and their four symptoms are given by,

$$A_1 = \begin{bmatrix} (3, 4, 5) & (8, 9, 10) \\ (6, 6.5, 7) & (4, 5.5, 7) \\ (2, 3.5, 5) & (5, 6, 7) \\ (7, 8.5, 10) & (6, 7.5, 9) \end{bmatrix}.$$

Step (ii). Let us define A_2 be the patient-symptom triangular fuzzy number matrix with mapping $F_2 : S \to F(P)$. The imprecise value of medical information of our symptoms and the corresponding three patients of relation A_2 is given by,

$$A_2 = \begin{bmatrix} (7, 8.5, 10) & (8, 9, 10) & (4, 5, 6) & (6, 7.5, 9) \\ (4, 5, 6) & (3, 4, 5) & (4, 6, 8) & (3, 4, 5) \\ (3, 4.5, 6) & (6, 7, 8) & (3, 5, 7) & (6, 7, 8) \end{bmatrix}.$$

Step (iii).

$$mem(A_1) = \begin{bmatrix} (0.3, 0.4, 0.5) & (0.8, 0.9, 1) \\ (0.6, 0.65, 0.7) & (0.4, 0.55, 0.7) \\ (0.2, 0.35, 0.5) & (0.5, 0.6, 0.7) \\ (0.7, 0.85, 1) & (0.6, 0.75, 0.9) \end{bmatrix}$$

$mem(A_2)$

$$= \begin{bmatrix} (0.7, 0.85, 1) & (0.8, 0.9, 1) & (0.4, 0.5, 0.6) & (0.6, 0.75, 0.9) \\ (0.4, 0.5, 0.6) & (0.3, 0.4, 0.5) & (0.4, 0.6, 0.8) & (0.3, 0.4, 0.5) \\ (0.3, 0.45, 0.6) & (0.6, 0.7, 0.8) & (0.3, 0.5, 0.7) & (0.6, 0.7, 0.8) \end{bmatrix}$$

$$mem(A_1)^c = \begin{bmatrix} (0.7, 0.6, 0.5) & (0.2, 0.1, 0) \\ (0.4, 0.35, 0.3) & (0.6, 0.45, 0.3) \\ (0.8, 0.65, 0.5) & (0.5, 0.4, 0.3) \\ (0.3, 0.15, 0) & (0.4, 0.25, 0.1) \end{bmatrix}$$

$mem(A_2)^c$

$$= \begin{bmatrix} (0.3, 0.15, 0) & (0.2, 0.1, 0) & (0.6, 0.5, 0.4) & (0.4, 0.25, 0.1) \\ (0.6, 0.5, 0.4) & (0.7, 0.6, 0.5) & (0.6, 0.4, 0.2) & (0.7, 0.6, 0.5) \\ (0.7, 0.55, 0.4) & (0.4, 0.3, 0.2) & (0.7, 0.5, 0.3) & (0.4, 0.3, 0.2) \end{bmatrix}$$

Step (iv).

$B = mem(A_1) \odot mem(A_2)^c$

$$= \begin{bmatrix} (0.4, 0.35, 0.4) & (0.5, 0.5, 0.4) \\ (0.7, 0.6, 0.5) & (0.6, 0.6, 0.5) \\ (0.4, 0.4, 0.4) & (0.7, 0.55, 0.4) \end{bmatrix}$$

$C = mem(A_1)^c \odot mem(A_2)$

$$= \begin{bmatrix} (0.7, 0.6, 0.5) & (0.6, 0.45, 0.3) \\ (0.4, 0.6, 0.5) & (0.4, 0.4, 0.3) \\ (0.4, 0.5, 0.5) & (0.6, 0.45, 0.3) \end{bmatrix}$$

$$B\Delta_T C = \begin{bmatrix} (0.49, 0.36, 0.25) & (0.36, 0.2025, 0.09) \\ (0.16, 0.36, 0.25) & (0.16, 0.16, 0.09) \\ (0.16, 0.25, 0.25) & (0.36, 0.2025, 0.09) \end{bmatrix}$$

Step (v). $MAX\,(B\Delta_T C) = \begin{bmatrix} 0.49 & 0.36 \\ 0.36 & 0.16 \\ 0.25 & 0.36 \end{bmatrix}.$

Thus the minimum values of the ij^{th} row denotes the strong confirmation of disease to the patient.

5. Conclusion

The medical analysis representation on triangular fuzzy number matrix is executed in this paper. We conclude by the implementation on triangular fuzzy number matrix under the medical diagnostic model with fuzzy membership matrix values. And it is simple to determine which individual is suffering from which ailment using triplet operator on triangular fuzzy number matrix.

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