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# The Effect of Fourier Transform on Functions in Hilbert Space

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## ABSTRACT

*The effect of Fourier transform on functions in Hilbert space was clearly stated. The smaller the variance of a quantity such as position or momentum, the more accurate will be its measurement.*

*Thus, the Heisenberg inequality implies precisely ideas that the more accurately we are able to measure the momentum  $p$ , the less accurate will be any measurement of its position  $x$ , and vice versa.*

**Keywords:** *Fourier Transform: Heisenberg inequality: Hilbert Space: Fourier Series : Uncertainty Principle.*

## 1. INTRODUCTION

The Fourier transform is, like Fourier series, completely compatible with the calculus of generalized functions. We have already noted that the Fourier transform, when defined, is a linear map, taking functions  $f(x)$  on physical space to functions  $f(k)$  on frequency space. A critical question is precisely which function space should the theory be applied to. Not every function admits a Fourier transform in the classical sense† — the Fourier integral is required to converge, and this places restrictions on the function and its asymptotics at large distances.

It turns out the proper setting for the rigorous theory is the Hilbert space of complex valued square-integrable functions — the same infinite-dimensional vector space that lies at the heart of modern quantum mechanics.

The Hilbert space  $L^2 = L^2(\mathcal{R})$  is the infinite-dimensional vector space consisting of all complex-valued functions  $f(x)$  which are defined for all  $x \in \mathcal{R}$  and have finite  $L^2$  norm:

$$f = \int_{-\infty}^{\infty} f(x)^2 dx < \infty \quad 1$$

Hilbert space contains many more functions, and the precise definitions and identification of its elements is quite subtle. The Hermitical inner product on the complex Hilbert space  $L^2$  is prescribed in the usual manner,

$$f; g = \int_{-\infty}^{\infty} f(x) \overline{g(x)} dx, \tag{2}$$

so that  $f^2 = f; f$ . The Cauchy–Schwarz inequality

$$|f; g| \leq \sqrt{f^2} \sqrt{g^2} \tag{3}$$

ensures that the inner product integral is finite whenever  $f, g \in L^2$ .

The goal of this paper is to see effect of Fourier transform between position and momentum, or multiplication and differentiation, related to uncertainty principle. And also, to explain the mean value of any  $f(x)$  the position variable is given by its integral against the system's probability density.

### 2. SOME PROPERTIES OF FOURIER TRANSFORM

If  $f(x) \in L^2$  is square-integrable, then its Fourier transform  $f(k) \in L^2$  is a well-defined, square-integrable function of the frequency variable  $k$ .

If  $f(x)$  is continuously differentiable at a point  $x$ , then its inverse Fourier transform equals its value  $f(x)$ . More generally, if the right and left hand limits  $f(x^-)$ ,  $f(x^+)$  and also  $f'(x^-)$ ,  $f'(x^+)$  exist, then the inverse Fourier transform integral converges to the average value  $\frac{1}{2} [f(x^-) + f(x^+)]$ .

Thus, the Fourier transform  $f(k) = \mathcal{F}[f(x)]$  defines a linear transformation from  $L^2$  functions of  $x$  to  $L^2$  functions of  $k$ . In fact, the Fourier transform preserves inner products. This important result is known as Parseval's formula.

Theorem 2.1. If  $f(k) = \mathcal{F}[f(x)]$  and  $g(k) = \mathcal{F}[g(x)]$ , then  $f; g = f, g$ , i.e.,

$$\int_{-\infty}^{\infty} f(x) \overline{g(x)} dx = \int_{-\infty}^{\infty} f(k) \overline{g(k)} dk \tag{4}$$

Proof: Let us sketch a formal proof that serves to motivate why this result is valid. By using the definition of the Fourier Transform to evaluate

$$\begin{aligned} \int_{-\infty}^{\infty} f(k) \overline{g(k)} dk &= \int_{-\infty}^{\infty} \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx \right] \overline{\left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} g(y) e^{iky} dy \right]} dk \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x) \overline{g(y)} \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ik(x-y)} dk \right] dx dy \end{aligned}$$



Accordingly, the inner  $k$  integral can be replaced by a delta function  $\delta x - y$ , and hence

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x) g(y) \delta(x - y) dx dy = \int_{-\infty}^{\infty} f(x) g(x) dx.$$

This completes the proof.

In particular, orthogonal functions, satisfying  $\int f, g = 0$ , will have orthogonal Fourier transforms,  $\int f, g = 0$ . Choosing  $f = g$  in Parseval's formula (4) results in the Plancherel formula

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |f(k)|^2 dk \quad 5$$

Therefore, the Fourier transform  $\mathcal{F}: L^2 \rightarrow L^2$  defines a unitary or norm-preserving transformation on Hilbert spaces, mapping  $L^2$  functions of the physical variable  $x$  to  $L^2$  functions of the frequency variable.

### 3. QUANTUM MECHANICS AND THE UNCERTAINTY PRINCIPLE

The Heisenberg Uncertainty Principle states that in a physical system, certain quantities cannot be simultaneously measured with complete accuracy. For instance, the more precisely one measures the position of a particle, the less accuracy there will be in the measurement of its momentum; vice versa, the greater the accuracy in the momentum, the less certainty in its position. A similar uncertainty couples energy and time. Experimental verification of the uncertainty principle can be found even in fairly simple situations. Consider a light beam passing through a small hole.

The position of the photons is constrained by the hole; the effect of their momenta is in the pattern of light diffused on a screen placed beyond the hole. The smaller the hole, the more constrained the position, and the wider the image on the screen, meaning the less certainty there is in the observed momentum. This is not the place to discuss the philosophical and experimental consequences of Heisenberg's principle. What we will show is that the Uncertainty Principle is, in fact, a mathematical property of the Fourier transform!

In quantum theory, each of the paired quantities, e.g., position and momentum, are interrelated by the Fourier transform. This Fourier transform-based duality between position and momentum, or multiplication and differentiation, lies at the heart of the Uncertainty Principle. In quantum mechanics, the wave functions of a quantum system are characterized as the elements of unit norm,  $\| \phi \| = 1$ , belonging to the underlying state space, which, in a one-dimensional model of a single particle, is the Hilbert space  $L^2 = L^2 \mathbb{R}$  consisting of square integrable, complex valued functions of  $x$ . The squared modulus of the

wave function,  $\phi(x)^2$ , represents the probability density of the particle being found at position  $x$ . Consequently, the mean value of any function  $f(x)$  of the position variable is given by its integral against the system's probability density, and denoted by.

$$\langle f(x) \rangle = \int_{-\infty}^{\infty} f(x) \phi(x)^2 dx. \tag{6}$$

In particular,

$$\langle x \rangle = \int_{-\infty}^{\infty} x \phi(x)^2 dx \tag{7}$$

is the mean or average measured position of the particle, while  $\Delta x$ , defined by

$$\Delta x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \int_{-\infty}^{\infty} x^2 \phi(x)^2 dx - \langle x \rangle^2 \tag{8}$$

is the variance, that is, the statistical deviation of the particle's measured position from the mean. We note that  $\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \phi(x)^2 dx = \langle x \phi(x)^2 \rangle$ . 9

On the other hand, the momentum variable  $p$  is related to the Fourier transform frequency via the de Broglie relation  $p = \hbar k$ , where

$$\hbar = \frac{h}{2\pi} \approx 1.055 \times 10^{-34} \text{ Joule seconds} \tag{10}$$

is Planck's constant, whose value governs the quantization of physical quantities. Therefore, the mean value of any function of momentum  $g(p)$  is given by its integral against the modulus of the Fourier transformed wave function:

$$\langle g(p) \rangle = \int_{-\infty}^{\infty} g(\hbar k) \phi(k)^2 dk. \tag{11}$$

In particular, the mean of the momentum measurements of the particle is given by

$$\langle p \rangle = \hbar \int_{-\infty}^{\infty} k \phi(k)^2 dk = -i \hbar \int_{-\infty}^{\infty} \phi'(x) \phi(x) dx = -i \hbar \langle \phi' \phi \rangle; \tag{12}$$

Where we used Parseval's formula formula 4 to convert to an integral over position, and by using the fact that the Fourier Transform of the derivative  $f'(x)$  of the function is obtained by multiplication of its Fourier transform by  $ik$ :

$$\mathcal{F}\{f'(x)\} = ik \mathcal{F}\{f(x)\} \tag{13}$$

Based on 13, we can infer that  $k \phi(k)$  is the Fourier transform of  $-i \phi'(x)$ . Similarly,

$$\Delta p^2 = \langle p^2 \rangle - \langle p \rangle^2 = \langle p^2 \rangle - \langle p \rangle^2 \tag{14}$$

is the squared variance in the momentum, where, by a similar computation,

$$\langle p^2 \rangle = \hbar^2 \int_{-\infty}^{\infty} k^2 \phi(k)^2 dk = -\hbar^2 \int_{-\infty}^{\infty} \phi(x) \phi''(x) dx = \hbar^2 \int_{-\infty}^{\infty} \phi'(x)^2 dx = \hbar^2 \langle \phi' \phi' \rangle. \tag{15}$$

With this interpretation, the uncertainty principle can be stated as follows.

Theorem 3.1 If  $\psi(x)$  is a wave function, so  $\int \psi^2 dx = 1$ , then the variances in position and momentum satisfy the inequality

$$\Delta x \Delta p \geq \frac{1}{2} \hbar. \tag{16}$$

The smaller the variance of a quantity such as position or momentum, the more accurate will be its measurement. Thus, the more accurate will be its measurement. Thus, the Heisenberg inequality (16) quantifies the statement that the more accurately we are able to measure the momentum  $p$ , the less accurate will be any measurement of its position  $x$ , and vice versa.

Proof: For any value of the real parameter  $t$ ,

$$0 \leq \int_{-\infty}^{\infty} t x \psi^2 dx + \int_{-\infty}^{\infty} \psi'^2 dx = t^2 \int_{-\infty}^{\infty} x^2 \psi^2 dx + 2t \int_{-\infty}^{\infty} x \psi \psi' dx + \int_{-\infty}^{\infty} \psi'^2 dx \tag{17}$$

The middle term can be evaluated as follows:

$$\begin{aligned} \int_{-\infty}^{\infty} x \psi \psi' dx &= \int_{-\infty}^{\infty} x \psi' \psi dx + \int_{-\infty}^{\infty} x \psi \psi' dx \\ &= \int_{-\infty}^{\infty} x \frac{d}{dx} \psi^2 dx = \int_{-\infty}^{\infty} \psi^2 dx = -1; \end{aligned}$$

where we employed integration by parts, noting that the boundary terms vanish since  $\psi \rightarrow 0$  as  $x \rightarrow \infty$ . Thus, as indicated in (9), (15) & (17) reads  $x^2 t^2 - t + \frac{p^2}{\hbar^2} \geq 0$  for all real  $t$ .

The minimum value of the left hand side of this inequality occurs at  $t_* = \frac{1}{2x^2}$ , where its value is

$$\frac{p^2}{\hbar^2} - \frac{1}{4x^2} \geq 0 \text{ and hence } x^2 p^2 \geq \frac{1}{4} \hbar^2.$$

To obtain the Uncertainty Relation, one performs same calculation, but with  $x \rightarrow x - \langle x \rangle$  replacing  $x$  and  $p \rightarrow p - \langle p \rangle$  replacing  $p$ .

The result is

$$(x - \langle x \rangle)^2 t^2 - t + \frac{(p - \langle p \rangle)^2}{\hbar^2} = \Delta x^2 t^2 - t + \frac{\Delta p^2}{\hbar^2} \geq 0. \tag{18}$$

Substituting  $t = \frac{1}{2 \Delta x^2}$  gives  $\Delta x \Delta p \geq \frac{1}{2} \hbar$ , which is the Heisenberg inequality.

#### 4. CONCLUSIONS

This paper is based on the use of Fourier Transform for determining Heisenberg inequality . The relationship between measurement of the position of a particle and accuracy of the measurement of the momentum, were inter related in such way that in reverse way. The linear equation which gives the Heisenberg inequality explained the inverse relation to between measurement of position and accuracy measurement momentum.

The method of Fourier Transform is efficient and accurate in relation to evaluate Hilbert spaces and Heisenberg inequality .Given theorem which are given in 4 and 16 supports the effect of Fourier Transform on Hilbert Spaces.

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# $g^* \gamma$ - Closed Sets in Topological Spaces

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## **ABSTRACT**

*In this paper, we introduce and study a new class of generalized closed sets, namely,  $g^* \gamma$  - closed sets in topology. Also, we characterize some basic properties of these sets.*

*Mathematics subject classification(2010); 54A05, 54C08, 54D10.*

**Key words:**  $\gamma$ -open sets,  $\gamma$ -closed sets,  $g \gamma$ -closed sets,  $g \gamma$ -closed sets,  $g^* \gamma$ -closed sets,  $g^* \gamma$ -open sets,  $g^* \gamma$ - $R_0$  space,  $g^* \gamma$ - $R^1$  space.

## **1. INTRODUCTION**

In 1996 D. Andrijevic[1] defined and studied the concepts of b-open sets in topological spaces. b-open sets are also called as sp-open sets. Later in 1997, A.A. El-Atik[5], has introduced and studied the concept of  $\gamma$  -open sets in topology. It is known that b-open sets or sp-open sets are same as  $\gamma$  -open sets. In 2007, E. Ekici[10] has defined and studied the concept of  $\gamma$  -normal spaces in topology. Also, author has introduced and studied  $g \gamma$  -closed sets,  $\gamma g$  - closed sets,  $g \gamma$  -closed function and  $\gamma$  - $g \gamma$  -closed function. In 2009, AynurKeskin et.al.[12] has defined and studied the concept of  $\gamma$  - $R_0$  and  $\gamma$  - $R_1$  spaces using  $\gamma$  -open sets. Recently, in 2013, A.I. El-Maghrabi[6] has introduced a concept of  $\gamma g$ -closed sets in topology which is same as that of  $g \gamma$ -closed set defined by E. Ekici in 2007. The purpose of this paper is to introduce a new class of generalized closed sets, namely,  $g^* \gamma$  -closed sets in topology, also some separation axioms and some related functions.

## **2. PRELIMINARIES**

In this paper  $(X, \tau)$  and  $(Y, \sigma)$  (or  $X$  and  $Y$ ) we always mean topological spaces on which no separation axioms are assumed. Unless otherwise mentioned.

For a subset  $A$  of  $X$ ,  $Cl(A)$  and  $Int(A)$  represent the closure of  $A$  and the interior of  $A$  respectively.

The following definitions and results are useful in the sequel:

**Definition 2.1:** Let  $X$  be a topological space. A subset  $A$  is called

- (i) semiopen[13] if  $A \subset Cl(Int(A))$ ,
- (ii) preopen[14] if  $A \subset Int(Cl(A))$ ,
- (iii) b-open[1] or sp-open[3] or  $\gamma$ -open[5] if  $A \subset Cl(Int(A)) \cup Int(Cl(A))$ .

The complement of semiopen (resp. peropen, b-open or sp-open or  $\gamma$ -open) set is called semiclosed[2] (resp. preclosed[14], b-closed[1] or sp-closed[3] or  $\gamma$ -closed[5]).

The family of all semiopen (resp. preopen, b-open or sp-open or  $\gamma$ -open) sets of a space  $X$  is denoted by  $SO(X)$  (resp.  $PO(X)$ ,  $BO(X)$ ,  $SPO(X)$  or  $\gamma O(X)$ ). And the family of all semiopen (resp. preopen, b-open or sp-open or  $\gamma$ -open) sets containing a point  $x$  of  $X$  will be denoted by  $SO(X,x)$  (resp.  $PO(X,x)$ ,  $BO(X,x)$  or  $SPO(X,x)$  or  $\gamma O(X,x)$ ).

**Definition 2.2:** Let  $A$  be a subset of a space  $X$ , then semi-interior [2] (resp. pre-interior[15],  $\gamma$ -interior[5]) of  $A$  is the union of all semiopen (resp. preopen,  $\gamma$ -open) sets contained in  $A$  and is denoted by  $sInt(A)$  (resp.  $pInt(A)$ ,  $\gamma Int(A)$ ).

**Definition 2.3:** Let  $A$  be a subset of a space  $X$ , then the intersection of all semi-closed (resp. pre-closed,  $\gamma$ -closed) sets containing  $A$  is called semiclosure[2] (resp. preclosure[4],  $\gamma$ -closure[5]) of  $A$  and is denoted by  $sCl(A)$  (resp.  $pCl(A)$ ,  $\gamma Cl(A)$ ).

**Definition 2.4:** A subset  $A$  of a space  $X$  is said to be  $g\gamma$ -closed[10] if  $\gamma Cl(A) \subset U$  whenever  $A \subset U$  and  $U \in \tau$ .

The complement of  $g\gamma$ -closed set is said to be  $g\gamma$ -open.

**Definition 2.5:** A subset  $A$  of a space  $X$  is said to be  $\gamma g$ -closed[6] if  $\gamma Cl(A) \dot{\subset} U$  whenever  $A \subset U$  and  $U \in \gamma O(X)$ .

The complement of  $\gamma g$ -closed set is said to be  $\gamma g$ -open.

The definitions of be  $g\gamma$ -closed set and  $\gamma g$ -closed set respectively, defined by E. Ekici [10] and El-Maghrabi [6] are the same.

**Definition 2.6:** A function  $f: X \rightarrow Y$  is said to be  $\gamma$ -closed [10], if the image of each closed set of  $X$  is  $\gamma$ -closed set in  $Y$ .

**Definition 2.7:** A function  $f: X \rightarrow Y$  is said to be strongly  $\gamma$ -open [10], if the image of each  $\gamma$ -open set of  $X$  is  $\gamma$ -open set in  $Y$ .

**Definition 2.8:** A function  $f: X \rightarrow Y$  is said to be strongly  $\gamma$ -closed [10], if the image of each  $\gamma$ -closed set of  $X$  is  $\gamma$ -closed set in  $Y$ .

**Definition 2.9:** A function  $f: X \rightarrow Y$  is said to be  $\gamma$ -continuous [5], if the inverse image of each open set of  $Y$  is  $\gamma$ -open set in  $X$ .

**Definition 2.10:** A function  $f: X \rightarrow Y$  is said to be  $\gamma$ -irresolute [7], if the inverse image of each  $\gamma$ -open set of  $Y$  is  $\gamma$ -open set in  $X$ .

**Definition 2.11:** A topological space  $X$  is called  $\gamma$ - $R_0$  [8] if its every  $\gamma$ -open set contains the  $\gamma$ -closure of each of its singletons.

**Definition 2.12 :** A topological space  $X$  is called  $\gamma$ - $R_1$  [9] if for  $x$  and  $y$  in  $X$  with  $\gamma Cl(\{x\}) \neq \gamma Cl(\{y\})$ , there exist disjoint  $\gamma$ -open sets  $U$  and  $V$  such that  $\gamma Cl(\{x\})$  is a subset of  $U$  and  $\gamma Cl(\{y\})$  is a subset of  $V$ .

The following are proved in [1]:

**Theorem 2.13:** For a subset  $A$  of a space  $X$  the following are equivalent:

- (i)  $A$  is  $\gamma$ -open
- (ii)  $A = pInt(A) \cup sInt(A)$
- (iii)  $A \subset pCl(pInt(A))$ .

**Theorem 2.14:** Let  $A$  be a subset of a space  $X$ . Then

- (i)  $\gamma Cl(A) = sCl(A) \cap pCl(A)$
- (ii)  $\gamma Int(A) = sInt(A) \cup pInt(A)$
- (iii)  $x \in \gamma Cl(A)$  iff  $A \cap U \neq \phi$  for every  $\gamma O(X, x)$
- (iv)  $\gamma Cl(A) = A \cup [Int(Cl(A)) \cap Cl(Int(A))]$
- (v)  $\gamma Int(A) = A \cap [Cl(Int(A)) \cup Int(Cl(A))]$

**Theorem 2.15:** Let  $A$  be a subset of a space  $X$ . Then

- (i)  $\gamma \text{Cl}(\text{Int}(A)) = \text{Int}(\gamma \text{Cl}(A)) = \text{Int}(\text{Cl}(\text{Int}(A)))$
- (ii)  $\gamma \text{Int}(\text{Cl}(A)) = \text{Cl}(\gamma \text{Int}(A)) = \text{Cl}(\text{Int}(\text{Cl}(A)))$
- (iii)  $\gamma \text{Cl}(\text{sInt}(A)) = \text{sCl}(\text{sInt}(A))$
- (iv)  $\gamma \text{Int}(\text{sCl}(A)) = \text{sInt}(\text{sCl}(A))$ .

### 3. $g^* \gamma$ -CLOSED SETS

In this section, we introduce the following:

**Definition 3.1:** A subset  $A$  of a space  $X$  is called generalized star  $g$ -closed (in brief,  $g^* \gamma$ -closed) set if  $\text{Cl}(A) \subset U$  whenever  $A \subset U$  and  $U$  is  $\gamma$ -open set in  $X$ .

**Definition 3.2:** A subset  $A$  of a space  $X$  is called generalized star  $\gamma$ -open (in brief,  $g^* \gamma$ -open) set if  $F \subset \text{Int}(A)$  whenever  $F \subset A$  and  $F$  is  $\gamma$ -closed set in  $X$ .

The family of all  $g^* \gamma$ -open sets in topological space  $X$  is denoted by  $g^* \gamma O(X)$  and that of, the family of all,  $g^* \gamma$ -closed sets in topological space  $X$  is denoted by  $g^* \gamma F(X)$ . And the family of all  $g^* \gamma$ -open sets containing  $x$  of  $X$  will be denoted by  $g^* \gamma O(X, x)$

In view of definitions 2.1(iii), 2.4 and 2.5, we have the following implications:

**Remark 3.3:** For any subset of a topological space  $X$ , we have

- (i) closed  $\Rightarrow$  semiclosed  $\Rightarrow \gamma$ -closed
- (ii) closed  $\Rightarrow$  preclosed  $\Rightarrow \gamma$ -closed
- (iii)  $\gamma$ -closed  $\Rightarrow \gamma g$ -closed  $\Rightarrow g \gamma$ -closed
- (iv)  $\gamma$ -closed  $\Rightarrow g^* \gamma$ -closed
- (v)  $g^* \gamma$ -closed  $\Rightarrow \gamma g$ -closed

Clearly, every  $g \gamma$ -closed set and  $g^* \gamma$ -closed set are independent.

We define the following:

**Definition 3.4:** Let  $A$  be a subset of a space  $X$ , then the intersection of all  $g^* \gamma$ -closed sets containing  $A$  is called the  $g^* \gamma$ -closure of  $A$  and is denoted by  $g^* \gamma \text{Cl}(A)$ .



**Definition 3.5:** Let  $A$  be a subset of a space  $X$ , then the union of all  $g^* \gamma$ -open sets contained in  $A$  is called the  $g^* \gamma$ -interior of  $A$  and is denoted by  $g^* \gamma \text{Int}(A)$

In view of theorem 2.14 and 2.15 we have the following:

**Theorem 3.6:** Let  $A$  be a subset of a space  $X$ . Then

- (i)  $g^* \gamma \text{Cl}(A) = s\text{Cl}(A) \cap p\text{Cl}(A)$
- (ii)  $g^* \gamma \text{Int}(A) = s\text{Int}(A) \cup p\text{Int}(A)$
- (iii)  $x \in g^* \gamma \text{Cl}(A)$  iff  $A \cap U \neq \emptyset$  for every  $g^* \gamma O(X, x)$
- (iv)  $g^* \gamma \text{Cl}(A) = A \cup [\text{Int}(\text{Cl}(A)) \cap \text{Cl}(\text{Int}(A))]$
- (v)  $g^* \gamma \text{Int}(A) = A \cap [\text{Cl}(\text{Int}(A)) \cup \text{Int}(\text{Cl}(A))]$ .

**Theorem 3.7:** Let  $A$  be a subset of a space  $X$ . Then

- (i)  $g^* \gamma \text{Cl}(\text{Int}(A)) = \text{Int}(g^* \gamma \text{Cl}(A)) = \text{Int}(\text{Cl}(\text{Int}(A)))$
- (ii)  $g^* \gamma \text{Int}(\text{Cl}(A)) = \text{Cl}(g^* \gamma \text{Int}(A)) = \text{Cl}(\text{Int}(\text{Cl}(A)))$
- (iii)  $g^* \gamma \text{Cl}(s\text{Int}(A)) = s\text{Cl}(s\text{Int}(A))$
- (iv)  $g^* \gamma \text{Int}(s\text{Cl}(A)) = s\text{Int}(s\text{Cl}(A))$ .

We define the following:

**Theorem 3.8:** A topological space  $X$  is called  $g^* \gamma$ - $R_0$ , if its every  $g^* \gamma$ -open set contains the  $g^* \gamma$ -closure of each of its singletons.

Clearly every  $\gamma$ - $R_0$  spaces is  $g^* \gamma$ - $R_0$  spaces.

We characterize the following:

**Theorem 3.9:** A topological space  $X$  is a  $g^* \gamma$ - $R_0$  space if and only if for any  $x$  and  $y$  in  $X$ ,  $g^* \gamma \text{Cl}(\{x\}) \neq g^* \gamma \text{Cl}(\{y\})$  implies  $g^* \gamma \text{Cl}(\{x\}) \cap g^* \gamma \text{Cl}(\{y\}) = \emptyset$ .

Proof: Obvious.

We define the following:

**Definition 3.10:** A function  $f: X \rightarrow Y$  is said to be strongly  $g^* \gamma$ -closed, if the image of each  $g^* \gamma$ -closed set of  $X$  is closed set in  $Y$ .

Every  $\gamma$ -closed functions and strongly  $g^* \gamma$ -closed functions are independent.

However we have the following:

**Lemma 3.11:** If  $f: X \rightarrow Y$  be  $\gamma$ -closed function and  $g: Y \rightarrow Z$  be strongly  $g^* \gamma$ -closed function, then  $g \circ f$  is closed function.

**Proof:** Let  $F$  be any closed subset of  $X$ . Then  $f(F)$  be  $\gamma$ -closed set in  $Y$ , since  $f: X \rightarrow Y$  is  $\gamma$ -closed function. Again,  $g: Y \rightarrow Z$  is strongly  $g^* \gamma$ -closed function and  $f(F)$  is  $\gamma$ -closed set in  $Y$ , but we know that every  $\gamma$ -closed set is  $g^* \gamma$ -closed set. And hence  $f(F)$  is  $g^* \gamma$ -closed set of  $Y$ . Then  $g(f(F)) = g \circ f(F)$  is closed set in  $Z$ . This shows that  $g \circ f$  is closed function.

We define the following:

**Definition 3.12:** A function  $f: X \rightarrow Y$  is called  $(g^* \gamma, \gamma)$ -closed if the image of each  $g^* \gamma$ -closed set of  $X$  is  $\gamma$ -closed in  $Y$ .

Now we give the following:

**Lemma 3.13:** Let  $f: X \rightarrow Y$  be strongly  $g^* \gamma$ -closed function and  $g: Y \rightarrow Z$  be  $\gamma$ -closed function, then  $g \circ f$  is  $(g^* \gamma, \gamma)$ -closed function.

**Proof:** Obvious.

We define the following:

**Definition 3.14:** A function  $f: X \rightarrow Y$  is said to be  $g^* \gamma$ -continuous if the inverse image of each open set of  $Y$  is  $g^* \gamma$ -open set in  $X$ .

**Definition 3.15:** A function  $f: X \rightarrow Y$  is said to be  $g^* \gamma$ -irresolute if the inverse image of each  $g^* \gamma$ -open set of  $Y$  is  $g^* \gamma$ -open set in  $X$ .

We give the following:

**Theorem 3.16:** If  $X$  is a  $g^* \gamma$ - $R_0$  space and  $f: X \rightarrow Y$  is  $g^* \gamma$ -irresolute and strongly  $g^* \gamma$ -closed surjection, then  $Y$  is  $g^* \gamma$ - $R_0$  space.

**Proof:** Let  $V$  be a  $g^* \gamma$ -open set of  $Y$  and  $y$  be any point of  $V$ . Since  $f$  is  $g^* \gamma$ -irresolute,  $f^{-1}(V)$  is  $g^* \gamma$ -open set in  $X$ . Since  $X$  is  $g^* \gamma$ - $R_0$  space, for a point  $x \in f^{-1}(\{y\})$ , by definition 3.4,  $g^* \gamma Cl(\{x\}) \subset f^{-1}(V)$ . But by the strongly  $g^* \gamma$ -closedness of  $f$ ,  $g^* \gamma Cl(\{y\}) = g^* \gamma Cl(\{f(x)\}) \subset f(g^* \gamma Cl(\{x\})) \subset V$ . Therefore,  $Y$  is  $g^* \gamma$ - $R_0$  space.

**Theorem 3.17:** Let  $X$  be a  $g^* \gamma$ - $R_0$  space and  $f: X \rightarrow Y$  is a strongly  $g^* \gamma$ -closed and  $g^* \gamma$ -continuous surjection, then  $Y$  is an  $R_0$  space.

**Proof:** The proof is similar to the theorem 3.16 and is thus omitted.

Now we define the following:

**Definition 3.18:** A topological space  $X$  is called  $g^* \gamma$ - $R_1$  if for  $x$  and  $y$  in  $X$  with  $g^* \gamma Cl(\{x\}) \neq g^* \gamma Cl(\{y\})$ , there exist disjoint  $g^* \gamma$ -open sets  $U$  and  $V$  such that  $g^* \gamma Cl(\{x\})$  is a subset of  $U$  and  $g^* \gamma Cl(\{y\})$  is a subset of  $V$ .

We prove the following:

**Lemma 3.19:** Every  $g^* \gamma$ - $R_1$  space is  $g^* \gamma$ - $R_0$ .

**Proof:** Let  $U$  be a  $g^* \gamma$ -open set such that  $x \in U$ . If  $y \notin U$ , since  $x \notin g^* \gamma Cl(\{y\})$ , we have  $g^* \gamma Cl(\{x\}) \neq g^* \gamma Cl(\{y\})$ . So, there exists a  $g^* \gamma$ -open set  $V$  such that  $g^* \gamma Cl(\{y\}) \subset V$  and  $x \notin V$ , which implies  $y \notin g^* \gamma Cl(\{x\})$ . Hence  $g^* \gamma Cl(\{x\}) \subset U$ .

Therefore,  $X$  is  $g^* \gamma$ - $R_0$ .

Now, we define the following:

**Definition 3.20:** A function  $f: X \rightarrow Y$  is called :

- (i) slightly  $g^* \gamma$ -continuous at a point  $x \in X$  if for each clopen subset  $V$  in  $Y$  containing  $f(x)$ , there exists a  $g^* \gamma$ -open subset  $U$  in  $X$  containing  $x$  such that  $f(U) \subset V$ .
- (ii) slightly  $g^* \gamma$ -continuous if it has this property at each point of  $X$ .

We give the following:

**Theorem 3.21:** Let  $X$   $Y$  be topological spaces. The following statements are equivalent for a function  $f: X \rightarrow Y$  :

- (i)  $f$  is slightly  $g\gamma$ -continuous  
(ii) for every clopen set  $V \subset Y$ ,  $f^{-1}(V)$  is a  $g^*\gamma$ -open  
(iii) for every clopen set  $V \subset Y$ ,  $f^{-1}(V)$  is a  $g^*\gamma$ -closed  
(iv) for every clopen set  $V \subset Y$ ,  $f^{-1}(V)$  is a  $g^*\gamma$ -clopen.

**Proof:** (i)  $\Rightarrow$  (ii): Let  $V$  be a clopen subset of  $Y$  and let  $x \in f^{-1}(V)$ . Since  $f(x) \in V$ , by (i), there exists a  $g^*\gamma$ -open set  $U$  in  $X$  containing  $x$  such that  $U \subset f^{-1}(V)$ . We obtain that  $f^{-1}(V) = \bigcup U$ . Thus  $f^{-1}(V)$  is  $g^*\gamma$ -open.

(ii)  $\Rightarrow$  (iii): Let  $V$  be a clopen subset of  $Y$ . Then,  $Y \setminus V$  is clopen. By (ii),  $f^{-1}(Y \setminus V) = X \setminus f^{-1}(V)$  is a  $g^*\gamma$ -open. Thus,  $f^{-1}(V)$  is  $g^*\gamma$ -closed.

(iii)  $\Rightarrow$  (iv): Obvious.

(iv)  $\Rightarrow$  (i): Let  $V$  be a clopen subset of  $Y$  containing  $f(x)$ . By (iv),  $f^{-1}(V)$  is  $g^*\gamma$ -clopen. Take  $U = f^{-1}(V)$ . Then  $f(U) \subset V$ . Hence,  $f$  is slightly  $g^*\gamma$ -continuous.

We define the following:

**Definition 3.22:** A function  $f: X \rightarrow Y$  is said to be  $g^*\gamma$ -open if the image of open set of  $X$  is  $g^*\gamma$ -open in  $Y$ .

**Definition 3.23:** A function  $f: X \rightarrow Y$  is said to be  $(\gamma, g^*\gamma)$ -continuous if the inverse image of each  $\gamma$ -open set of  $Y$  is  $g^*\gamma$ -open in  $X$ .

We recall the following:

**Definition 3.24:** A function  $f: X \rightarrow Y$  is called slightly continuous [11] if the inverse image of each clopen set of  $Y$  is open set in  $X$ .

Now, We have the following:

**Theorem 3.25:** Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be functions. Then the following statements are valid:

- (i) If  $f$  is  $g\gamma$ -irresolute and  $g$  is slightly  $g\gamma$ -continuous, then  $g \circ f$  is slightly  $g^*\gamma$ -continuous.  
(ii) If  $f$  is  $g^*\gamma$ -irresolute and  $g$  is  $g^*\gamma$ -continuous, then  $g \circ f$  is  $g^*\gamma$ -continuous.  
(iii) If  $f$  is  $g^*\gamma$ -irresolute and  $g$  is  $(\gamma, g^*\gamma)$ -continuous, then  $g \circ f$  is  $(\gamma, g^*\gamma)$ -continuous.  
(iv) If  $f$  is  $g^*\gamma$ -continuous and  $g$  is slightly continuous, then  $g \circ f$  is slightly  $g^*\gamma$ -continuous.

**Proof:** (i) Let  $V$  be any clopen set in  $Z$ . Since  $g$  is slightly  $g^*\gamma$ -continuous,  $g^{-1}(V)$  is  $g^*\gamma$ -open in  $Y$ . Since,  $f$  is  $g^*\gamma$ -irresolute and  $g^{-1}(V)$  is  $g^*\gamma$ -open, then  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $g^*\gamma$ -open in  $X$ . Therefore,  $g \circ f$  is slightly  $g^*\gamma$ -continuous.

(ii) Let  $V$  be any open set in  $Z$ . Since  $g$  is  $g^* \gamma$ -continuous,  $g^{-1}(V)$  is  $g^* \gamma$ -open in  $Y$ . Again,  $f$  is  $g^* \gamma$ -irresolute and  $g^{-1}(V)$  is  $g^* \gamma$ -open in  $Y$ , then  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $g^* \gamma$ -open set in  $X$ . Therefore,  $g \circ f$  is  $g^* \gamma$ -continuous.

(iii) Let  $V$  be any  $\gamma$ -open subset in  $Z$ . Since  $g$  is  $(\gamma, g^* \gamma)$ -continuous,  $g^{-1}(V)$  is  $g^* \gamma$ -open in  $Y$ . Again,  $f$  is  $g^* \gamma$ -irresolute and  $g^{-1}(V)$  is  $g^* \gamma$ -open set in  $Y$ , then  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $g^* \gamma$ -open set in  $X$ . Hence  $g \circ f$  is  $(\gamma, g^* \gamma)$ -continuous.

(iv) Let  $V$  be any clopen set in  $Z$ . Since  $g$  is slightly continuous,  $g^{-1}(V)$  is open set in  $Y$ . Again,  $f$  is  $g^* \gamma$ -continuous and  $g^{-1}(V)$  is open set in  $Y$ , then  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $g^* \gamma$ -open in  $X$ . Then  $g \circ f$  is slightly  $g^* \gamma$ -continuous.

**Theorem 3.26:** Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be functions. If  $f$  is  $g^* \gamma$ -open and surjective and  $g \circ f: X \rightarrow Z$  is slightly  $g^* \gamma$ -continuous, then  $g$  is slightly  $g^* \gamma$ -continuous.

**Proof:** Let  $V$  be any clopen set in  $Z$ . Since  $g \circ f$  is slightly  $g^* \gamma$ -continuous,  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is  $g^* \gamma$ -open in  $X$ . Since,  $f$  is  $g^* \gamma$ -open, then  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is  $g^* \gamma$ -open in  $Y$ . Hence,  $g$  is slightly  $g^* \gamma$ -continuous.

We define the following:

**Definition 3.27:** A function  $f: X \rightarrow Y$  is said to be always  $g^* \gamma$ -open, if the image of each  $g^* \gamma$ -open set of  $X$  is  $g^* \gamma$ -open in  $Y$ .

**Definition 3.28:** A function  $f: X \rightarrow Y$  is said to be always  $g^* \gamma$ -closed, if the image of each  $g^* \gamma$ -closed set of  $X$  is  $g^* \gamma$ -closed in  $Y$ .

**Definition 3.29:** A function  $f: X \rightarrow Y$  is said to be strongly  $g^* \gamma$ -open, if the image of each  $g^* \gamma$ -open set of  $X$  is open in  $Y$ .

We have the following:

**Theorem 3.30:** Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be functions such that  $g \circ f: X \rightarrow Z$  is slightly  $g^* \gamma$ -continuous

(i) if  $f$  is strongly  $g^* \gamma$ -open surjective, then  $g$  is slightly continuous function.

(ii) if  $f$  is always  $g^* \gamma$ -open surjective, then  $g$  is slightly  $g^* \gamma$ -continuous.

**Proof:** (i) Let  $V$  be clopen set in  $Z$ . Since  $g \circ f$  is slightly  $g^* \gamma$ -continuous,  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is  $g^* \gamma$ -open set in  $X$ . Since  $f$  is strongly  $g^* \gamma$ -open surjective, then  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is open set in  $Y$ . Hence  $g$  is slightly continuous function.

(ii) Let  $V$  be clopen set in  $Z$ . Since  $g \circ f$  is slightly  $g^* \gamma$ -continuous,  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is  $g^* \gamma$ -open set in  $X$ . Since  $f$  is always  $g^* \gamma$ -open surjective, then  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is  $g^* \gamma$ -open set in  $Y$ . Hence  $g$  is slightly  $g^* \gamma$ -continuous.

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# Mathematical Modeling to Assess Littoral Drift and Shoreline Changes at a Marine Terminal

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## ABSTRACT

*Natural course of littoral drift can be seriously impacted by the construction of coastal hydraulic structures. If due consideration is not given during construction of breakwaters, it may result in severe siltation on one side of breakwater and erosion on the other side. Further, siltation in the vicinity of harbour area interrupts smooth operation of ports. To prevent such situation, mathematical modeling tools can play a significant role in estimating quantum and extent of littoral drift and assessing shoreline changes. However, emphasis should also be given to corroborate the findings of mathematical modeling with actual field data. In this paper, mathematical modeling of a typical case of marine terminal on west coast of India, which was facing severe problem of siltation in and around the terminal due to littoral drift is discussed and remedial measures in such cases are suggested.*

**Keywords:** *Littoral drift, siltation, groyne, shoreline evolution*

## 1. INTRODUCTION

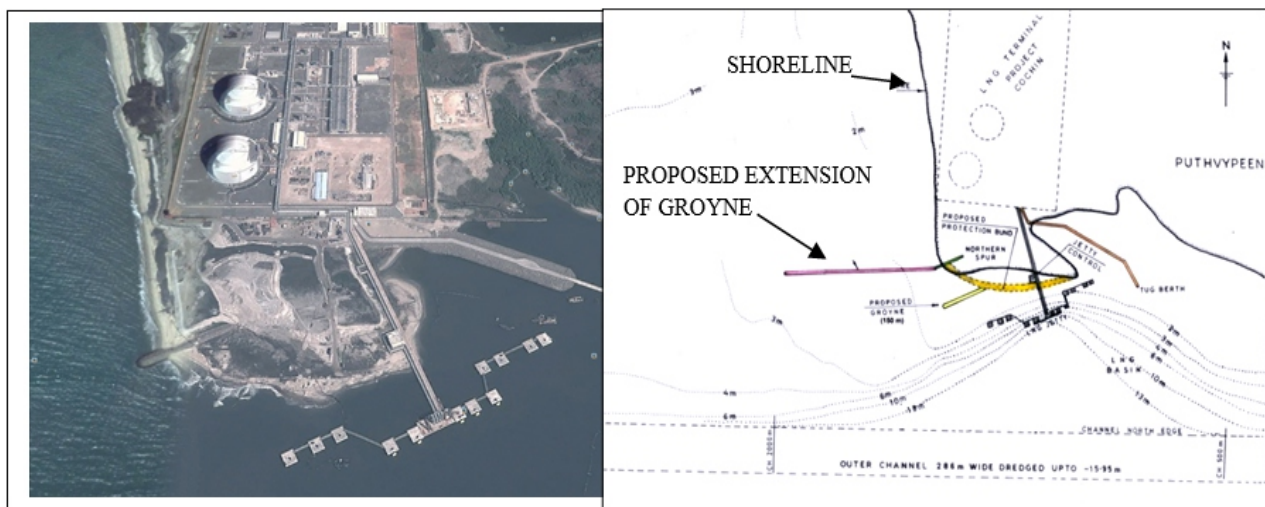
Natural course of littoral drift can be seriously impacted by the construction of coastal hydraulic structures. If due consideration is not given during construction of breakwaters, it may result in severe siltation on one side of breakwater and erosion on the other side. Further, siltation in the vicinity of harbour area interrupts smooth operation of ports.

There is a marine terminal for import of LNG on accreted land at Vypeen near the entrance of Cochin Port in Kerala on the west coast of India. The LNG terminal was facing severe problem of siltation in and around its terminals due to littoral drift. As this region falls under the moderate wave regime, wave action is the primary source of energy available in the nearshore zone and this could be the main factor responsible for the coastal processes including the longshore sediment transport influencing this region particularly during the southwest monsoon season when the wave activity is at its peak. The phenomenon of excessive longshore sediment transport was observed at the LNG site in the year 2011, when a groyne of 130 m length was constructed in the north of the terminal to prevent siltation in the vicinity of terminal caused due to littoral drift. It is reported that the accretion of about 70 m occurred on



the northern side of the groyne within a period of the first four months of monsoon season and during subsequent seasons, the shoreline further advanced by 20 to 25 m (Fig. 1(a)). In present condition the groyne does not play significant role in arresting the southward littoral drift as most of the drift is being bypassed and getting deposited in the vicinity of the terminal.

In order to prevent or minimise the siltation due to southward littoral drift in the vicinity of LNG terminal, it is proposed to extend the length of the existing groyne by 500 m (Fig. 1(b)). In the present study of seasonal and annual littoral drift rates were estimated and shoreline changes due to extension of existing groyne were assessed.



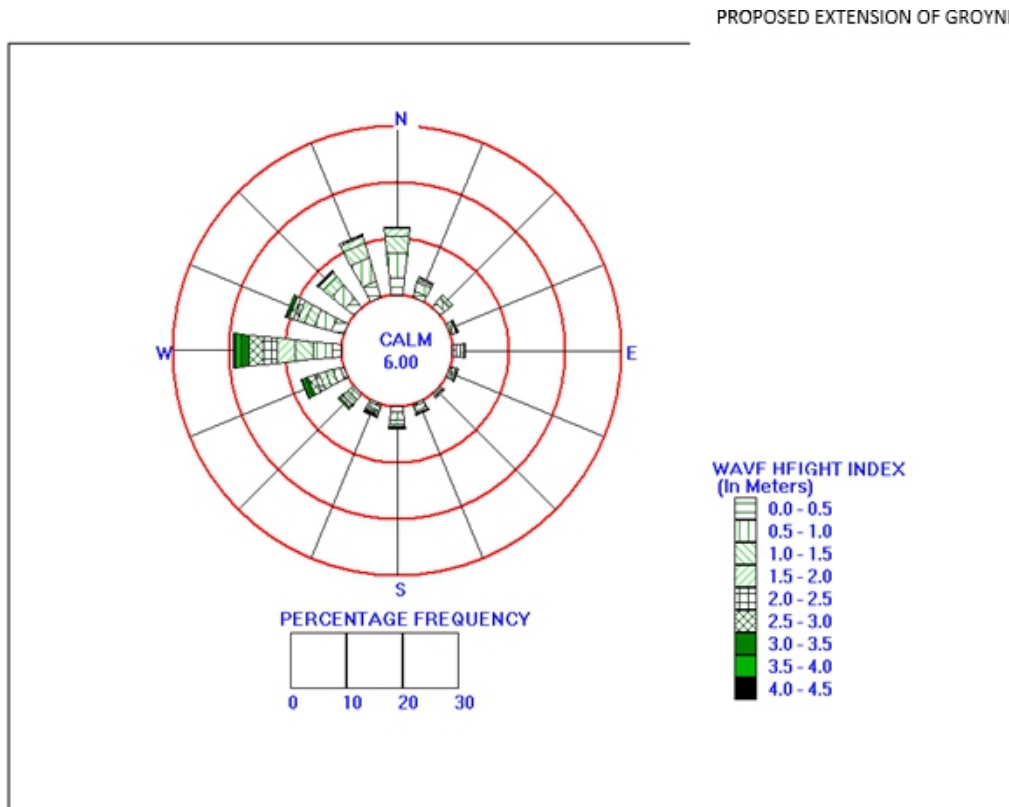
**Figure 1. (a) Layout of LNG terminal at Cochin (b) Proposed protection works at LNG Terminal**

## 2 LITERATURE REVIEW AND SITE CONDITIONS

### 2.1 Offshore wave climate

For simulation of littoral drift and shoreline changes, nearshore wave data is essential. Due to unavailability of near shore wave data, generally, the wave data observed by the ships plying in offshore region of Cochin reported by India Meteorological Department (IMD) during the past 30 years were considered to arrive at nearshore wave data. The wave climate during the entire year Fig.2 indicates that the predominant wave directions in deep water are West, North, NNW and WNW with the maximum wave height of the order of 4.5 m. In order to get the nearshore wave climate, the deep water wave data were transformed at a location of -10 m depth contour near the LNG terminal using the SW module of MIKE 21 software.





**Figure 2 Rose diagram for wave heights off Cochin for annual period (Jan-Dec)**

## 2.2 Tidal levels and currents

The maximum tidal range at the study area is of the order of 0.6 m. It is also observed that the maximum current in the vicinity of LNG terminal is of the order of 0.33 m/s while the average current is of the order of 0.25 m/s.

## 2.3 Sediment characteristics

The bed material in the vicinity of LNG terminal is mainly composed of fine sand and silt, whose mean grain size,  $D_{50}$  varies from 0.025 mm to 0.25 mm.

## 2.4 Littoral drift

Longshore sediment transport rates for Cochin coastal regions are estimated and reported in CWPRS Technical Report No.3633. It is reported that the littoral drift having annual net transport of 0.27 million  $m^3$  (southward) and the gross transport of 0.930 million  $m^3$  occurs near the Cochin Port, which is close to the LNG terminal site. Several other literatures (Sajeev et al., 1997 and Sanilkumar et al., 2006) are available in which longshore sediment transport rate of nearby locations is mentioned. They estimated the annual net transport of 0.11 million cum (southward) and the gross transport of 0.26 million  $m^3$  at Kozhikode, which is about 192 km north of Cochin port. Chandramohan et al. (1990) computed

longshore transport along south Indian coast using Energy Flux Method (Shore Protection Manual, 1984). At Cochin, they predicted southward and northward longshore transport of 0.98 and 0.69 million m<sup>3</sup> respectively. Furthermore, it is mentioned here that the Energy Flux Method, in general, gives higher values of longshore transport

### 3. MODELLING TECHNIQUES

Bathymetry in the offshore region of Cochin is fairly simple and consists of almost parallel contours from -10 m depth upto -65 m depth. The bathymetric data in the offshore were taken from CMAP database. It was found that that the bed slope in the vicinity of LNG terminal is very mild (1:650).

For computation of wave transformation from deep water to nearshore region the MIKE 21 Spectral Wave (SW) model was used. The model is capable of simulating the growth, decay and transformation of wind generated waves and swells in both offshore and coastal areas. MIKE 21 SW is a new generation spectral wind-wave model based on unstructured meshes, which takes into account all the important phenomena like wave growth by influence of wind, non-linear wave-wave interaction, dissipations due to white-capping, bottom friction and depth-induced breaking. It can also model diffraction effects due to the presence of large structures which becomes increasingly important in the presence of coastal structures like breakwaters, groins etc. The effect of refraction and shoaling of waves due to depth variations and wave-current interaction are also considered in the model. The outputs from the model area the regular wave parameters which include the significant wave height, mean wave period, mean wave direction directional standard deviation and the wave radiation stresses. The governing equation used in the model is the wave action balance equation. The equation is solved using cell centred finite volume method. In horizontal Cartesian co-ordinates the conservation equation for wave action is

$$\frac{\partial N}{\partial t} + \nabla \cdot (\vec{v} N) = \frac{S}{\sigma} \quad (1)$$

#### Nomenclature

|   |   |
|---|---|
| $N(x, \sigma, \theta, t)$               | = the action density,   |
| $t$                                     | = time (sec),   |
| $x(x, y)$                               | = Cartesian co-ordinates,   |
| $\vec{v}(C_x, C_y, C_\sigma, C_\theta)$ | = propagation velocity of a wave group in four dimensional phase space $x, \sigma$ and $\theta$ |
| $S$                                     | = source term for the energy balance equation.  |
| $\nabla$                                | = the four dimensional differential operator in $x, \sigma$ and $\theta$ .                      |

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An area of 90 km alongshore and 40 km cross-shore with an unstructured mesh was considered for the simulation of MIKE 21 SW model. In the cross-shore, it extends from high water line near the shore to -65 m depth contour in deep sea. The model was run for incident waves of wave height 4 m for all the predominant wave directions in deep sea viz. SE, SSE, South, SSW, SW, WSW, West, WNW, NW, NNW and North.

Wave heights and wave directions, at -10 m depth contour near the location of the existing groyne proposed to be extended by 500 m. were extracted from the model results for all the incident wave directions. The extracted information of wave heights and directions at -10 m depth contour was applied to seasonal and yearly offshore wave climate to obtain frequency distribution of wave heights and wave directions near the location of the existing groyne.

The wave climate during the entire indicates that the predominant wave directions are from WNW and West with the maximum wave height of the order of 3.5 m and the percentage occurrence of 32% and 30% respectively for the annual period.

#### **4. MODEL SIMULATIONS AND RESULTS**

In order to simulate littoral drift & its distribution, shoreline changes etc, LITPACK software was used. In LITPACK model, the cross-shore distribution of longshore current, wave height and setup for an arbitrary beach profile are computed by solving the longshore and cross-shore momentum balance equations. The longshore current model includes effect of regular and irregular waves, influence of tidal currents, wind stress and non-uniform bottom friction, wave refraction, shoaling and breaking.

Calculation of longshore sediment transport is based on the local wave, current and sediment characteristics. The sediment transport model is an intra-wave period model, which takes into account time-varying distribution of both suspended load and bed load within the wave period in combined wave and current motion including the effect of wave breaking. The model gives cross-shore distribution of longshore sediment transport for an arbitrary, non-uniform bathymetry and sediment profile. The annual sediment budget is calculated based on the contribution of transport from each of the wave incidents occurring during the year. Thus the total annual drift is the sum of the contribution of transport from all incident waves.

The model assumes the depth contours parallel to the seashore. shows the beach profile (cross-shore bed profile) near the LNG terminal site at Cochin which is used for the littoral drift computation. Profile covers a distance of 6.5 km extending upto about -10 m depth contour. It was found that -5 m depth

contour is about 2.5 km from the coastline. Profile is discretised into 650 grid points with uniform grid size of 10 m. Littoral drift at the LNG site was simulated using LITDRIFT module of LITPACK, which takes into account the cross-shore profile, the sediment characteristics and the wave climate as input.

The general orientation of the coastline at the proposed site is NW-SE making an angle of about 3400 with respect to north. Further, depending on the wave direction with respect to the coastline, the littoral drift will be either directed northward or southward along the shoreline. The model was simulated for the cross-shore profiles for the seasonal and the annual wave climates.

Based on the simulation of littoral drift, the annual net and the gross transports were estimated to be of the order of 0.66 and 1.19 million m<sup>3</sup> respectively. The model was also used to compute seasonal transport rates. The distribution of littoral transport rates for the three seasons and for the annual period. It may be noted that the northward drift is indicated as negative and the southward drift is indicated as positive in the figures. It could be seen that during SW monsoon, southward transport is more than the northward transport and the transport occurs within a range of 3000 m from the shoreline. It could also be seen that main drift moves within a range of 1000 m from coastline. During NE monsoon also, southward transport is more than northward transport and the transport is mainly confined to a range of 500 m from the shoreline. During non-monsoon period also, the southward transport is predominant and the major transport occurs within a range of 500 m from the shoreline. The distribution of annual transport indicates that the sediment drift moves within a range of 2500 m from the shoreline as the cross-shore profile has mild slope. Further, it could be seen that the southward transport is more than the northward transport during all the seasons. It could also be seen that the peak transport occurs at about 200 m from the shoreline and about 75% transport occurs between 50 m to 1000 m from shoreline i.e. between +0.5 m to -2.5 m depth contours.

Annual and seasonal net and gross littoral drift were estimated using LITDRIFT model. The northward and the southward drift are calculated based on the gross and the net littoral drift. The littoral drifts are reported in the following table:

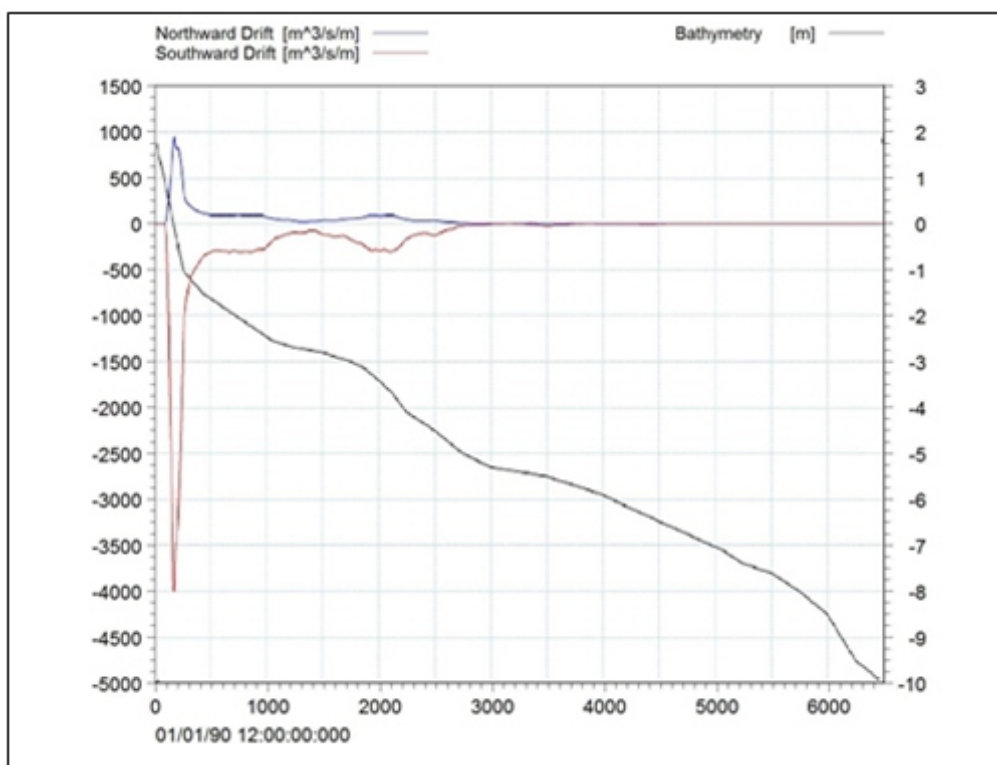
**Table 1 Transport rates (million cum)**

| Period      | Northward | Southward | Net    | Gross |
|-------------|-----------|-----------|--------|-------|
| SW Monsoon  | 0.19      | 0.65      | -0.46  | 0.84  |
| NE Monsoon  | 0.042     | 0.107     | -0.065 | 0.149 |
| Non-Monsoon | 0.037     | 0.18      | -0.14  | 0.213 |
| Annual      | 0.265     | 0.925     | -0.66  | 1.19  |

It could be seen from the table that the annual northward and southward drift are 0.265 million cum and 0.925 million cum respectively while the net and the gross transports are 0.66 million cum and 1.19 million cum respectively. The net transport is towards south. It is also observed that the major southward transport occurs during SW monsoon.

Coastline evolution analyses are performed with the LITLINE module of LITPACK modelling system. LITLINE simulates the coastal response to gradients in the longshore sediment transport capacity resulting from natural features and a wide variety of coastal structures. LITLINE calculates the coastline evolution by solving a continuity equation for the sediment in the littoral zone. The influence of structures, sources and sinks are also included. The basic input data for running the model are the longshore relative coastline alignment, cross-shore profile description and bathymetry, active depth of transport and depth contour angles at each grid point, environmental data with wave properties, tidal currents and water levels position and size of structures etc.

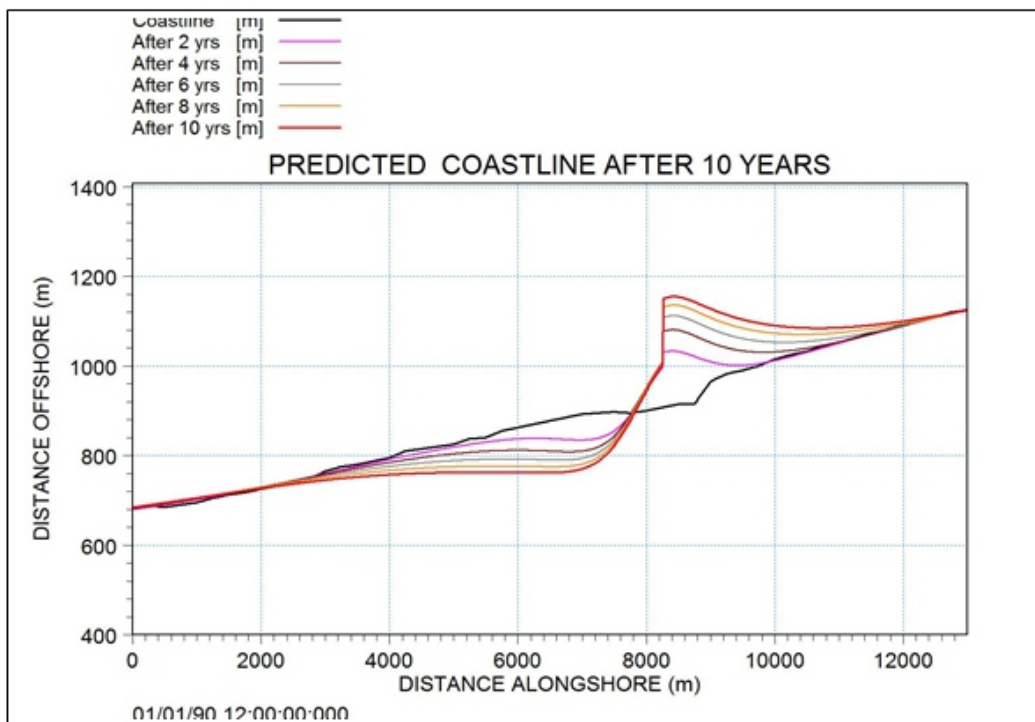
For the shoreline evolution model, a shoreline of 16.5 km length was considered. The length was divided into 1650 grid points with uniform grid size of 10 m. The proposed location of the extension of the existing groyne was schematically located in the middle of the coastline. The transport rates were computed for the above mentioned profile, which was used as input to the shoreline evolution model for LITLINE module of LITPACK software.



**Figure 3 Cross-shore distribution of littoral drift for annual period**

As mentioned earlier, the LNG terminal site was naturally prone to acute siltation due to high southward littoral drift in the area and a groyne of length 130 m was constructed in the year 2011 to arrest siltation. However, it was observed that during SW monsoon in the same year the shoreline advanced by about 70 m on the northern side of the groyne. In subsequent seasons, the shoreline further advanced by 20 to 25 m. The advancement of shoreline during SW monsoon by 70 m has been used to calibrate the LITLINE model. In order to calibrate the model, the existing groyne of 130 m length was considered. The model was simulated for SW monsoon period and it was found that the model results match well with the field data. The calibrated model was used to simulate for the proposed condition in which the existing groyne was extended further by 500 m. The model was run with the schematic layout of the extended groyne for a period of 10 years. The predicted shoreline change obtained from the model simulation is shown in Fig.4 . As the net transport is directed towards south, the deposition on the north side of the extended groyne is noticed while on the southern side erosion is observed, if the coastline is assumed to be straight. However, due to the presence of the approach channel in the Cochin Gut in the southern side of the extended groyne (500m), the coastal stretch has a breach. Thus the extension of the groyne would not change the conditions at the beach in front of the Fort Kochi. The effect of erosion will also not be felt in the LNG Terminal area as well as in the Port area. Furthermore, it is also proposed to provide a suitable protection along the shore in the vicinity of the Terminal area which would help in protecting this area from erosion.

The extension of groyne has been carried out at the site using sand-filled geo-textile tubes, which has shown positive results by arresting sand on the northern side of the groyne



**Figure 4 Shoreline evolution after 2<sup>nd</sup>, 4<sup>th</sup>, 6<sup>th</sup>, 8<sup>th</sup> and 10<sup>th</sup> year**



Further, model was simulated subsequently for a period of 2, 4, 6, 8 and 10 years and the maximum cross-shore advancement for each period would be obtained as 64 m, 105 m, 135 m, 155 and 175 m respectively from the prevailing coastline position. It may be noted that during the LITPACK model simulation, it has been assumed that the shoreline consisted of erodible material. Further, the corresponding longshore effect of deposition was felt for a length of about 4250 m. The maximum cross-shore advancement of the shoreline in 10 years period would be of the order of 175 m. Although, accretion on the northern side of the groyne is limited upto one third of the extended length (500 m) of the existing groyne, it would arrest only partial littoral drift as the surf zone extended upto 2.5 km during SW monsoon.

## 5. CONCLUSIONS

Mathematical model studies have been carried out to estimate littoral drift at the marine terminal and to examine the likely changes in the coastline due to the proposed extension. It was seen that the annual southward and northward transports would be 0.925 million cum and 0.265 million cum respectively with the net and the gross transports being 0.66 million cum and 1.19 million cum respectively. The net longshore transport is towards south. Also, the major southward transport occurs during SW monsoon. Further, based on the distribution of annual longshore transport, the transport is confined within a range of 3000 m from the shoreline and the southward transport is more than the northward transport. The peak transport occurs at about 200 m from the shoreline and about 75% transport occurs in the range of 50 m to 1000 m from shoreline i.e. between +0.5 m to - 2.5 m depth contours. As the net transport is directed towards south, the deposition is expected on the north side of the existing groyne. After a period of 2, 4, 6, 8 and 10 years, the maximum cross-shore advancement would be about 64 m, 105 m, 135 m, 155 and 175 m respectively from the prevailing coastline position. The corresponding longshore effect on northern side of the groyne would be felt upto 4250 m. In 10 years period, accretion on the northern side of the groyne is limited upto one third of the extended length of the groyne, but it would arrest only partial littoral drift as the surf zone would be extended up to 2.5 km especially during SW monsoon.

## ACKNOWLEDGEMENT:

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# Comparative Analysis of Different Categories of Anomaly Detection System.

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## **ABSTRACT**

*Intrusion detection is so much popular since the two decades where intrusion attempted to break into or misuse the system. This IDS system has ability to detect the virus, malware, spy ware & different form of viruses. IDS classifying the two categories one is structure based & another is according to detection techniques. The network based & host based are included in structure based IDS. The detection techniques are used to detect suspicious activity both at anomaly based and signature based. The general scheme of anomaly preprocessor & its types of categories are explained in this paper.*

**Keyword:-Anomaly, Anomaly detection techniques, common anomaly based IDS, Intrusion Detection System, Signature based IDS.**

## **1. INTRODUCTION**

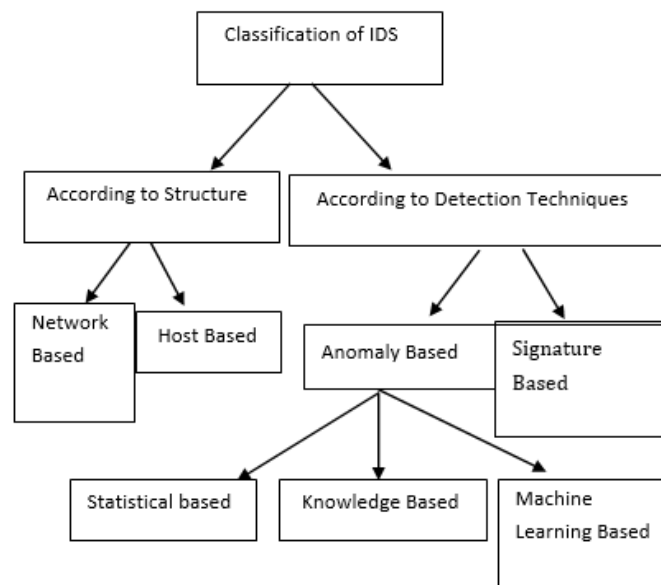
### **1.1. Intrusion Detection System**

Intrusion detection systems are security tools like other measures such as antivirus software, firewall etc. It proposed to improve computer security because it is not feasible to build completely secure systems [23]. In particular IDSs are used to identify, assess and report unauthorized network activities, so that appropriate actions may be taken to prevent any future damage. Intrusion detection systems are the „burglar alarms“ of the computer security field [3]. The aim is to defend a system by using a combination of an alarm that sounds whenever the site’s security has been compromised and an entity –most often a site security officer (SSO) that can respond to the alarm and take the action. There is often the need to update an installed Intrusion Detection System (IDS) due to new attack methods or upgraded computing environments. Since many current IDSs are constructed by manual encoding of expert knowledge, changes to IDSs are expensive and slow. Intrusion detection systems are classified into anomaly based or signature based. Signature based uses specifically known patterns of unauthorized behavior to predict and detect subsequent similar attempts [23]. It is also known as misuse detection. These specific patterns are called signature. On the other hand, anomaly based detectors attempt to estimate the “normal” behavior of the system to be protected and generate an anomaly alarm. Another possibility is to model the “abnormal” behavior of the system and to raise an alarm when the difference between the observed behavior and the expected one falls below a given limit.

## 1.2. Types of Intrusion Detection System

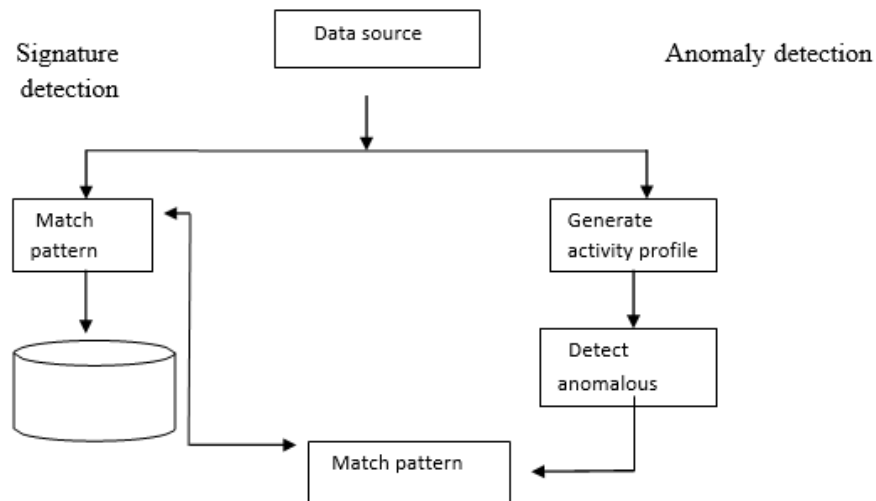
IDS can also be categorized according to the detection approaches they use. Basically, there are two detection methods: anomaly detection and signature detection [25]. The major difference between the two methods is that anomaly detection analyzes the properties of normal behavior while signature detection identifies intrusion based on features of known attacks. Signature based uses specifically known patterns of unauthorized behavior to predict and detect subsequent similar attempts [2]. It is also known as misuse detection. These specific patterns are called signature. On the other hand, anomaly based detectors attempt to estimate the “normal” behavior of the system to be protected and generate an anomaly alarm [20]. The following subsections explain the two detection approaches.

**Fig1. Classification of IDS**



### 1.2.1. Signature Detection

Any action that conforms to the pattern of a known attack is measured in it. The main issues in signature detection system are now to write a signature that encompasses all possible variations of attack. In contrast do not match non-intrusion activity [21]. It identifies intrusion by matching monitored events to patterns. These specific patterns are called signatures. For host-based intrusion detection one example of signature is “Three failed logins”. For network intrusion detection, a signature can be as simple as a specific pattern that matches a portion of a network packet. The major advantage is that the high accuracy in detecting known attacks. However, its detection ability is limited by signature database [25]. Otherwise new attacks are transformed into signature and added to database. Signature IDS cannot detect any attack of this. Different techniques such as expert system, signature analysis and state transition analysis are utilized in misuse detection.



**Fig2.General operation of IDS**

### 1.2.1 Anomaly Detection

It is based on the normal behavior of the subject. Any action that significantly deviates from the normal behavior is considered as intrusive. An example: if a user logs on and off of machine more than 10 times a day, instead of the normal 1 or 2. Also if a computer is used at 2:00 am when normally no one outside of business hours should have access, this should raise some suspicious [8, 2]. At another level anomaly detection can investigate user patterns such as profiling the program executed daily. If a user in the management department suddenly starts accessing programs or compiling code the system can properly alert its administrator [8].

## 2. RELATED WORK

Asmaa Shaker Ashore [2] examines the importance of Intrusion Detection Systems its categories and a classification .It also concludes that IDS is basically detects attacks signs and then alerts. In terms of performance, IDS becomes more accurate as it detects more attacks raises fewer positive alarms.

P.Garcia-Teodoro [23] discusses the main A-NIDS technologies, together with their general operational architecture and provides a classification for according to the type of processing related to the “behavioral model for the target system. The main features of several currently available IDS systems platforms.

J. Gomez [15] presents a new anomaly pre-processor that extends the functionality of Snort IDS, making it a hybrid IDS. It has been verified that when the number of elements increases it has less sensitivity and detect few attacks. Results also denote the importance of training the system during a long time to reduce the number of false alarms.

V. Jyothisna,"A [27] presents anomaly-based approaches are efficient, signature-based detection is preferred for mainstream implementation of intrusion detection systems. High detection rate of 98% at a low alarm rate of 1% can be achieved by using these techniques. It elaborates the main anomaly based network intrusion detection technologies along with their operational architectures and also presents a classification based on the type of processing that is related to the "behavioral" model for the target system.

Mueen Uddin [18] explains the IDS system is how to keep up with large volume of incoming traffic. When each packet needs to be compared with every signature in database. In contrast compare with anomaly detection technique. It introduced a new model of dynamic multilayer signature based IDS.

Augustine Soule [3] explains here how any anomaly detection method can be viewed as a problem in statistical hypothesis testing. It compares different method for analyzing residual. These methods focus on different aspects of the traffic pattern change.

Bar ford at al[6] presented a frame work for detecting and localizing performance anomalies based on using an active – probe-enabled measurement infrastructure deployed on periphery of a network. Their frame work has three components; an algorithm for detecting performed anomalies on a path, algorithm for selecting which paths to probe at a given time in order to detect performance anomalies and an algorithm for identifying the links that are causing an identified anomaly on path (i.e. localizing).

### 3. INTRODUCTION TO ANOMALY

#### 3.1 What is Anomaly

Anomalies can be treated as pattern not observed before. [23] Anomaly Detection refers to detection patterns in a given data set that do not conform to an established normal behavior. The patterns thus detected are called anomalies [16]. It translates to critical and actionable information in several application domains. Anomaly Intrusion detection system is ineffective in detecting insider attacks, an intrusion detection system that employs only one of that method will have a limited range of intrusions .The main benefit of anomaly-based detection techniques is their potential to detect previously unseen intrusion events.Fig3.Shows the general scheme of anomaly detection module .Using two different operation modes: training mode and anomaly detection mode. Using the training mode the system records in a database the network traffic considered as normal and expected. Both operation modes share the same functionality. When the pre-processor of Snort receive a package, it is classified according to its class. (if the package is primary/secondary and if the package belongs to a network server or a client) and

it stores the vector-class package, i.e. the system is recording and counting the network traffic. When the system is in training mode it stores the recorded information in the database and later it obtains a profile of the normal activity. The information store in the database is used when the system is in detection mode. Daily and each time the system is executed the activity profiles of the most active clients and servers in the network are loaded from the database. Therefore as the expected traffic is recorded in the database and compared with the real traffic passing through the network. If it is detected a deviation in the traffic higher than a certain percentage it means that something abnormal is happening and an incidence of abnormality is registered by the system

### 3.2 Anomaly Detection Categories

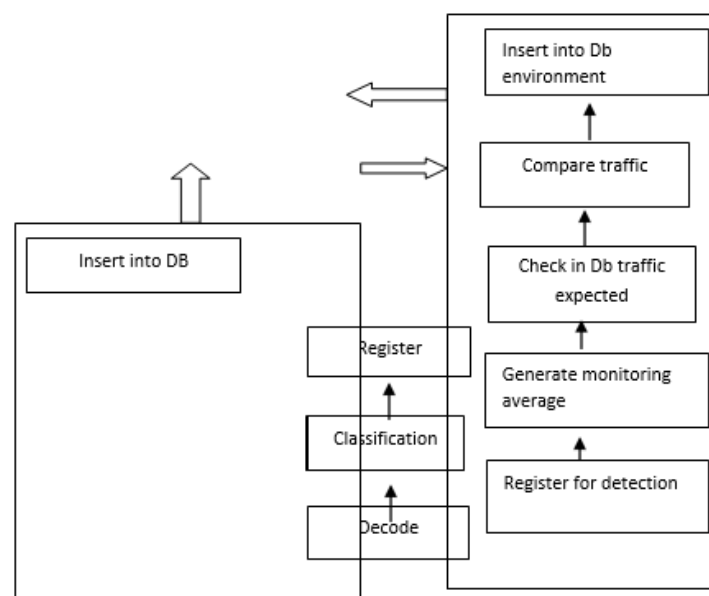
Anomaly detection is based on a host or network. Many different techniques are used based on type of processing related to behavioral model [23]. They are: Statistical based, knowledge model, Machine Learning based. [16]. (See Fig. 4)

#### 1) Statistical-based

The behavior of the system is represented from a random viewpoint. In statistical-based techniques, the network traffic activity is captured and a profile representing its behavior is created. This profile is based on metrics such as the traffic rate, the number of packets for each protocol, the rate of connections, the number of different IP addresses, etc. As the network events occur, the current profile is determined [21].

#### Operational Model (or) Threshold Metric

The count of events that occur over a period of time determines the alarm to be raised if fewer than  $m$  or more than  $n$  events occur. This can be visualized in Win2k lock, where a user after  $n$  unsuccessful login attempts here lower limit is „0“ and upper limit is  $n$ . Executable files size downloaded is restricted in some organizations about 4MB. The difficulty in this sub model is determining „ $m$ “ and „ $n$ “.



**Fig3. General Scheme of Anomaly Pre-processor**

### Markov Process or Marker Model

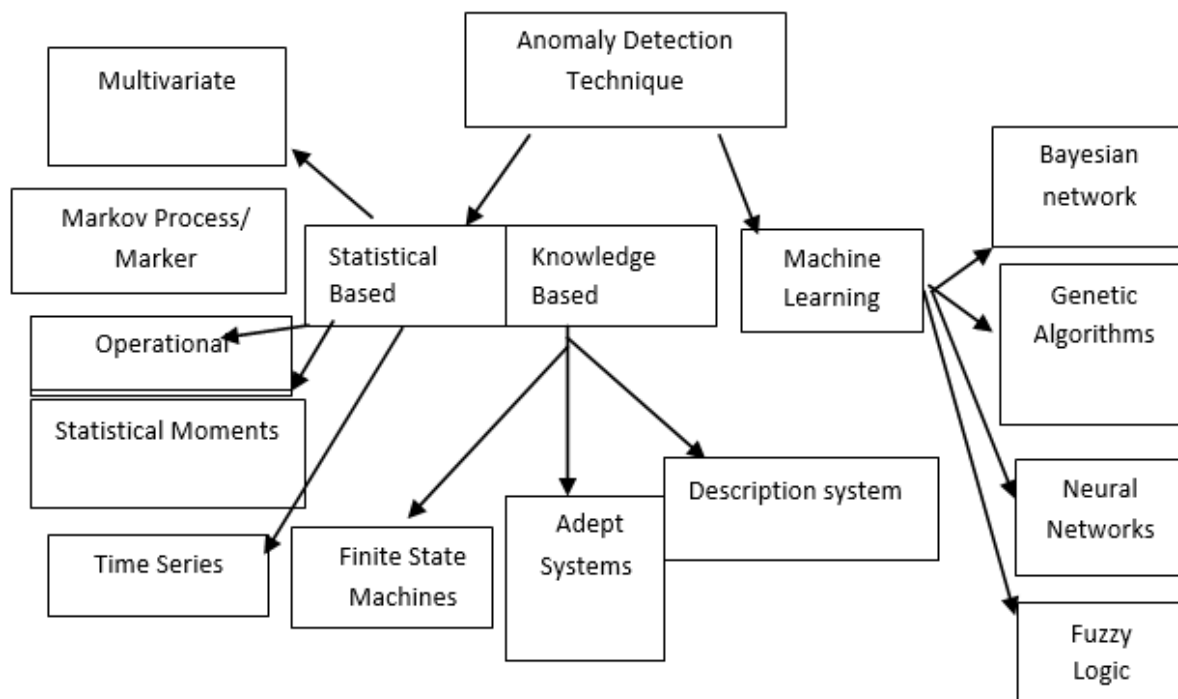
The Intrusion detection in this model is done by investigating the system at fixed intervals and keeping track of its state; a probability for each state at a given time interval. The change of the state of the system occurs when an event happens and the behavior is detected as anomaly if the probability of occurrence of that state is low.

### Statistical Moments

In statistical mean, standard deviation, or any other correlations are known as a moment. If the event that falls outside the set interval above or below the moment is said to be anomalous. The system is subjected to change by considering the aging data [13]. There is two major advantages over an operational model. First, prior knowledge is not required determining the normal activity in order to set limits; Second, determining the confidence intervals depends on observed. User data, as it varies from user to user. Threshold model lacks this flexibility. The major variation on the mean and standard deviation model is to give higher weights for the recent activities.

### Multivariate Model

The major difference between the mean and standard deviation model is based on correlations among two or more metrics. If experimental data reveals better judicious power can be achieved from combinations of related measures rather than treating them individually [13].



**Fig 4. Classification of Anomaly Based Intrusion Detection**

### Time Series Model

Interval timers together with an event counter or resource measure are major components in this model. Order and inter-arrival times of the observations as well as their values are stored [25]. If the probability of occurrence of a new observation is too low then it is considered as anomaly. The disadvantage of this model is that it is more computationally expensive.

## **2) Knowledge based**

Expert system approach is one of the most widely used knowledge-based IDS schemes. Expert systems are intended to classify the audit data according to a set of rules [23], involving three steps. First, different attributes and classes are identified from the training data. Second, a set of classification rules, parameters or procedures are deduced. Third, the audit data are classified accordingly. More restrictive/particular in some senses are specification-based anomaly methods, for which the desired model is manually constructed by a human expert, in terms of a set of rules (the specifications) that seek to determine legitimate system behavior. If the specifications are complete enough [3], the model will be able to detect illegitimate behavioral patterns. Moreover, the number of false positives is reduced, mainly because this kind of system avoids the problem of harmless activities, not previously observed, being reported as intrusion.

### Finite State Machine

A finite state machine (FSM) or finite automation is a model of behavior captured in states, transitions and actions [25]. A state contains information about the past, i.e. any changes in the input are noted and based on it transition happens [19]. An action is a description of an activity that is to be performed at a given moment. There are several action types: entry action, exit action, and transition action.

### Adept Systems

Human expertise in problem solving is used in adept systems. It solves uncertainties where generally one or more human experts are consulted. These systems are efficient in certain problem domain, and also considered as a class of artificial intelligence (AI) problems [13]. Adept Systems are trained based on extensive knowledge of patterns associated with known attacks provided by human experts.

### Description Scripts

Numerous proposals for scripting languages, which can describe signatures of attacks on computers and networks, are given by the Intrusion Detection community [13]. All of these scripting languages are capable of identifying the sequences of specific events that are indicative of attacks.

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### 3) Machine Learning Based Detection Techniques

Machine learning techniques are based on establishing an explicit or implicit model that enables the patterns analyzed to be categorized. A singular characteristic of these schemes is the need for labeled data to train the behavioral model, a procedure that places severe demands on resources. The main advantages are Flexibility and adaptability, Capture of interdependencies [21]. Their main drawback is High dependency on the assumption about the behavior accepted for the system. There has some sub types Markov models (stochastic Markov theory), Neural networks (human brain foundations), Fuzzy logic (approximation and uncertainty), Genetic algorithms (evolutionary biology inspired), Clustering and outlier detection (data grouping).

#### Bayesian networks

A Bayesian network is a model that encodes probabilistic relationships among variables of interest. This technique is generally used for intrusion detection in combination with statistical schemes, a procedure that has several advantages[15], including the capability of encoding interdependencies between variables and of predicting events, as well as the ability to incorporate both prior knowledge and data.

#### Genetic algorithms

They are a particular class of evolutionary algorithms that use techniques inspired by mutation, selection and recombination. These algorithms constitute another type of machine learning –based technique, capable of deriving classification rules and / or selecting features parameter for the detection process [16].The main advantage of this algorithm is the use of a flexible and robust global search method that coverage to a solution from multiple directions. The disadvantage is the resource consumption is involved in it.

#### Neural

Neural network have been adopted in the field of anomaly intrusion detection, mainly because of their flexibility and adaptability to environment changes. This detection approach has been employed to create user profiles [13]to identify the intrusive behavior of traffic patterns[8] etc. common characteristics is that they do not provide a descriptive model that why a particular detection a decision has been taken.

#### Fuzzy logic techniques

Fuzzy logic is derived from fuzzy. Fuzzy techniques are thus used in the field of anomaly detection mainly because the features to be considered can be seen as fuzzy variables.[7] This kind of processing schemes considers an observation as normal if it lie with in a given interval.[12]



## Clustering

It distances measure. The procedure most commonly used for this consists in selecting are representatives point for each cluster. Then, each new data point is classified as belonging to a given cluster according to the proximity to the corresponding representative point [16]. Some points may not belong to any cluster; these are representing the anomalies in the detection process [5].

## 4. CONCLUSION

The present paper discusses the foundations of main intrusion detection system technologies. All anomaly – based intrusion detection systems works on the assumption that normal activities differ from the abnormal activities (intrusions) substantially. It provides classification of intrusion detection system. The categories of anomaly technique are elaborate in it.

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