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# A Correlation Analyzer for Vestibular Disorders

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Paritala, Andhra Pradesh, India.

## ABSTRACT

*The vestibular system is the sensory apparatus of the inner ear that helps the body maintain its postural equilibrium. The information furnished by the vestibular system is also essential for coordinating the position of the head and the movement of the eyes. The vestibular system includes the parts of the inner ear and brain that process the sensory information involved with controlling balance and eye movements. If disease or injury damages these processing areas, vestibular disorders can result. Vestibular disorders can also result from or be worsened by genetic or environmental conditions, or occur for unknown reasons.*

*The severity of the problem can be identified with the knowledge based system that intern uses domain specific knowledge leads to develop an Expert system. This expert system will analyses the details of the patient through the software based questionnaire and checks the severity of the problem and suggests suitable cognitive and behavioral therapies to mitigate the problem of vertigo. The expert system also suggests suitable rehabilitation exercises and diet in considering usage, duration, precautions based on the case and the severity and develops a comprehensive response, which can be used as supplement to the doctor.*

**Keywords—***Positional Vertigo, dizziness, inflammation, cognitive therapy, expert system.*

## I. INTRODUCTION

The expert systems are developed to assist the experts in making decisions for domain specific problems. The major components of an expert system are knowledge base and the inference engine. The inference engine uses the domain knowledge that was already fed to the system as knowledge base and controls the questionnaire depending on the age group and gives the conclusion that specifies further treatment to be given. The proposed expert system is a rule based expert system that uses —if<antecedent> then<consequent>|| rules to work out with the answer given to the questionnaire. This clinical expert system for vertigo disorders concentrates mainly on 3 age groups. They are 1.Adults, 2.Middle aged and 3.Aged people. The person falls under the age group 21-45 is treated as adults and the age between 45 and 60 then it is treated as middle aged group and above 60 is treated as Senior citizen's group or aged group. There will be different sequence of questions for different age groups. Depending upon the age group separate questionnaire is prepared. So the proposed system will estimate the problem with the help of these questions and the problem severity will be estimated. Depending on the data provided the systems will generate a comprehensive report stating the severity of the problem. The severe cases will be sent to the respective doctors for the further diagnosis. This information of the doctors can be seen in the links of expert system. The cases with moderate severity will be suggested with the needed cognitive and behavioral therapies and if the problem is not solved then cases will sent to the doctors. The mild cases are also suggested with the rehabilitation exercises and diet control. The videos that help in daily exercises can be seen in the system.

## **II. RELATED WORK**

### **SYMPTOMS OF POSITIONAL VERTIGO**

The symptoms of positional vertigo include the following. They are sudden dizziness that lasts less than a minute, Feeling of spinning, Dizziness with certain movements, Loss of balance, Nausea, Vomiting, Feeling unsteady, Fatigue, Swaying. The symptoms may last for one minute or two minutes and they may last for weeks, months and even for years in rarest of the rare cases

### **CAUSES OF POSITIONAL VERTIGO**

The inner ear contains tiny crystals. These crystals can sense movement and help you keep your balance. BPPV occurs because of a shift in location of these crystals. When these crystals move to the wrong location or clump in one spot, BPPV can result.

#### **BPPV may be caused by:**

Head injury, Viral infection, such as Labyrinthitis (infection of the nerve to the ear), Disorders of the inner ear, Prolonged immobility of the head, Age-related changes to inner ear

### **TESTS FOR POSITIONAL VERTIGO**

To diagnose benign positional vertigo, the health care provider will often perform a test called the Dix-Hallpike maneuver. The doctor holds your head in a certain position and asks you to lie quickly backward over a table. As you do this, the doctor will look for abnormal eye movements and ask if you feel a spinning sensation. The doctor may use various methods to help evaluate your eye movements A physical exam is otherwise normal. A complete medical history and careful neurological exam should be done to rule out other reasons for your symptoms. Tests that may be done include EEG, Electronystagmography, Head CT, Head MRI, Magnetic resonance angiography of the head, Warming and cooling the inner ear with water (caloric stimulation) or air to test eye movements.

### **DIET RECOMMENDATIONS FOR POSITIONAL VERTIGO**

#### **FOODS TO AVOID**

Foods high in sodium, such as lunch meats, canned soups, frozen foods and other processed foods have high sodium content and can therefore be dehydrating, according to the Mayo Clinic. If you are curious as to how much sodium intake is appropriate for you, you should consult a dietitian or physician to determine an appropriate level. You should also avoid foods known to trigger migraines, as these contain high levels of tyramine, which can cause dizziness. These foods and drinks include red wine, chicken livers, smoked meats, chocolate, citrus fruits, Brie, cheddar cheese and nuts

#### **FOODS TO CHOOSE**

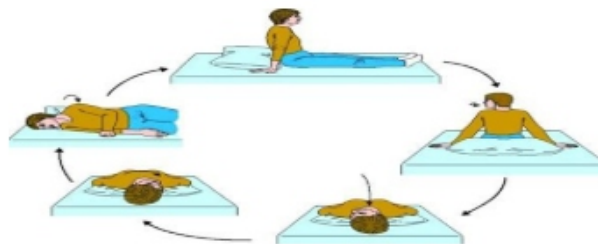
Instead of foods high in sugar, choose those high in complex sugar, such as whole-grain breads and legumes. Also, you should try the following to lower your salt intake while cooking by substituting salt-free seasonings or low-sodium vinegar, lemon juice or herbs. Choose fresh fruits and vegetables instead of canned whenever possible, Avoid processed foods or choose those with less than 35 mg sodium per serving. Opt for reduced-salt or reduced-sodium versions of products you enjoy whenever possible. As a general rule, consume 2,000 mg or less of sodium per day, according to the American Academy of Family Physicians.

## REMEDIAL THERAPIES

The positional vertigo disorders are sometimes treated in wait-and-see manner because it will be cured on its own. However there are some therapies that help in mitigating the positional vertigo disorders. They are Epley maneuver and the Brandt-Daroff exercise.

### EPLEY MANEUVER EXERCISE

This maneuvers involve a series of specifically patterned head and trunk movements performed by a trained professional who closely watches eye movements with each position change. This maneuver can be performed in the doctor's office with medication such as diazepam to help block nausea during the procedure, which takes about 15 minutes to accomplish. Precautions are provided for the days immediately following the procedure in order to ensure that the canaliths don't have the opportunity to return to their formerly problematic location in one of the semi-circular canals.

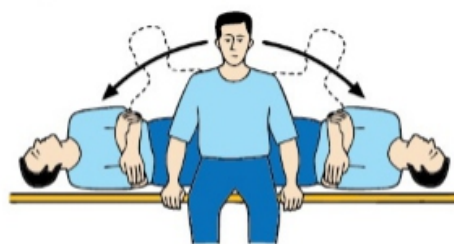


**Fig: Steps involved in the epley maneuver exercise**

### BRANDT-DAROFF EXERCISE.

These exercises are designed to break up this material and unblock the canal. Sitting on the edge of a bed and turning head slightly to the left (approximately 45 degrees). While maintaining this head position, lie down quickly on right side so that the back of your head is resting on the bed. Wait for 20 to 30 seconds or for any giddiness to resolve. Sit up straight, and again wait for 20 to 30 seconds or for any giddiness to resolve. Turn the head slightly to the right and repeat the sequence in the opposite direction. Continue this process for 10 minutes (5 or more repetitions to each side). Perform the exercises 3 times daily if possible.

The symptoms of giddiness need to be reproduced by the exercises if any benefit is to occur. If the exercises are done regularly, the symptoms should resolve over a period of several days in most cases.



**Fig: Steps involved in the brandt daroff exercise**

## III. OBJECTIVES OF THE PROPOSED SYSTEM

The proposed clinical expert system is to fulfill the following objectives.

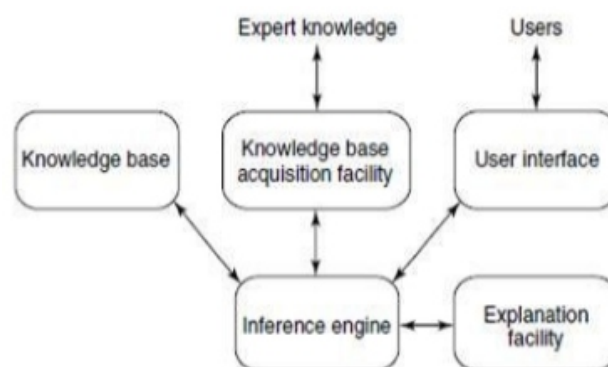
- To develop an expert system with domain specific knowledge that interacts with the users. To collect and to create, maintain the medical records of the patient's consulted.
- To estimate the severity of the problem based on the symptoms expressed.

- To give the relevant data and information of doctors for particular solutions for the vertigo disorders,
- To assist doctors in various symptoms and providing information that helps the doctor to make a decision concerning a patient's health.
- To suggest various cognitive, behavioral therapies and rehabilitation exercises.

### WORKING OF THE PROPOSED SYSTEM

The proposed clinical expert system works with the help of domain specific knowledge which is used in the process of analyzing the severity of the problem. The system which is interactive in nature first collects the data with the help of software based questionnaire which contains both the personal data and the information about the problem and the symptoms. The collected data will be analyzed with help of knowledge fed to the system and generates a comprehensive report that specifies the severity of the problem as well as the suitable therapies that are helpful in solving the problem. The report also contains the rehabilitation exercises that help to mitigate the problem. The data that was collected from the patients will be saved and maintained in the database for the further investigation of the clinical treatment of the patient.

### ARCHITECTURE OF AN EXPERT SYSTEM



The purpose of the inference engine is to seek information and relationships from the knowledge base and to provide answers, predictions, and suggestions in the way a human expert would. The inference engine must find the right facts, interpretations, and rules and assemble them correctly. Two types of inference methods are commonly used – Backward chaining is the process of starting with conclusions and working backward to the supporting facts. Forward chaining starts with the facts and works forward to the conclusions. The explanation facility allows a user to understand how the expert system arrived at certain results. The overall purpose of the knowledge acquisition facility is to provide a convenient and efficient means for capturing and storing all components of the knowledge base. Very often specialized user interface software is used for designing, updating, and using expert systems. The purpose of the user interface is to ease use of the expert system for developers, users, and administrators.

### THE STAKEHOLDERS

The expert systems are developed mainly for four different groups of users. They are 1. Clinicians, 2. Medical researchers, 3. Patients and 4. Administrators. Clinicians could include physicians (including junior physicians) and nurses to use their knowledge and expert systems to make decisions on behalf of others by, for example, diagnosing diseases or evaluating drug interactions from a treatment or adopting a nursing strategy to alleviate pain. General research has shown that expert physicians do make use of explanation facilities, but their requirements are very different to that of other users. Experts tend to use



feedback rule- trace explanations and are more likely—than no experts—to use explanations for resolving anomalies, such as disagreements with system advice, exploring alternative diagnoses, and verification of assumptions. However, no experts such as trainee physicians are more likely to use arrange of explanations types for short- and long-term learning. Administrators do not have direct clinical interaction with patients but are responsible for the management of health care options and facilities. They may therefore use expert systems to manage health care options and could benefit from explanation facilities by aiding clinicians, through, for example, the managing of patient referrals by finding out whether a patient is suitable for a referral. All of the explanation types discussed previously may benefit administrators depending on the nature of the application task. Furthermore, administrators may use expert system for patient care.

#### IV. CONCLUSION

The vertigo disorders are very common in aged people. But in the case of adults it is very rare and if it arises it can be mitigated through some simple exercises and some cognitive, behavioral therapies. This system helps in estimating the severity of the problem and also suggests the needed clinical aid to mitigate the problem. The expert system gives the list of doctors who treats for these types of disorders. This expert system also suggests suitable therapies and exercises.

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# Observations on the Hyperbola $y^2 = 10x^2 + 6$

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## ABSTRACT

The binary quadratic equation  $y^2 = 10x^2 + 6$  representing hyperbola is considered and a few interesting properties among the solutions are presented. Employing the integral solutions of the equation under consideration, a few special Pythagorean triangle each of which satisfying certain relations among its sides, area and perimeter, are obtained.

**Keywords:** binary quadratic, hyperbola, integral points, Pell equation. 2010 Mathematics subject classification No: 11D09

## INTRODUCTION

$y^2 = Dx^2 + 1$  where  $D$  is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when  $D$  takes different integral values [1, 2, 3, 4]. In [6] infinitely many Pythagorean triangles in each of which hypotenuse is four times the product of the generators added with unity are obtained by employing the non-integral solutions of binary quadratic equation  $y^2 = 3x^2 + 1$ . In [7], a special Pythagorean triangle is obtained by employing the integral solutions of  $y^2 = 10x^2 + 1$ . In [13], different patterns of infinitely many Pythagorean triangles are obtained by employing the non-integral solutions of  $y^2 = 12x^2 + 1$ . In this context one may also refer [8 – 16]. These results have motivated us to search for the integral solutions of yet another binary quadratic equation  $y^2 = 10x^2 + 6$  representing a hyperbola and the corresponding properties.

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## Notations Used:

$tm_n$  = Polygonal number of rank  $n$  with size  $m$ .

$P_{nm}$  = Pyramidal number of rank  $n$  with size  $m$ .

$nCP_m$  = Centered pyramidal number or rank  $n$  with size  $m$ .

$CP_{m,n}$  = Centered polygonal number or rank  $n$  with size  $m$ .

$GNO_n$  = Gnomonic number of rank  $n$ .

$Sn$  = Star number of rank  $n$ .

## METHOD OF ANALYSIS:

The binary quadratic equation representing hyperbola under consideration is

$$y^2 = 10x^2 + 6 \quad (1)$$

whose smallest positive integer solution is  $x_0 = 1, y_0 = 4$ .

To obtain the other solutions of (1), consider the Pellian equation  $y^2 = 10x^2 + 1$  whose general solution  $(\overline{x_n}, \overline{y_n})$  is given by

$$\overline{x_n} = \frac{g}{2\sqrt{10}} \text{ and } \overline{y_n} = \frac{f}{2} \text{ in which}$$

$$f = (19 + 6\sqrt{10})^{n+1} + (19 - 6\sqrt{10})^{n+1} \text{ and } g = (19 + 6\sqrt{10})^{n+1} - (19 - 6\sqrt{10})^{n+1}, n = -1, 0, 1, 2, 3, \dots$$

Applying Brahmagupta lemma between the solutions of  $(x_0, y_0)$  and  $(x_n, y_n)$  the general solution of (1) is found to be

$$x_{n+1} = \frac{f}{2} \pm \frac{2g}{\sqrt{10}} \tag{2}$$

$$y_{n+1} = 2f \pm \frac{5g}{\sqrt{10}} \tag{3}$$

where  $n = -1, 0, 1, 2, 3, \dots$

Thus, (2) and (3) represent non-zero distinct integral solutions of hyperbola (1).

The recurrence relations satisfied by  $x$  and  $y$  are given by

$$x_{n+3} = 38x_{n+2} - x_{n+1}, \quad x_0 = 1, x_1 = 43$$

$$y_{n+3} = 38y_{n+2} - y_{n+1}, \quad y_0 = 4, y_1 = 136$$

Taking the positive signs in (2) and (3), some numerical examples of  $x$  and  $y$  satisfying (1) are given in the following table:

<b>n</b>	<b>x<sub>n</sub></b>	<b>y<sub>n</sub></b>
0	1	4
1	43	136
2	1633	5164
3	62011	196096
4	2354785	7446484
5	89429819	282770296

Taking the negative signs in (2) and (3), some numerical examples are represented below:

<b>n</b>	<b>x<sub>n</sub></b>	<b>y<sub>n</sub></b>
0	1	4
1	-5	16
2	-191	604
3	-7253	22936
4	-275423	870964
5	-10458821	33073696

From the above table, it is seen that the values of  $x_n$  are odd whereas that of  $y_n$  are even and  $\equiv 0 \pmod{4}$

Also, a few interesting properties between the solutions and special numbers are given below:

$$1. S_f = 4[(2y_{2n+2} - 5x_{2n+2}) - (2y_{n+1} - 5x_{n+1})] + 13$$

$$2. 3GNO_f = 4(2y_{n+1} - 5x_{n+1}) - 3$$

$$3. 2t_{m,f} = \frac{2}{3} \{ (m-2)[2y_{2n+2} - 5x_{2n+2}] - (m-4)[2y_{n+1} - 5x_{n+1}] \} + 2(m-2), m \geq 3$$

$$4. 6P_f^m = \frac{2}{3} \{ (m-2)(2y_{3n+3} - 5x_{3n+3}) + 3(2y_{2n+2} - 5x_{2n+2}) + (2m-1)(2y_{n+1} - 5x_{n+1} + 1) \} + 6, m \geq 3$$

$$5. 6CP_f^m = \frac{2}{3} \{ m(2y_{3n+3} - 5x_{3n+3}) + 2(m+3)(2y_{n+1} - 5x_{n+1}) \}, m \geq 3$$

6.

$$2CP_{m,f} = \frac{2}{3} \{ m[(m-2)(2y_{2n+2} - 5x_{2n+2}) + m(4-m)(2y_{n+1} - 5x_{n+1})] + 2m(m-2) + 2 \}, m \geq 3$$

$$7. \frac{2}{3}[2y_{2n+2} - 5x_{2n+2}] + 2 \text{ is a Perfect square.}$$

$$8. \frac{2}{3}[2y_{3n+3} - 5x_{3n+3}] + 2[2y_{n+1} - 5x_{n+1}] \text{ is a Cubic integer.}$$

$$9. 4[2y_{2n+2} - 5x_{2n+2}] + 12 \text{ is a Nasty number.}$$

**Remarkable Observations:**

1. Let N be any non-zero positive integer write  $N = \frac{x_{n+1}-1}{2}$ .

Treating N as the rank of the triangular number  $t_{3,N}$  it is observed that  $80t_{3,N} + 16$  is a Perfect square.

2. Let m and n be any two non-zero distinct positive integers such that  $m = x_{s+1} + y_{s+1}$  and  $n = x_{s+1}$ . Treat m and n as the generators of the Pythagorean triangle  $T(\alpha, \beta, \gamma)$  where  $\alpha = 2mn, \beta = m^2 - n^2, \gamma = m^2 + n^2, m > n > 0$ . Then the Pythagorean triangle T satisfies the relations

$$(i) 5\beta - 4\gamma - \alpha = 6$$

$$(ii) \gamma - \frac{4A}{P} = 5(\gamma - \beta) + 6$$

$$(iii) 20\frac{A}{P} + \gamma = 6(\alpha + 1)$$

$$(iv) (\gamma - \alpha) = 10t_{4,x} + 6$$

where A and P represent the Area and Perimeter of the Pythagorean

Triangle T

3. Employing the solutions (x, y) of (1), following relations among the special polygonal and Pyramidal numbers are obtained:

$$(i) \quad \left| \frac{3P^3 - 3}{y-2} \right| = 10 \left| \frac{P^5}{x} \right| + 6$$

$$\left| \frac{3P^3 - 3}{y-2} \right| = 10 \left| \frac{2P^5 - 1}{x-1} \right| + 6$$

## CONCLUSION:

In this paper, we have presented non-zero distinct integer solutions of the hyperbola  $y^2 = 10x^2 + 6$  and employing these solutions, a special Pythagorean triangle has been obtained. As the binary quadratic equations are rich in variety due to their definition, one may search for other choices of hyperbola for their corresponding pattern of solutions along with the properties involving special numbers.

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# Common Fixed Point Theorems in Complex Valued Metric Space using Implicit Relation

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## ABSTRACT

*Banach contraction principle in gives appropriate and simple conditions to establish the existence and uniqueness of a solution of an operator equation  $Tx = x$ . Later, a number of papers were devoted to the improvement and generalization of that result. Most of these results deal with the generalizations of the different contractive conditions in metric spaces. The aim of this paper is to prove the existence and uniqueness of a common fixed point for a pair of mappings satisfying occasionally weakly compatible maps in complex valued metric space using implicit relations. The obtained results generalize and extend some of the well-known results in the literature.*

**Keywords:** *Complex metric space, weakly compatible, occasionally weakly compatible, implicit relation.*

## INTRODUCTION:

Azam et al.[1] introduced the notion of complex valued metric spaces and established some fixed point results for a pair of mappings for contraction condition satisfying a rational expression. Though complex valued metric spaces from a special class of cone metric space, yet this idea is intended to define rational expressions which are not meaningful in cone metric spaces and thus many results of analysis cannot be generalized to cone metric spaces. Indeed the definition of a cone metric space banks on the underlying Banach space which is not a division ring. However, in complex valued metric spaces, one can study improvements of a host of result of analysis involving division. One can refer related results in [3, 7]. Jungck generalized the concept of weak commuting mapping given by Sessa [12], by introducing the concept of compatible mapping in different. Many authors [2, 5, 6] proved fixed point theorems for compatible mappings in different types.

In this paper, we introduced some new common fixed point theorems for generalized contractive maps in complex-valued metric space by using these new properties.

For the sake of completeness, we recall some definitions and known results in complex valued metric space.

## BASIC DEFINATIONS AND PRELIMINARIES

An ordinary metric  $d$  is a real-valued function from a set  $X \times X$  into  $\mathbb{R}$ , where  $X$  is a non-empty set. That is  $\rho : X \times X \rightarrow \mathbb{R}$ . A Complex number  $z \in \mathbb{C}$  is an ordered pair of real number, whose first co-ordinate is called,  $\text{Re}(z)$  and second co-ordinate is  $\text{Im}(z)$ . Thus a complex-valued metric  $d$  would be a function from a set  $X \times X$  into  $\mathbb{C}$ , where  $X$  is a non-empty set and  $\mathbb{C}$  is the set of complex number. That is  $\rho: X \times X \rightarrow \mathbb{C}$ .

Suppose  $\mathbb{C}$  be the set of complex numbers throughout this section and  $z_1, z_2 \in \mathbb{C}$ , recall a natural partial order relation  $\preceq$  on  $\mathbb{C}$  as follows:  $z_1 \preceq z_2$  if and only if  $\operatorname{Re}(z_1) \leq \operatorname{Re}(z_2)$  and  $\operatorname{Im}(z_1) \leq \operatorname{Im}(z_2)$ . Consequently, one can infer that  $z_1 \preceq z_2$  if one of the following conditions is satisfied:

- (i)  $\operatorname{Re}(z_1) = \operatorname{Re}(z_2), \operatorname{Im}(z_1) < \operatorname{Im}(z_2)$
- (ii)  $\operatorname{Re}(z_1) < \operatorname{Re}(z_2), \operatorname{Im}(z_1) = \operatorname{Im}(z_2)$
- (iii)  $\operatorname{Re}(z_1) < \operatorname{Re}(z_2), \operatorname{Im}(z_1) < \operatorname{Im}(z_2)$
- (iv)  $\operatorname{Re}(z_1) = \operatorname{Re}(z_2), \operatorname{Im}(z_1) = \operatorname{Im}(z_2)$

In particular, we write  $z_1$  not  $\preceq z_2$  if  $z_1 \neq z_2$  and one of (i), (ii), and (iii) is satisfied and we write  $z_1 \prec z_2$  if only (iii) is satisfied. Notice that  $0 \preceq z_1$  not  $\preceq z_2 \Rightarrow |z_1| < |z_2|$ , and  $z_1 \preceq z_2, z_2 \prec z_3 \Rightarrow z_1 \prec z_3$ .

**Definition 2.1. [1].** Let  $X$  be a nonempty set, whereas  $\mathbb{C}$  be the set of complex numbers. Suppose that the mapping  $d: X \times X \rightarrow \mathbb{C}$  satisfies the following conditions:

- (C<sub>1</sub>)  $0 \preceq (x, y)$  for all  $x, y \in X$  and  $\rho(x, y) = 0$  if and only if  $x = y$ ;
- (C<sub>2</sub>)  $\rho(x, y) = \rho(y, x)$  for all  $x, y \in X$ ;
- (C<sub>3</sub>)  $\rho(x, y) \preceq \rho(x, z) + \rho(z, y)$  for all  $x, y, z \in X$ .

Then  $\rho$  is called a complex-valued metric on  $X$ , and  $(X, \rho)$  is called a complex-valued metric space.

**Example 2.1.** Define complex valued metric  $\rho: X \times X \rightarrow \mathbb{C}$  by  $(z_1, z_2) = e^{3i}|z_1, z_2|$ . Then  $(X, \rho)$  is a complex valued metric space.

**Definition 2.2. [1]** Let  $(X, \rho)$  be a complex valued metric space and  $B \subseteq X$ .

- (i)  $b \in B$  is called an interior point of a set  $B$  whenever there is  $0 \prec r \in \mathbb{C}$  such that  $N(b, r) \subseteq B$ , where  $N(b, r) = \{y \in X: \rho(b, y) \prec r\}$ .
- (ii) A point  $x \in X$  is called a limit point of  $B$  whenever for every  $0 \prec r \in \mathbb{C}$ ,  $N(x, r) \cap (B \setminus \{x\}) \neq \emptyset$ .

(iii) A subset  $A \subseteq X$  is called open whenever each element of  $A$  is an interior point of  $A$  whereas a subset  $B \subseteq X$  is called closed whenever each limit point of  $B$  belongs to  $B$ . The family  $\mathcal{F} = \{N(x, r): x \in X, 0 \prec r\}$  is a sub-basis for a topology on  $X$ . We denote this complex topology by  $\tau_c$ . Indeed, the topology  $\tau_c$  is Hausdorff.

**Definition 2.3. [1]** Let  $(X, d)$  complex-valued metric space and  $x \in X$ . Then sequence  $\{x_n\}$  in  $X$  is

- (i) Convergent if  $\{x_n\}$  converges to  $x$  and  $x$  is the limit point of  $\{x_n\}$ , if for every  $0 \prec c \in \mathbb{C}$ , there is a natural number  $N$  such that  $\rho(x_n, x) \prec c$ , for all  $n > N$ . We denote it by  $\lim_{n \rightarrow \infty} x_n = x$ .
- (ii) a Cauchy sequence, if for every  $c \in \mathbb{C}$ , with  $0 \prec c$  there is a natural number  $N$  such that  $\rho(x_n, x_m) \prec c$ , for all  $n, m > N$ .



- (iii) The metric space  $(X, \rho)$  is a complete complex valued metric space if every Cauchy sequence is convergent.

In a metric space, every convergent sequence is a Cauchy sequence but the converse is not true. For instance, Euclidean  $n$ -space with the Euclidean distance is complete metric space where as the set of rational numbers with metric  $\rho(x, y) = |x - y|$  is not a complete metric space.

In 1968, Jungck [8] defined the concept of compatible mappings which is more general than that of commuting and weakly commuting mappings.

**Definition 2.4.**[9] A pair  $(f, g)$  of self-mappings of a metric space  $(X, \rho)$  into itself, is called compatible mapping if  $\lim_{n \rightarrow \infty} \rho(fg x_n, gf x_n) = 0$  whenever  $\{x_n\}$  is a sequence in  $X$  such that  $\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g x_n = z$  for some  $z \in X$ .

**Example 2.2** Let  $X = [0, \infty)$  be endowed with usual metric  $d$  and  $f, g : X \rightarrow X$  such that  $fx = x^3$  and  $gx = 2x^3$ . Then  $fgx \neq gfx$ . So,  $f$  and  $g$  are not commuting on  $X$  and  $|fgx - gfx| > |fx - gx|$ . Therefore,  $f$  and  $g$  are not weakly commuting on  $X$ . Also, for any sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g x_n = u \in X$  then  $\lim_{n \rightarrow \infty} (fg x_n, gf x_n) = \lim_{n \rightarrow \infty} |fg x_n - gf x_n| = 0$ . Therefore,  $f$  and  $g$  are compatible.

In 1998, Jungck and Rhoades [10] introduced the notion of weakly compatible mappings which is more general than that of compatibility as follows:

**Definition 2.5.**[10] A pair  $(f, g)$  of self-mappings of a metric space  $(X, \rho)$  into itself, is called weakly compatible mapping if they commute at all of their coincidence point i.e.  $fx = gx$  for some  $x \in X$  implies  $fgx = gfx$ . Also, compatible mapping are weakly compatible but converse is not true.

**Example 2.3.** Define complex-valued metric  $\rho : X \times X \rightarrow \mathbb{C}$  by such that  $\rho(z_1, z_2) = e^{ia} |z_1 - z_2|$  where  $a$  is any real constant. Then  $(X, \rho)$  is a complex valued metric space. Suppose self-maps  $A$  and  $S$  be defined as:

$$Az = 2e^{i\pi/4} \text{ if } \operatorname{Re}(z) \neq 0, \quad Az = 3e^{i\pi/3} \text{ if } \operatorname{Re}(z) = 0, \text{ and}$$

$$Sz = 2e^{i\pi/4} \text{ if } \operatorname{Re}(z) \neq 0, \quad Sz = 4e^{i\pi/3} \text{ if } \operatorname{Re}(z) = 0,$$

Then maps  $A$  and  $S$  are weakly compatible at all  $z \in \mathbb{C}$  with  $\operatorname{Re}(z) \neq 0$ .

In 2008, Al Thagafi and Shahzad [2] introduced the concept of occasionally weakly compatible (owc) mappings which is a proper generalization of weakly compatible mappings.

**Definition 2.6.**[2]. Two self mappings  $f$  and  $g$  of a complex-valued metric space  $(X, \rho)$  are said to be occasionally weakly compatible (owc) if there is a point  $x$  in  $X$  which is a coincidence point of  $f$  and  $g$  at which  $f$  and  $g$  commute.

**Definition 2.6.[2].** Two self mappings  $f$  and  $g$  of a complex  $\rho$ -valued metric space  $(X, \rho)$  are said to be occasionally weakly compatible (owc) if there is a point  $x$  in  $X$  which is a coincidence point of  $f$  and  $g$  at which  $f$  and  $g$  commute.

**Example 2.4.** Let  $X = [0, \infty)$  with usual metric. Define  $f, g: X \rightarrow X$  by  $fx = 2x$  and  $gx = x^2$ , for all  $x \in X$ . Then  $fx = gx$  at  $x=0, 2$  but  $fg(0) = gf(0)$  and  $fg(2) \neq gf(2)$ . Therefore, mappings  $f$  and  $g$  are occasionally weakly compatible but not weakly compatible.

**Definition 2.7.[11]** A pair  $(f, g)$  of self-mappings of a metric space  $(X, \rho)$  is said to satisfy property (E.A), if there exists a sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z$  for some  $z \in X$ .

**Example 2.5.** Let  $X = [0, \infty)$ . Define  $f, g: X \rightarrow X$  by  $fx = \frac{2x}{4}$  and  $gx = \frac{5x}{4}$ , for all  $x \in X$ . Consider the sequence  $\{x_n\} = \frac{2}{n}$  clearly,  $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = 0 \in X$ . Then  $f$  and  $g$  satisfy property (E.A).

**Example 2.6.** Let  $X = \mathbb{C}$  and  $d$  be any complex valued metric. Define self maps  $A$  and  $S$  by  $Az = 1-z$  and  $Sz = 1+z$ , for all  $z \in X$ . Consider a sequence in  $X$  as  $\{x_n\} = \{1/n\}$  where  $n = 1, 2, 3, \dots$  then  $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = 0$ .

Hence the pair  $(A, S)$  satisfies property (E.A) for the sequences  $\{x_n\}$  in  $X$ .

**Definition 2.8.[11].** Two pairs of self-maps  $(A, S)$  and  $(B, T)$  on a complex valued metric space  $(X, \rho)$  Satisfies common property (E.A) if there exists two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that

$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ty_n = \lim_{n \rightarrow \infty} By_n = p$  for some  $p \in X$ .

**Definition 2.1.9.[11].** Two finite families of self maps  $\{A\}_{i=1}^m$  and  $\{B\}_{j=1}^m$  on a set  $X$  are pair

wise commuting if

$$(i) \quad A_i A_j = A_j A_i, \quad i, j \in \{1, 2, 3, \dots, m\},$$

$$(ii) \quad B_i B_j = B_j B_i, \quad i, j \in \{1, 2, 3, \dots, n\},$$

Implicit relations play important role in establishing of common fixed point results.

Let  $M_\phi$  be the set of all continuous functions satisfying the following conditions:

$$(A) \quad \phi(u, 0, u, 0, 0, u) \leq 0 \implies u \leq 0$$

$$(B) \quad \phi(u, 0, 0, u, u, 0) \leq 0 \implies u \leq 0$$

$$(C) \quad \phi(u, u, 0, 0, u, u) \leq 0 \implies u \leq 0 \text{ for all } 0 \leq u.$$

**Example 3.1.** Define  $\emptyset: (C)^6 \rightarrow C$  as  $\emptyset(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \emptyset_1(\min\{t_2, t_3, t_4, t_5, t_6\})$ , where  $\emptyset_1: C \rightarrow C$  is increasing and continuous function such that  $\emptyset_1(s) > s$  for all  $s \in C$ , clearly,  $\emptyset$  satisfies all conditions (A), (B) and (C). Therefore,  $\emptyset \in M_6$ .

Our main theorem runs as follows.

**Theorem 3.1.1.** Let  $A, B, S, T, P$  and  $Q$  be six self mappings of a complex-valued metric space  $(X, \rho)$  satisfying the following conditions:

- (i)  $(X) \subseteq AB(X), Q(X) \subseteq ST(X)$ ,
- (ii) The pair  $(P, AB)$  and  $(Q, ST)$  share the common (E.A) property.
- (iii) For any  $x, y \in X, \emptyset$  in  $M_6$ .

$$\emptyset \left\{ \begin{array}{l} (Px, Qy), \rho(ABx, STy), \rho(ABx, Qy), \\ (STy, Px), \rho(ABx, Px), \rho(STy, Qy) \end{array} \right\} \leq 0$$

- (iv)  $AB = BA, ST = TS, PB = BP, SQ = QS, QT = TQ$ .

Then the pair  $(P, AB)$  and  $(Q, ST)$  have a point of coincidence each. Moreover  $A, B, S, T, P$  and  $Q$  have a unique common fixed point provided both the pairs  $(P, AB)$  and  $(Q, ST)$  are occasionally weakly compatible.

**Proof.** In view of (ii), there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Px_n = \lim_{n \rightarrow \infty} ABx_n = \lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} STy_n = z \text{ for some } z \in X.$$

since  $P(X) \subset AB(X)$ , there exist a point  $u \in X$  such that  $ABu = z$ .

Put  $x = u$  and  $y = y_n$  in (iii), we have

$$\emptyset \left\{ \begin{array}{l} (Pu, Qy_n), \rho(ABu, STy_n), \rho(ABu, Qy_n), \\ (STy_n, Pu), \rho(ABu, Pu), \rho(STy_n, Qy_n) \end{array} \right\} \leq 0$$

$$\emptyset \left\{ \begin{array}{l} (Pu, z), \rho(z, z), \rho(z, z), \\ (z, Pu), \rho(z, Pu), \rho(z, z) \end{array} \right\} \leq 0$$

$$\emptyset \left\{ \begin{array}{l} (Pu, z), 0, 0, \\ (z, Pu), \rho(z, Pu), 0 \end{array} \right\} \leq 0$$

Using implicit relation (B), we get

$$(Pu, z) \leq 0.$$

This gives  $Pu = z$ . Therefore  $Pu = ABu = z$ .

Since  $Q(X) \subset ST(X)$ , there exist a point  $v \in X$  such that  $STv = z$ .

Put  $x = u$  and  $y = v$  in (iii), we have

$$\emptyset \left\{ \begin{array}{l} (Pu, Qv), \rho(ABu, STv), \rho(ABu, Qv), \\ (STv, Pu), \rho(ABu, Pu), \rho(STv, Qv) \end{array} \right\} \leq 0$$

$$\begin{aligned} \emptyset \left\{ \begin{matrix} (z, Qv), \rho(z, z), \rho(z, Qv), \\ (z, z), \rho(z, z), \rho(z, Qv) \end{matrix} \right\} &\leq 0 \\ \emptyset \left\{ \begin{matrix} (z, Qv), 0, \rho(z, Qv), \\ 0, 0, \rho(z, Qv) \end{matrix} \right\} &\leq 0 \end{aligned}$$

Using implicit relation (A), we get

$$(z, Qv) \leq 0.$$

This gives  $Qv = z$ . Therefore  $Qv = ABu = Pu = STv = z$ .

Since  $(P, AB)$  is occasionally weakly compatible therefore  $Pu = ABu$  implies that

$PABu = ABPu$  that is  $Pz = ABz$

Now we show that  $z$  is a fixed point of  $P$  so we put  $x=z$  and  $y=v$  in (iii), we get

$$\begin{aligned} \emptyset \left\{ \begin{matrix} (Pz, Qv), \rho(ABz, STv), \rho(ABz, Qv), \\ (STv, Pz), \rho(ABz, Pz), \rho(STz, Qz) \end{matrix} \right\} &\leq 0 \\ \emptyset \left\{ \begin{matrix} (Pz, z), \rho(z, z), \rho(z, z), \\ (z, Pz), \rho(z, Pz), \rho(z, z) \end{matrix} \right\} &\leq 0 \\ \emptyset \left\{ \begin{matrix} (Pz, z), 0, 0, \\ \rho(z, Pz), \rho(z, Pz), 0 \end{matrix} \right\} &\leq 0 \end{aligned}$$

Using implicit relation (B), we get

$$(z, Pz) \leq 0.$$

This gives  $Pz = z$ . Hence  $Pz = z = ABz$ .

Similarly  $(Q, ST)$  is occasionally weakly compatible we have  $Qz = STz = z$ .

Now we show that  $Bz = z$ .

Put  $x = Bz$  and  $y = y_n$  in (iii), we have

$$\begin{aligned} \emptyset \left\{ \begin{matrix} (PBz, Qy_n), \rho(ABBz, STy_n), \rho(ABBz, Qy_n), \\ (STy_n, PBz), \rho(ABBz, PBz), \rho(STy_n, Qy_n) \end{matrix} \right\} &\leq 0 \\ \emptyset \left\{ \begin{matrix} (Bz, z), \rho(Bz, z), \rho(Bz, z), \\ (z, Bz), \rho(Bz, Bz), \rho(z, z) \end{matrix} \right\} &\leq 0 \\ \emptyset \left\{ \begin{matrix} (Bz, z), \rho(Bz, z), \rho(Bz, z), \\ (z, Bz), 0, 0 \end{matrix} \right\} &\leq 0 \end{aligned}$$

Using implicit relation (B), we get

$$(z, Bz) \leq 0.$$

This gives  $Bz = z$ .

Since  $ABz = z$  therefore  $Pz = ABz = Bz = Qz = STz = z$

Finally we show that  $Tz = z$ .

Put  $x = z$  and  $y = Tz$  in (iii), we get

$$\begin{aligned} \emptyset \{ & (Pz, QTz), \rho(ABz, STTz), \rho(ABz, QTz), \\ & (STTz, Pz), \rho(ABz, Pz), \rho(STTz, QTz) \} \leq 0 \\ \emptyset \{ & (z, Tz), \rho(z, Tz), \rho(z, Tz), \\ & (Tz, z), \rho(z, z), \rho(Tz, Tz) \} \leq 0 \\ \emptyset \{ & (z, Tz), \rho(z, Tz), 0, \\ & 0, \rho(z, Tz), \rho(z, Tz) \} \leq 0 \end{aligned}$$

Using implicit relation (B), we get

$$(z, Tz) \leq 0.$$

This gives  $Tz = z$ .

Hence  $ABz = Bz = STz = Tz = Pz = Qz = z$ . Uniqueness follows easily.

If we put  $B = T = I$ , identity map on  $X$ , in Theorem 3.1, we have the following:

**Corollary 3.1.** Let  $A, S, P$  and  $Q$  is six self mappings of a complex-valued metric space  $(X, \rho)$  satisfying the following conditions:

- (i)  $(X) \subseteq A(X), Q(X) \subseteq S(X)$ ,
- (ii) The pair  $(P, A)$  and  $(Q, S)$  share the common (E.A) property.
- (iii) For any  $x, y \in X, \emptyset$  in  $M_6$ .

$$\emptyset \{ \begin{aligned} & (Px, Qy), \rho(Ax, Sy), \rho(Ax, Qy), \\ & (Sy, Px), \rho(Ax, Px), \rho(Sy, Qy) \end{aligned} \} \leq 0$$

Then the pair  $(P, A)$  and  $(Q, S)$  have a point of coincidence each. Moreover  $A, S, P$  and  $Q$  have a unique common fixed point provided both the pairs  $(P, A)$  and  $(Q, S)$  are occasionally weakly compatible.

As an application of the theorem 3.2., we prove a common fixed point theorem for six finite families of maps on metric space, while proving our results; we utilize definitions of finite families which is natural extension of commutativity condition to two finite families.

**Theorem 3.2.** Let  $\{A_1, A_2, \dots, A_m\}, \{B_1, B_2, \dots, B_n\}, \{S_1, S_2, \dots, S_p\}, \{T_1, T_2, \dots, T_q\}, \{P_1, P_2, \dots, P_r\}$ , and  $\{Q_1, Q_2, \dots, Q_s\}$  be six finite families of self maps of a complex-valued metric space  $(x, d)$  such that  $A = A_1, A_2, \dots, A_m, B = B_1, B_2, \dots, B_n, S = S_1, S_2, \dots, S_p, T = T_1, T_2, \dots, T_q, P = P_1, P_2, \dots, P_r$  and  $Q = Q_1, Q_2, \dots, Q_s$  satisfy the following conditions.

- (1)  $P(X) \subset AB(X)$  (or  $Q \subset ST(X)$ )
- (2) The pair  $(P, AB)$  (or  $(Q, ST)$ ) satisfy property (E.A).

Then the pairs  $(P, AB)$  and  $(Q, ST)$  have a point of coincidence each. Moreover finite families of self maps  $P_r, A_i, B_n$  and  $Q_i, S_p, T_q$  have a unique common fixed point provided that the pairs of families  $(\{P_r,$

$\}, \{A_1\}, \{B_n\}$  and  $\{Q_i\}, \{S_p\}, \{T_q\}$  commute pair-wise for all  $i = 1, 2, \dots, m, k = 1, 2, \dots, n, t = 1, 2, \dots, o, v = 1, 2, \dots, r, p = 1, 2, \dots, s$  and  $q = 1, 2, \dots, x$ .

**Proof.** Since self maps  $A, B, S, T, P$  and  $Q$  satisfy all the conditions of above theorem, the pairs  $(P, AB)$  and  $(Q, ST)$  have a point of coincidence. Also the pairs of families  $(\{P_r\}, \{A_i B_n\})$ , and  $(\{Q_t\}, \{S_p T_q\})$  commute pair wise, we first show that  $PAB = ABP$  as

$$\begin{aligned} PAB &= (P_1 P_2 \dots P_r)(A_1 A_2 \dots A_m)(B_1 B_2 \dots B_n) = (P_1 P_2 \dots P_{r-1})(A_1 A_2 \dots A_m)(B_1 B_2 \dots B_n) \\ &= (P_1 P_2 \dots P_{r-2})(A_1 A_2 \dots A_m B_1 B_2 \dots B_n P_{r-1} P_r) = \dots = P_1(A_1 A_2 \dots A_m B_1 B_2 \dots B_n P_2 \dots P_r) \\ &= (A_1 A_2 \dots A_m)(B_1 B_2 \dots B_n)(P_1 P_2 \dots P_r) = ABP \end{aligned}$$

Similarly one can prove that  $QST = STQ$ . Hence, obviously the pair  $(P, AB)$  and  $(Q, ST)$  are occasionally weakly compatible. We conclude that  $A, B, S, T, P$  and  $Q$  have a unique common fixed point in  $X$ , say  $z$ .

Now, one needs to prove that  $z$  remains the fixed point of all the component maps. For this consider

$$\begin{aligned} A(A_i z) &= ((A_1 A_2 \dots A_m)_i) z = ((A_1 A_2 \dots A_{m-1}) A_m A_i) z \\ &= (A_1 A_2 \dots A_{m-1})(A_i A_m) z = (A_1 A_2 \dots A_{m-2})(A_i A_m A_{m-1}) z \\ &= (A_1 A_2 \dots A_{m-2})(A_i A_{m-1} A_m) z = \dots = A_1(A_1 A_2 \dots A_m) z \\ &= (A_1 A_i)(A_2 \dots A_m) z \\ &= (A_i A_1)(A_2 \dots A_m) z = A_i(A_1 A_2 \dots A_m) z = A_i A z = A_i z. \end{aligned}$$

Similarly, one can prove that

$$\begin{aligned} P(B_k z) &= B(Pz) = B_k z, B(B_k z) = B_k(Bz) = B_k z, \\ P(P_v z) &= P_v(Pz) = P_v z \\ P(A_i z) &= A(Pz) = A_i z, A(A_i z) = A_i(Az) = A_i z \\ ((A_i B_k) z) &= (A_i B_k)(Pz) = (A_i(B_k Pz)) = (A_i Pz) = A_i Pz. \end{aligned}$$

Similarly one can prove that  $QST = STQ$ . Hence, obviously the pair  $(P, AB)$  and  $(Q, ST)$  are occasionally weakly compatible. We conclude that  $A, B, S, T, P$  and  $Q$  have a unique common fixed point in  $X$ , say  $z$ .

Now, one needs to prove that  $z$  remains the fixed point of all the component maps. For this consider

$$\begin{aligned} Q(S_p z) &= S(Qz) = S_p z, Q(T_q z) = T_q(Qz) = T_q z, \\ Q(Q_t z) &= Q_t(Qz) = Q_t z \\ Q((S_p T_q) z) &= (S_p T_q)(Qz) = (S_p(T_q Qz)) = (S_p Qz) = S_p z, \end{aligned}$$

Which shows that (for all  $k, i, q, p, v$  and  $t$ )  $P_v z$  and  $A_i B_k z$  are other fixed point of the pair  $(P, AB)$  whereas  $Q_t z$  and  $S_p T_q z$  are other fixed point of the pair  $(Q, ST)$ .

As  $A, B, S, T, P$  and  $Q$  have a unique common fixed point, so, we get

$$z = P_v z = A_i z = B_k z = Q_t z = S_p z = T_q z,$$

$$\text{for all } v = 1, 2, \dots, r, \quad i = 1, 2, \dots, m$$

$$k = 1, 2, \dots, n, \quad t = 1, 2, \dots, o$$

$$p = 1, 2, \dots, s, \quad q = 1, 2, \dots, x$$

Which shows that  $z$  is a unique common fixed point of  $\{P_v\}_{v=1}^s, \{A_i\}_{i=1}^m, \{B_k\}_{k=1}^n, \{Q_t\}_{t=1}^o, \{S_p\}_{p=1}^s$  and  $\{T_q\}_{q=1}^x$ .

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# Radio Access Technology Discovery in Mobile with Heterogeneous Environment

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## ABSTRACT

*Discovery in mobile in heterogeneous environment an available radio access technology (RATs) is one of the most challenging issues in the heterogeneous wireless network environment. Especially, discovering WLAN, which support high data rates but have limited service coverage, and high flexibility for the users. In this paper, we propose three WLAN discovery schemes which are based on the upcoming IEEE 802.21 standards. In the proposed schemes, we exploit the information on neighboring WLANs from a MIH information server to discover available WLANs as soon as possible. Our simulation results show that the proposed schemes can enhance the performance of RAT discovery. In the heterogeneous environment of mobile communication and the radio access technology that proposed to The protocol operation consists of four stages. Cognition (through external triggering), Discovery of RAN/RAT capabilities, Negotiation and allocation, Handoff decision-making and execution.*

**Keywords:** *Heterogeneous Environment, RAT discovery, Handoff decision, MIH, IEEE 802.21.*

## 1. INTRODUCTION

The integration of different wireless communication technologies is the important factor in the next generation wireless communications. The different network environments offer the Internet access to users, any where anytime, and with better Quality of Service (QoS). The integration of heterogeneous wireless networks has some issues such as vertical handoff, admission control, security guarantee, and power management for a mode terminal (MT) [3].

IEEE 802.21 standard defines Media Independent Handover (MIH) mechanisms that enable the enhancement of handovers in heterogeneous networks [1]. Each wireless technology in the join networks has its own characteristics that complement others.

For example, Wireless Local Area Networks (WLANs) which are based on the IEEE 802.11 standards support higher data rates than other wireless access technologies. In contrast, the IEEE 802.16 (WiMAX) or 3G networks cover relatively large areas, but they provide smaller data rates than WLANs. Due to such distinctive characteristics, the integration of WLANs with other wide area access networks is one of the most famous examples in network integration scenarios. WLANs with 3G systems has been studied actively by research communities and standardization bodies defined the partnership such as 3GPP/3GPP2 consortium, IEEE, and ETSI [2-4] between WLANs and the IEEE 802.16 based networks also has gained much attention. In this IEEE 802.16 proposed that 802.16e base stations (BSs) periodically broadcast the information on the density of WLAN APs [6].

A WLAN discovery scheme of utilizing 3GPP networks to broadcast the channel information on WLANs [7]. In the utilized the location service server (LSS) which stores the geographical information of WLAN AP locations [5]. With the help of the LSS, a RAT can turn on its WLAN interface only if the

WLAN is available. In this paper, we propose efficient WLAN discovery schemes for heterogeneous wireless networks to reduce the energy consumption and detection time of WLANs. The proposed schemes utilize the IEEE 802.21 MIH services to provide information on WLANs to a RAT in a heterogeneous environment. Since the IEEE 802.21 standard supports media independent services, a MT can obtain the information on WLANs regardless of the currently connected network type. A main strong point of our schemes compared to existing work is that we consider a case where some APs are not managed by an information server. Note that most of WLAN APs are independently owned and managed unlike commercial networks.

Mobility is the important feature of a wireless cellular communication system. Usually, continuous service is achieved by supporting handoff (or handover) from one cell to another. Handoff is the process of changing the channel (frequency, time slot, spreading code, or combination of them) associated with the current connection while a call is in progress [8]. Handoff is divided into two broad categories—hard and soft handoffs [2]. In soft handoffs, both recourse existing during the handoff process such as new resources and those resource which is used during the handoff process. Poorly designed handoff schemes generate very heavy signaling traffic and there by a decrease in quality of service (QoS).

The reason why handoffs are critical in cellular communication systems is that neighboring cells are always using a disjoint subset of frequency bands, so negotiations must take place between the mobile station (MS), the current serving base station (BS), and the next potential BS. Other related issues, such as decision making and priority strategies during overloading, might influence the overall performance.

### 1.1 INTRODUCTION TO CELLULAR NETWORKS

The cellular network consists of several cells with each cell covering a certain geographic area. In each cell one base station is present. The cell area is determined by the signal strength within the region, which in turn depends on many factors such as height of the transmitting antenna, presence of hills, valleys and tall buildings, and atmospheric conditions. Therefore, the actual shape of the cell may be in the form of zigzag shape. However, the cell is approximated as hexagon. Because the hexagon is a good approximation of the circular region and it allows a larger region to be divided into no overlapping hexagonal sub regions of equal size [10]. All mobile users in the cell are served by this base station (BS). These base stations allocate frequency channels to the mobile station (MS) or users for communication when a call is made. The mobile user is blocked when there the base station cannot allocate a free channel to it.

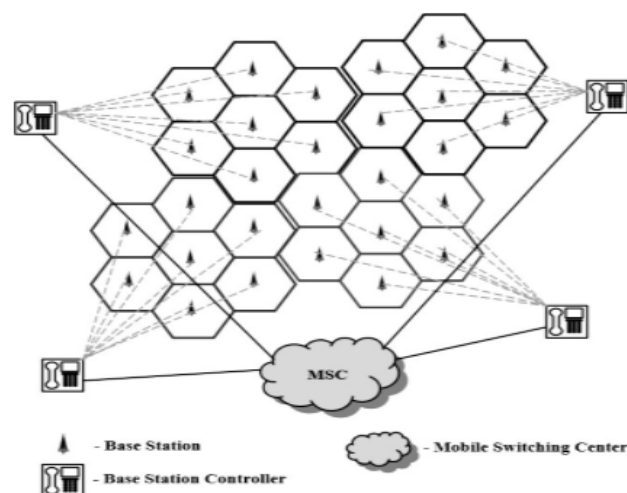


Fig 1.1

When a mobile user using a frequency channel of that particular base station reaches the boundary of the cell and moving to another cell, it needs to free the current channel and should acquire a channel from the neighboring cell to maintain connection. This procedure is called handoff or handover. These base stations are again linked to the base station controller (BSC). The set of base station controllers are connected to mobile switching center (MSC) which is responsible for controlling the calls and acting as a gateway to other networks. This cellular network scenario is shown in Fig. 1.1.

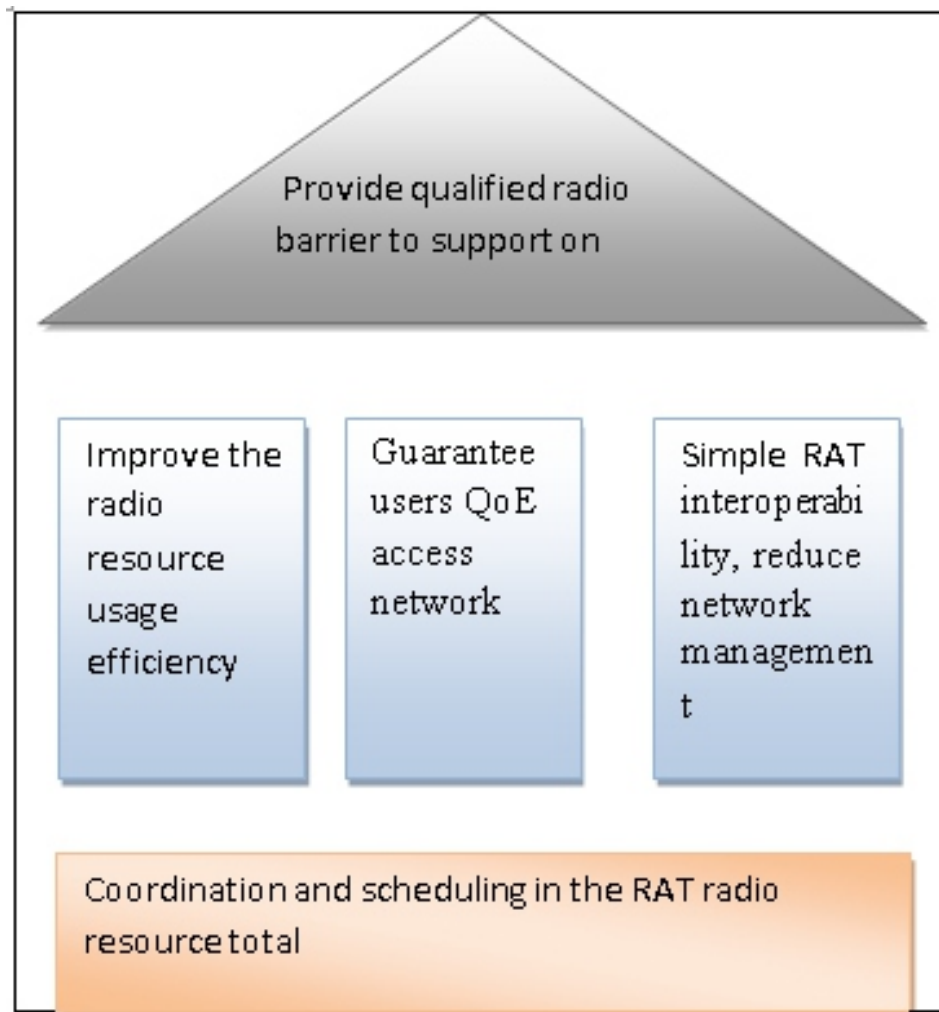
## RELATED WORK

In IEEE 802.11 WLANs that discover of potential WLAN APs is accomplished by means of a medium access control (MAC) layer function called scanning according to the IEEE 802.11 standard. There are two types of scanning mode: passive mode and active mode. In the passive scanning mode, a terminal listens to each channel of the physical medium one by one for beacon frames. During the passive scanning mode, the terminal waits for at least a period of time longer than the beacon interval, before it switches to the next channel. On the other hand, a terminal in the active scanning mode transmits a probe request frame containing the broadcast address as its destination and waits for probe responses from APs. If no response has been received by minimum channel Time, then the next channel is scanned. If one or more responses are received by minimum channel time, the terminal waits for more responses at most maximum channel time.

With mobile broadband (MBB) growth and evolution of the network, the network is becoming dense and complex, each site can be up to seven bands spectrum, five modes (GSM/UMTS/LTE- FDD/TD-LTE/WiFi), and five layers network architecture (Low-frequency macro coverage layer/ high-frequency capacity layer/ hotspot Micro capacity layer/ indoor Pico layer/ WiFi hotspots). If lack of effective coordination, it cannot effectively use all of the wireless network resources and cannot guarantee user experience. Meanwhile, intelligent terminals, pads and other types of users' equipment (UEs) spread quickly, and the service types of MBB increase rapidly. In such a multi-layer/multi-band/multi-mode wireless network, a question on how to improve the utilization efficiency of radio resources, to guarantee QoE of MBB service, to simplify multi- RAT network management, is a huge challenge of the Single-RAN.

At present, the coexistence of several networks assumes the existence of different capabilities. Not all of the existing terminals can support all radio systems, so multi-RAT networks cannot fully substitute for the different type of services to support performance. For example, throughput rate of data is faster than other RATs in the recent LTE network. In addition, the VoLTE (Voice over LTE Initiative) is proposed by GSMA [8] so as to ensure continuity of voice calls when a user moves from an LTE cell to a non-LTE cell. According to service requirements, SRC needs real-time unified management of all wireless resources so as to coordinate the usage of resources in different RATs (Figure 1.2), in order to meet the following targets:

- Improve the overall utilization of radio resources.
- Guarantee users that they get consistent service experience regardless the used system.
- Simplify the process of multi-RAT interoperability, reduce network management difficulty.

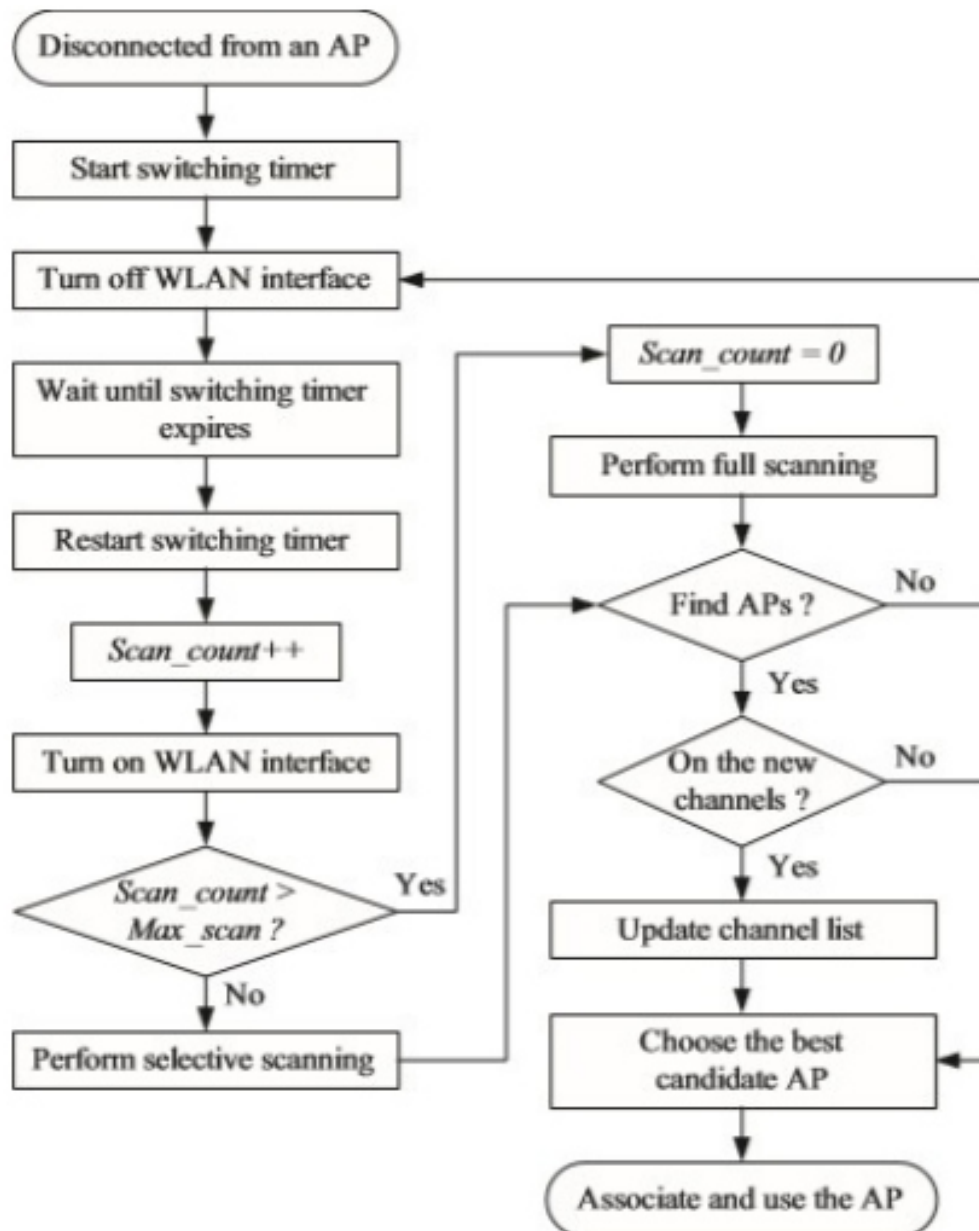


### PROPOSED WORK & APPROCHED FOR THE SOLUTION

Before proposing new RAT discovery schemes for a mobile in a heterogeneous environment, we introduce a conventional WLAN discovery scheme based on periodic switching. The timing model of the periodic switching scheme [1]. When a MT turns on its WLAN interface, it scans all channels via active or passive scanning modes. If one or more APs are detected after scanning, the MT associates with a new AP and starts to use the WLAN interface for data communications. Otherwise the WLAN interface is turned off during the switching interval  $T$ . Our aim is to design an advanced WLAN discovery scheme based on MIIS of the IEEE 802.21 standard. We assume that a MIH-capable to obtains the information on WLANs as follows.

Radio access technology discovery mainly selects the access network by dynamically considering a variety of network selection parameters such as: QoS offered by the network, cost of service, the QoS of applications running at the moment, the preferences of the end-user, wireless signal perception, power requirements of each network interface. Also, the method introduces a new network discovery approach, which provides the efficient performance when the network discovery is performed on devices such as smart phones, PDAs which are very sensitive to the power usage.

The Proposed method using mobile-controlled handover decision enables the mobile node to select the best network depending of various criteria's such as network conditions, costs, user preferences and so on.



### The handover process is divided in three phases:

- Network discovery.
- Handover decision.
- Handover execution.

In the network discovery the mobile node determines what wireless networks are available. The Handover decision is responsible for decision to which network to connect and the handover execution performs the actual switch to other network to transmit the data [10].

We propose a RAT discovery scheme that reduces the scanning time by limiting the number of scanned network. In this scheme, the receiving MIH Get Information response, a MT constructs a list of channels used by nearby APs to perform scanning for selected channels (selective scanning) rather than for all network (full scanning). There already have been several approaches that adopt selective scanning for the fast AP discovery. however, this designed to reduce the energy consumption of MT as well as the discovery time. Fig. 3 shows the overall operations of RAT discovery.

After disconnecting from an AP, the MT starts ‘switching timer’ with the switching interval  $T$  and turns off its WLAN interface. When the switching timer expires, the MT restarts the timer and increases the value of Scan count which is set to zero initially. Then it turns on the WLAN interface and decides whether to perform selective scanning or full scanning. In case Scan count does not exceed the pre-defined threshold Max scan, the MMT tries to discover APs by selective scanning. If the MMT fails to discover APs until Scan count exceeds Max scan, then it resets Scan count to zero and performs full scanning. (How to determine the optimal value of  $T$  and Max scan is outside the scope of this paper). After the scanning operation, if the MT fails to detect any APs, it turns off its WLAN interface again and waits until the next switching timer expires. If one or more APs are found, then it uses the WLAN interface for data transmission and voice transmission.

### Simulation & result

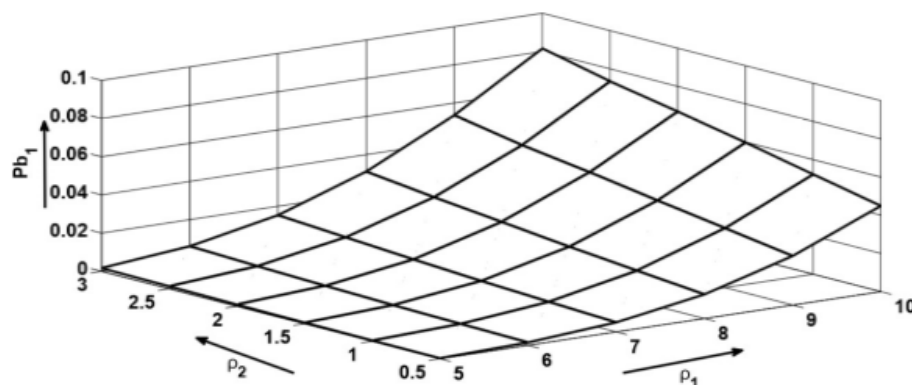
The performance of the system is evaluated under the following three different scenarios showed in Table 1.

Table

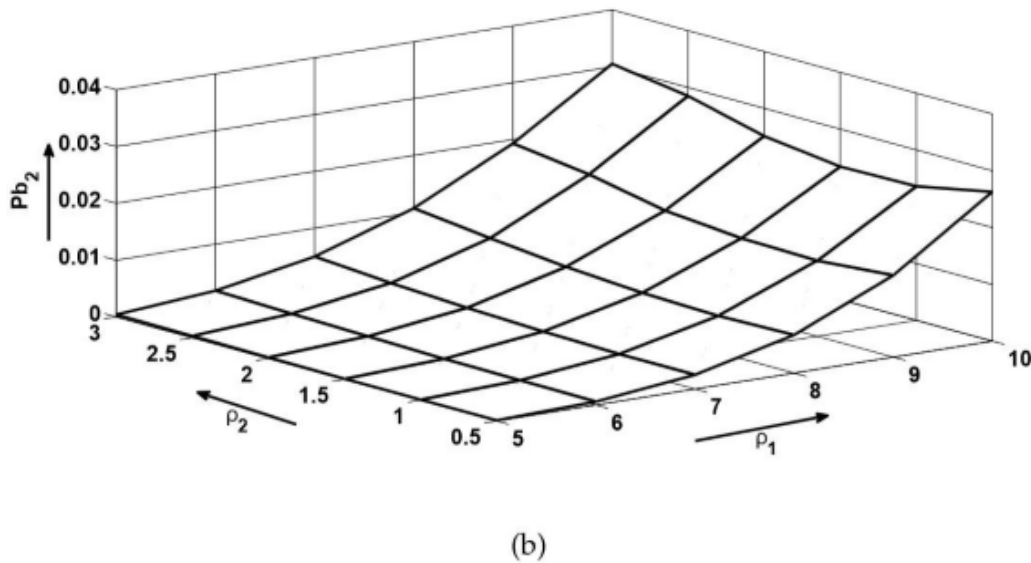
Scenario I		Scenario II				Scenario III					
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value		
$b_1$	2 channels	$\omega_1$	0.8	$\rho_1$	5	$\omega_1$	0.8	$b_1$	2 channels	$\rho_1$	5
$b_2$	1 channel	$\omega_2$	0.2	$\rho_2$	3	$\omega_2$	0.2	$b_2$	1 channel	$\rho_2$	3

- Scenario-I is for varying traffic intensity of class-1 and class-2 calls that gives a comprehensive view of the expected load on a HetNet. The traffic intensity describes the number of call requests received by the fixed network elements, in a unit area element during a time interval.
- Scenario-II is for varying required bandwidths for class-1 and class-2 calls. This scenario is to illustrate how varying bandwidth requirements are satisfied under limited resources and the impact on the blocking probability.
- Scenario-III is for varying weight for energy consumption cost in the total cost. This scenario is to illustrate the relative importance of each objective function on the system performance. We evaluate the performance of the system under the three above mentioned scenarios.

### Analysis of Results for Scenario I



(a)



**Figure 4.1: Scenario I: Blocking probability versus call intensities (a) blocking probability for class-1 call and (b) blocking probability for class-2.**

In Figure , it can be observed that the higher the call intensities, the higher the blocking probabilities. This is attributed to the fact that the system capacity is fixed. It is also observed that class-2 call intensity does not greatly impact on the blocking probabilities. This is due to the fact that the system blocks less class-2 calls due to its lower call intensity. On the other hand, the blocking probabilities of both calls sharply increase when class-1 call intensity is high (from 8 to 10), meaning that the system has less channels to accept new calls in a fast way. Figure (a) shows that the optimal policy blocks more class-1 calls compared to class-2 calls in figure (b). This is attributed to the fact that class 1 calls require more bandwidth which makes RATs consume more energy.

Figure (a) and (b) show that when the values of  $\rho_1$  and  $\rho_2$  are smaller, the initial RAT selection policy decides that it is better to accept more calls in less energy consuming RAT (i.e. RAT2), in order to save the overall energy. For this reason, the utilization of RAT2 is far better than that of RAT1 at this step. But when both call intensities are getting high, in order to tackle the traffic volume, the optimal policy starts taking more calls in RAT1 which makes its utilization high. Figure (b) shows that the RAT2 utilization goes down and figure (a) shows that the RAT1 utilization goes up when both call intensities are high.

This is attributed to the fact that the optimal policy accepts more calls in RAT1 due to its higher capacity.

## CONCLUSION

In this paper, we proposed RAT discovery for mobile in heterogeneous environment. The proposed schemes have been designed to discover available networks efficiently utilizing the MIIS of the IEEE 802.21 standard. In the proposed schemes, a MT obtains the information on neighboring WLANs from a MIIS server and exploits them to discover networks as soon as possible, while reducing the energy consumption. Our simulation results show that the proposed schemes outperformed the conventional discovery scheme in terms of the energy consumption and the discovery time of WLANs.

We have proposed an optimization model based on the Semi-Markov Decision Process (SMDP) framework for the problem of selecting the initial Radio Access Technology (RAT) in co-located

wireless networks. Our optimal initial RAT selection method considers a cost function that involves a blocking cost and an energy consumption cost associated with deferent weights to support the optimal JCAC decision. For the studied scenario with two co-located wireless networks, our simulation results demonstrate that variations in the weights of blocking cost and energy consumption cost can greatly impact both the system capacity and the network energy consumption. For the scenarios investigated in this thesis, our SMDP model generated 27,283 pairs of state-action for the two co-located wireless networks. As future work, we believe that our proposed SMDP-based model can be extended to support more sophisticated Het Nets architectures (i.e. with more than two co-located wireless networks and several service classes), which of course will involve a huge number of state actions that should be deal with.

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# A Survey Paper on Detection and Prevention of Black Hole in Manet

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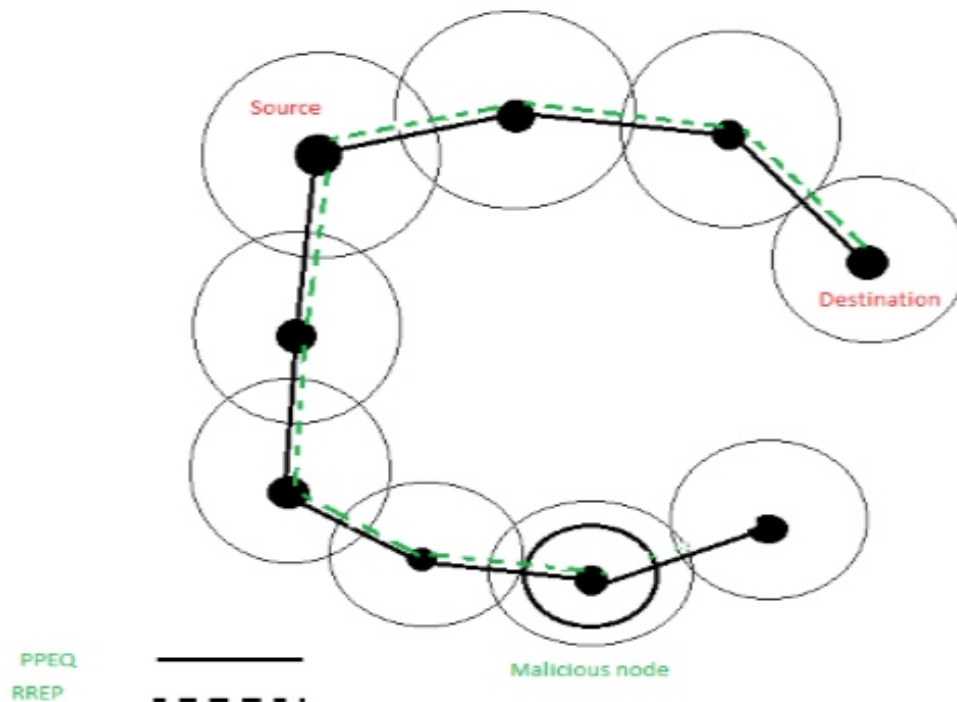
## ABSTRACT

*The black hole attack is one of the common security threats in wireless mobile ad hoc networks. The intruders utilize the loophole to carry out their malicious behaviors because the route discovery process is necessary and inevitable. Many researchers have conducted different detection techniques to propose different types of detection schemes. In this paper, we survey the existing solutions and discuss the state-of-the-art routing methods. We not only classify these proposals into single black hole attack and collaborative black hole attack but also analyze the categories of these solutions and provide a comparison table. We expect to furnish more researchers with a detailed work in anticipation.*

**Keywords :** *Mobile ad hoc networks, routing protocols .single black hole attack*

## 1. INTRODUCTION

Mobile Ad Hoc Network (MANET) consist of mobile devices that are capable communicate to each other without any centralized system. MANET is self organized not fixed infrastructure, automatic self configuration, quick deployment etc. It is particularly vulnerable due to its fundamental characteristics such as open medium, dynamic topology, distributed cooperation, and constrains capabilities [1] [2]. One of popular routing protocol is Ad-hoc on demanding vector (AODV) [1]. According the discussion in [2], it is suitable for applying in high mobility devices, which the network topology is changed frequently. Due to mobility of nodes, routing path for communication may be disrupted. Mobile nodes have to discover and setup a routing path first when data communication is needed. In such a environment, malicious nodes have many opportunities to join the process of setup routing path. Therefore security problems should be paid more attention. In AODV routing protocol, source node broadcasts a Route Request (RREQ) message including a sequence number to discover a routing path to destination. The purpose of sequence number is used to identify freshness of routing paths. That is the more large sequence number, the more fresh routing path. On receiving RREQ message, intermediate nodes will check where there exists a fresh routing path destined to destination nodes in their routing table. If the sequence number of routing path in routing table is larger than or equal to the sequence number of RREQ, then a Route Reply(RREP) will be sent and propagate back to source node. Otherwise, RREQ message will be broadcasted to neighboring nodes toward destination node. Therefore malicious node can easily reply a faked RREP message with large sequence number to spoof the source node. Then malicious node can be on the path from source to it, even there is not a real path from it to destination node. Packets from source node may be dropped by the malicious node; it is a well-known black holes problem [3].



**Figure 1. Malicious node behavior in a network**

## 2. BACKGROUND

Nodes communicate with each other by sending RREQ to their neighbors. Each neighbors reply with RREP when they got route from the destination node. On the basis of characteristic of route, they are classifying in three categories.

### a. Proactive (table driven) routing protocol:

In proactive protocol, mobile nodes periodically broadcast their routing information to the neighbors. Each node need to maintain the routing table which not only records the adjacent nodes and reachable nodes but also number of hops. Ex. - destination sequence distance vector (DSDV) and optimized link state routing (OLSR) [7].

### b. Reactive (on-demand) routing protocol:

Unlike the proactive routing, the reactive routing is simply started when nodes desired to transmit data packet. The strength is that the wasted bandwidth include from the cyclically broadcast can be reduced. Ex – ad hoc on demand distance vector (AODV) and dynamic source routing (DSR) [7].

### c. Hybrid routing protocol:

Hybrid protocol combines the advantages of proactive and reactive routing. The routing is initially established with some proactively prospected routes and then serves the demand from additionally activated nodes through reactive flooding. The choice of one or the other method requires predetermination for typical cases. Ex – Zone routing protocol (ZRP) [9].

Security issues research is very immerging field in the MANET. MANET supports to connectivity to internet so attackers can easily attack on the MANET, it not necessary to attackers are belong from the

same network or just neighbor node. There are several attacks in this so we can classify it in layered manner

Serial no.	Layers	Attacks
1	MAC layer	Malicious Behavior, Selfish Behavior, Active, Passive, Internal External
2	Physical layer attack	Jamming, interruption, eavesdropping
3	Network layer	Black hole attack, Gray hole attack, Wormhole attack, Information discloser, Message altering, Sending data to node out, Transmission range, Routing attacks.
4	Transport layer attacks	Session hijacking , SYN flooding
5	Application layer attacks	Repudiation, Data corruption.
6	Others	DOS, Impersonlasation, Device terming, reply, Man in the middle.

Table1- table contain the list of layered attach in the networks.

### 3. Black hole attack

Black hole attack is very common attack in the MANET. An attacker use the routing protocol to advertise it as having shortest path compare to all RREP (highest sequence number and lowest hope count). In the figure-[2] see that if a source node broadcast RREQ and wait for the RREP, malicious node reply RREP to sender with highest sequence number and lowest hope count, if the RREP come from the actual node then nothing going wrong but when malicious node reply first then source node think to this is shortest path and ignore all the later RREP so send data to malicious node and attackers perform a DOS on the data [6][9].

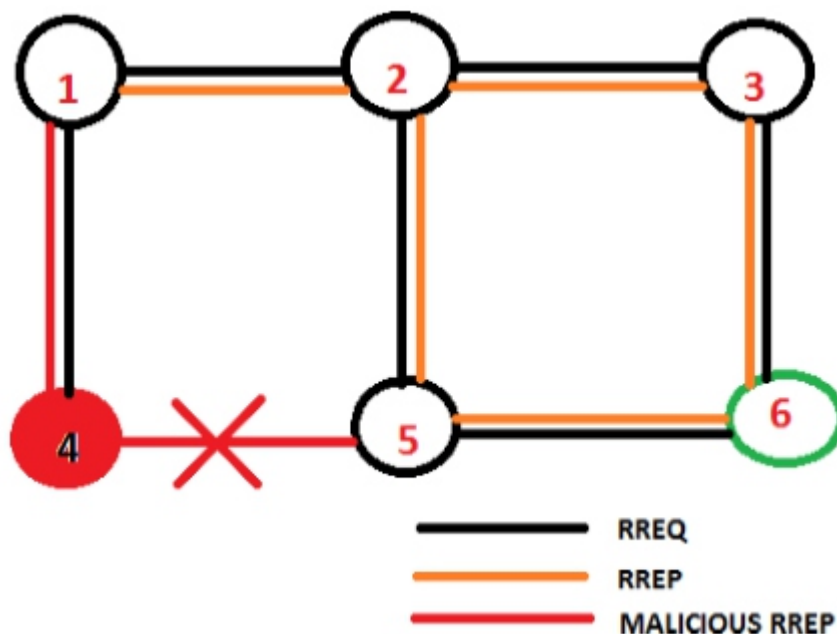


Figure 2- The black hole problem with 6 nodes where node 4 is a malicious node.

#### 4. RELATED WORK

A novel automatic security mechanism proposed by authors [1] to detect/find out the black hole attack (malicious node) by the help of the behavior/performance of the intermediate nodes. Authors proposed method called Support Vector Machine (SVM) on using AODV routing protocol. Technique in SVM method, behavior of the node on the basis of the PDED (Pocket delivery ratio), PMOR (pocket modifying ratio) and PMSIR (pocket misroute ratio). A standard ratio defines for this above ratio on the basis of previous. So if node not follows the standard, that node is malicious node or black hole.

A technique for prevention from the Black Hole attack on AODV routing protocol for MANET, created by authors [2]. Method use promiscuous mode to detect malicious node and if yes, broadcast the presence of malicious node in the network. If the intermediate node send a PPER to the source then a node providing to the node which sent RREP pocket switch on its promiscuous mode and send a hello message to the destination node through the intermediate node which is RREP. If it is legitimate node then message forward from it to the destination node else it's a malicious node and broadcast a message to it's a malicious node and source refresh its route table and find again new route by broadcasting RREQ.

Meenakshi et al. [3] introduced an approach to detect and prevent from the MANET flooding attack. Authors used a Pocket Delivery Ratio (PDER), Control Overhead (CO) and Pocket Misroute Ratio (PMIR) behavior of the node to define the flooding attack. If a node send RREQ without considering the RREQ\_RATELIMIT within per second. Due to flooding attack nodes route table full by the bogus RREQ, so it not perform operation on the actual RREQ and misrouting the packet is consider and read the behavior and create a matrix to read it, by it author defines nodes are legitimate or malicious.

Authors [4] developed a novel approach to avoid the malicious node attack in MANET. Malicious node adversities that I have the greatest sequence number and lowest hop count. Proposed approach, each RREP have extra three properties such as node ID, PPN number and cluster head ID. Each cluster head maintain a neighbor table which is keep information about the entire node. Neighbor table maintain the node ID and cluster head ID. Each node in the network has a specific prime number which acts as node Identity and this identity must not be change.

A novel method described in research paper [5] called source routing discovery to prevent black hole attacks (SRD-AODV). Author define the Threshold, minimum and maximum number of nodes on the network. The Threshold id also defines in small, medium and large environment and have malicious node in this environment respectively 6%, 4% and 2%. Malicious node always generate the greatest sequence number because malicious do not know the destination sequence number and if the sequence number is greater than the total number of node in the network so we easily find out the malicious node or black hole attack in the network. Here authors compare the malicious sequence number with the Threshold Number. If sequence number is greater than the Threshold Number, it's a malicious node and need to broadcast that node is malicious node and rebroadcast the RREQ.

In paper [6] authors proposed a method for identification of the black hole in the network by authentication mechanism. It's a prevention mechanism. In this authentication based on the Hash function, message authentication code (MAC), and pseudo random function (PRF) on the top of the AODV routing protocol. It's a faster approach to identification of malicious node. The technology used in this paper say that when RREP generated by the destination node at the time RREP message encrypt

and sends to the intermediate node with public key and private key. At each intermediate node take the packet and just forward it. When it reaches at the source node then using private key, open the packet. Before the open the packet it check the packet is same as the send packet. If the message packet (RREP) is affected then just discard it and broadcast the fresh RREQ to the neighbor node and broadcast a message to detect a malicious node in the network.

Authors Proff. Sanjeev Sharma et al [7] proposed a technique to detection of the black hole attack in manet called Secure-ZRP protocol which can used to prevent from black holes in zones and out zones. Authors divided the security in two group (a) local communication attack, inside the zones (b) when inter zone communication, outside the zones. In local communication source node broadcast the bluff probe packet. This contains the address of the destination but in actual this is the address of non existence node. This message is directly revised by the neighbor node. If the malicious node present in the zone it will give immediate response to the source node.

Authors Mr. Golak Panda et al [8], introduced a algorithm on the authentication manner to prevent the black hole in manet. Using AODV protocol authors sends a HELLO message is broadcast to its neighbor node and a routing table maintain by the all nodes for the temporary basis in active state. The total bits consume by this route discovery and route maintenance is 32 bit each. But here both RREQ and RREP packets the 9 bit is reserve sector will be there. This proposed algorithm is based on the key mechanism process. First is key generation process in this process use 12 digit left shift of binary number and AND operator for reshuffling and this will fitted in 9 bit reserve sector packet. Second is key authentication process here after comparison both the key the trust value will be decided.

### Caparison of all above related papers

Scheme	Routing Protocol	Simulator	Detection Type	Publish Year	Result	De fe cts
Behavioral approach	AODV	NS-3.14	Single detection	2013	Increase pocket delivery ratio	Take more time in comparison
Comparison technique	AODV	Qual net	Single detection	2013	Throughput increase 40%.	Also increase end-to-end delay and network overheads.
Behavioral matrices using SVM	AODV	NS-3	Single detection	2013	Increase pocket delivery ratio and high throughput	Decrease the network overheads.
Avoiding technique	AODV	MATLAB	Single detection	2013	By CRCMD&R scheme improve throughput 75% from 40% provided by AODV	More network overload and time delay.
Verification technique using SRD-AODV	AODV	NS-2	Single detection	2013	PDR increased by 88-97%	Need more comparison so time dely.
Encryption technique using hash-function	AODV	NS-2	Single detection	2008	Throughput increase	Also increase detection time so time delay.

**Table 2- contain the summary of the related work summery.**

## 5. CONCLUSION AND FUTURE WORK

Due to the inherent design disadvantages of routing protocol in MANETs, many researchers have conducted diverse techniques to propose different types of prevention mechanisms for black hole problem. In this paper, we first summary the pros and cons with popular routing protocol in wireless mobile ad hoc networks. Then, the state-of-the-art routing methods of existing solutions are categorized and discussed. The proposals are presented in a chronological order and divided into single black hole and collaborative black hole attack. According to this work, we observe that both of proactive routing and reactive routing have specialized skills. The proactive detection method has the better packet delivery ratio and correct detection probability, but suffered from the higher routing overhead due to the periodically broadcast packets. The reactive detection method eliminates the routing overhead problem from the event-driven way, but suffered from some packet loss in the beginning of routing procedure. Therefore, we recommend that a hybrid detection method which combined the advantages of proactive routing with reactive routing is the tendency to future research direction. However, we also discover that the attacker's misbehavior action is the key factor. The attackers are able to avoid the detection mechanism, no matter what kinds of routing detection used. Accordingly, some key encryption methods or hash-based methods are exploited to solve this problem. The black hole problem is still an active research area. This paper will benefit more researchers to realize the current status rapidly.

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