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Aim & Scope

The Advances and Applications in Mathematical Sciences (ISSN 0974-6803) is a monthly journal. The AAMS's coverage extends across the whole of mathematical sciences and their applications in various disciplines, encompassing Pure and Applied Mathematics, Theoretical and Applied Statistics, Computer Science and Applications as well as new emerging applied areas. It publishes original research papers, review and survey articles in all areas of mathematical sciences and their applications within and outside the boundary.

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Matching in Fuzzy Labeling Tree

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ABSTRACT

A graph is said to be a complete fuzzy labeling graph if it has every pair of adjacent vertices of the fuzzy graph. A matching is a set of non-adjacent edges. If every vertex of fuzzy graph is M-saturated then the matching is said to be complete or perfect. In this paper, we introduce the new concept of matching in fuzzy labeling tree and its spanning sub graph. We discussed some properties using these concepts and spanning sub graphs of labeling tree using matching and perfect matching.

1. INTRODUCTION

Graph theory is rapidly moving into mainstream of mathematics mainly because of its applications in diverse fields with include biochemistry (DNA double helix and SNP assembly Problem), chemistry (model chemical compounds) electrical engineering (communication networks and coding theory) computer science (algorithms and computations) and Operations Research (scheduling).

Many Problems of practical interest that can be modeled as graph theoretic problems may be uncertain. To deal with this uncertainty the concept of fuzzy theory was applied to graph theory.

A fuzzy set was defined by L. A. Zadeh in 1965. Every element in the universal set is assigned a grade of membership, a value in The elements in the universal set along with their grades of membership form a fuzzy set. In 1965 Fuzzy relations on a set was first defined by Zadeh [13]. Among many branches of modern mathematics, the theory of sets (which was founded by G. Cantor occupies a unique place. The mathematical concept of a set can be used as foundation for many branches of modern mathematics. [. 1 , 0]Rosenfeld first introduced the concept of fuzzy graphs. After that fuzzy relation on a set was first defined by Zadeh in 1965. Based on Zadeh fuzzy relation the first definition of a fuzzy graph was introduced by Kaufmann in 1973.

Azriel Rosenfeld in 1975 developed the structure of fuzzy graphs and obtained analogs of several graph theoretical concepts like bridges and tree [6]. A. Nagoorgani, D. Rajalaxmi [3] introduced the concept of fuzzy labelling tree and S. Yahya Mohamad, S. Suganthi [8] introduced matching in fuzzy labelling graph.

In this paper, we introduce the new concept of matching in fuzzy labeling tree and its spanning sub graph. We discussed some properties using these concepts and spanning sub graphs of labelling tree using matching and perfect matching. Here we consider the simple complete fuzzy graph with even number of vertices.

2. PRELIMINARIES

Definition 2.1. Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of $U \times V$. A fuzzy graph $G = (\alpha, \beta)$ is a pair of functions $\alpha : V \to [0, 1]$ and $\beta : V \times V \to [0, 1]$ where for all $u, v \in V$, we have $\beta(u, v) \leq \min \{\alpha(u), \alpha(v)\}$.

Definition 2.2. If every vertex of fuzzy graph is M-saturated then the matching is said to be complete or perfect. It is denoted by C_M.

Example

Definition 2.3. Let $G : (\alpha, \beta)$ be a fuzzy graph and F is a subset of G. If nodes of F is contained (or) equal to the nodes of G then F is said to be a fuzzy subgraph.

Definition 2.4. A fuzzy sub graph F of the fuzzy labeling graph G is said to be fuzzy spanning sub graph [FSS] of G if nodes of fuzzy sub graph is equal to the nodes of fuzzy graph.

Definition 2.5. A fuzzy graph G is said to be fuzzy simple labelling graph [FSG] if G does not contain a line with same ends and multiple lines.

Definition 2.6. A fuzzy simple graph G is said to be fuzzy complete labelling graph [FCLG] if every pair of nodes of the graph are joined by line. A FCLG with n nodes are denoted by k_n.

Definition 2.7. A fuzzy labelling graph G is said to be fuzzy connected labelling graph [FCG] if there exists a path between all pair of nodes of G.

Definition 2.8. A cyclic graph G is said to be fuzzy cyclic graph if it has fuzzy labeling.

3. MAIN RESULTS

Definition 3.1. A subset M of $\beta(v_i, v_{i+1}), 1 \leq i \leq n$ is called a matching in fuzzy graph if its elements are links and no two are adjacent in G. The two ends of an edge in M are said to be saturated under M.

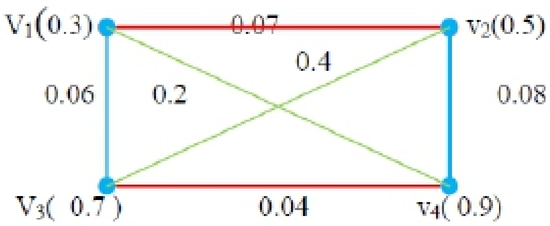
Definition 3.2. Let M be a matching in fuzzy labeling graph. An M-alternating path in G is a path whose edges alternatively in $\beta - M$ and M.

Definition 3.3. An M-Augmenting path is an M-alternating path whose origin and terminal vertices are M-unsaturated.

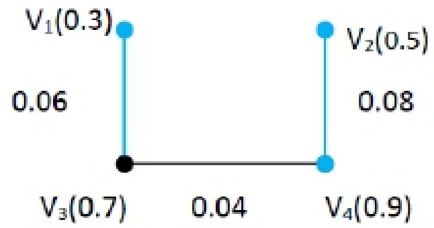
Definition 3.4. A graph $G = (\alpha, \beta)$ is said to be fuzzy labeling tree (FLT) if it has fuzzy labeling and a fuzzy spanning sub graph $T = (U, V)$ which is a tree in which every pair of nodes contains an alternating path.

Example:

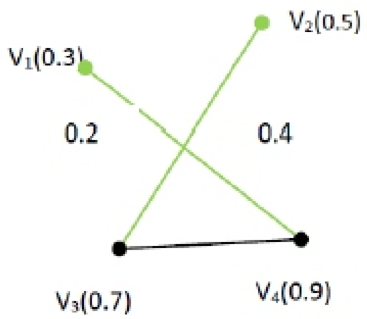
The fuzzy labelling trees of G are given below.



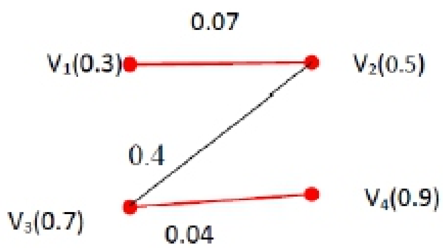
(FLG)



FLT_1



FLT_2



FLT_3

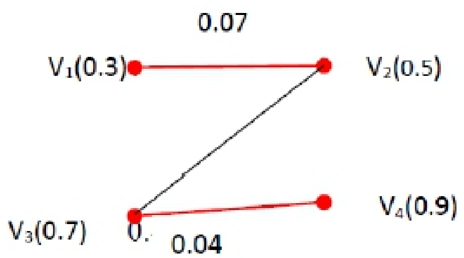
There are three distinct perfect matchings exists. Here the matchings are

$M_1 = \{0.06, 0.08\}$, $M_2 = \{0.07, 0.04\}$ and $M_3 = \{0.2, 0.4\}$.

Similarly we can find the remaining nine fuzzy labelling trees.

Definition 3.6. The weight of the fuzzy labelling tree is the sum of the membership value of the lines in the spanning subgraph.

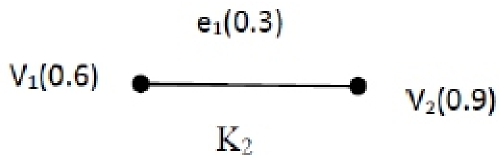
Example:



Here $W(FLT) = 0.07 + 0.4 + 0.04 = 0.51$.

Definition 3.7. An edge in a fuzzy labelling tree is said to be matching bridge if it belongs to any one of the perfect matching.

Example:



Here the line {0.3) is a matching bridge.

Theorem 3.8. Every complete fuzzy labelling graph with even number of vertices $(n \geq 2)$ s a fuzzy labelling tree.

Proof. Let G be a complete fuzzy labelling graph with even number of vertices and M be a perfect matching in G.

To prove G has a fuzzy labelling tree.

Since every fuzzy labelling graph has proper or improper subgraph, G always has the subgraph with fuzzy labeling.

The Matching is a set of non-adjacent edges. So every pair of nodes in spanning subgraph has an alternating path.

Therefore by the definition of Fuzzy labelling tree, G has fuzzy labeling and a fuzzy spanning sub graph $T = (U, V)$ which is a tree in which every pair of nodes contains an alternating path.

Hence always G has a fuzzy labelling tree.

Theorem 3.9. Every fuzzy labeling tree of a given fuzzy labelling graph has the same number of edges.

Proof. Let T_1 and T_2 are two fuzzy labelling trees of a given fuzzy labelling graph G.

To prove T_1 and T_2 have the same number of edges.

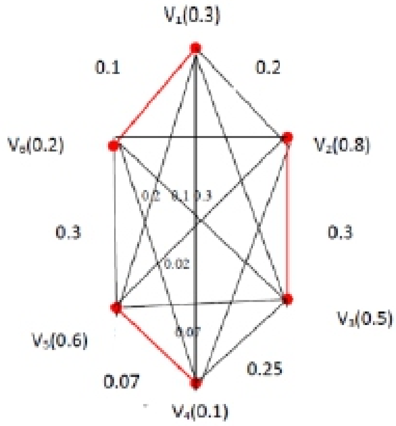
We know that every complete fuzzy labelling graph with even number of vertices $(n \geq 2)$ has a fuzzy labelling tree.

So G has fuzzy labelling trees. And also a tree is a connected acyclic graph.

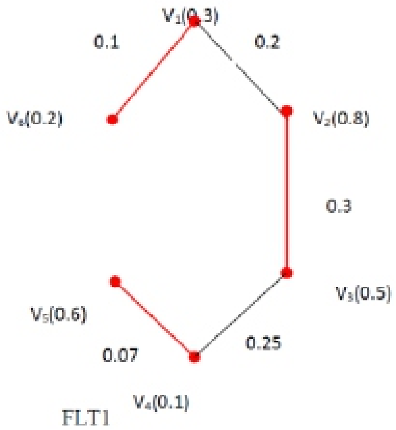
By the properties of a tree, "every tree with n vertices has $n-1$ edges" we have all fuzzy labelling trees have same number of edges.

Hence every fuzzy labeling tree of a given fuzzy labelling graph has the same number of edges.

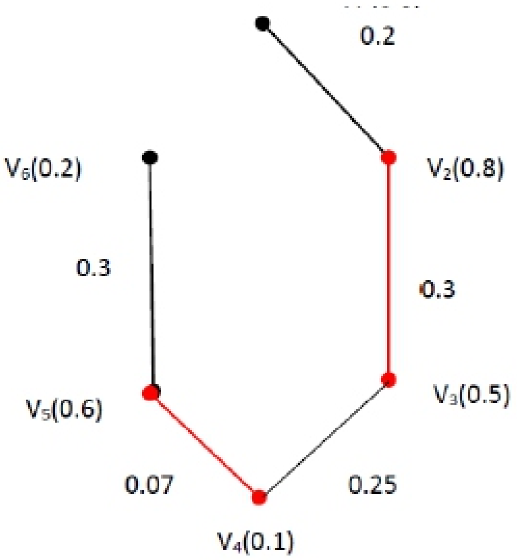
Example:



FLG



FLT1



Here FLT_1 and FLT_2 have five edges. Similarly we can find remaining fuzzy labelling trees with five edges.

Theorem 3.10. Every fuzzy labelling tree contains at least one matching bridge.

Proof. Let G be a fuzzy labelling graph and T be a fuzzy labelling tree of G.

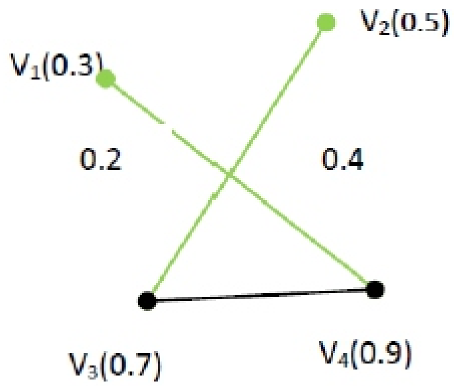
To prove T contains at least one matching bridge.

By the definition of fuzzy labelling tree, "A graph $G = (\alpha, \beta)$ said to be fuzzy labeling tree(FLT) if it has fuzzy labeling and a fuzzy spanning sub graph $T = (U, V)$ which is a tree in which every pair of nodes contains an alternating path", we have T has an alternating path between every pair of nodes in it.

In alternating path, the edges are alternatively in M and $\beta - M$. So T contains at least one edge from perfect matching M.

Therefore every fuzzy labelling tree contains at least one matching bridge.

Example:



Here the matching bridges are 0.2 and 0.4.

Theorem 3.11. *Let G be a fuzzy labeling graph and T be a fuzzy labelling tree of G. Then $G - G(T)$ is again a spanning sub graph which contains a matching.*

Proof. Let G be a fuzzy labeling graph and T be a fuzzy labelling tree of G. To prove $G - G(T)$ is again a spanning sub graph which contains a perfect matching.

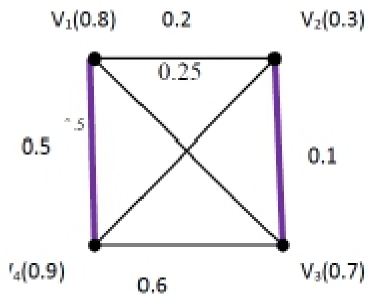
Since T be a fuzzy labelling tree. T contains a spanning subgraph s_1 in which every pair of vertices contains an alternating path.

Now we remove the edges of s_1 from G we obtain another spanning subgraph.

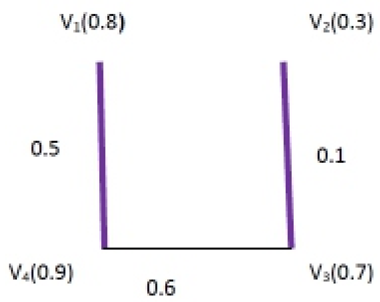
This spanning subgraph also contains an alternating path. It is also contains the edges in the matching.

Hence $G - G(T)$ is again a spanning sub graph which contains a matching.

Example:



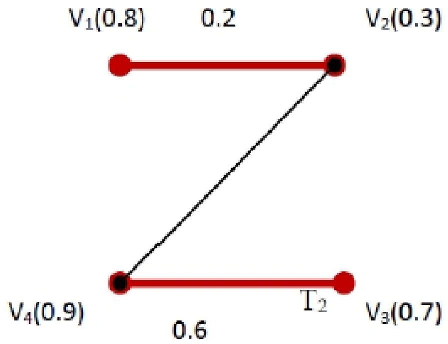
FLG



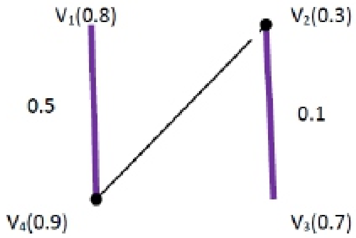
T_1

Therefore every fuzzy labelling tree contains at least one matching bridge.

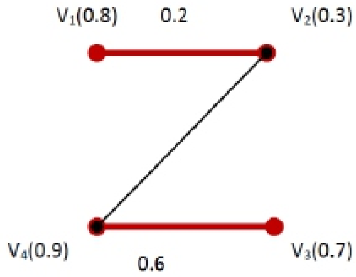
Example:



T_2



T_3



G-E (T_3)

4. CONCLUSION

In this paper, we introduced the new concept of matching in fuzzy labeling tree and its spanning sub graph. We discussed some properties using these concepts and spanning sub graphs of labelling tree using matching and perfect matching. In Future, we will find centre and eccentricity of fuzzy labelling tree using matching and perfect matching.

REFERENCES

[1] Frank Harary, Graph Theory, Indian Student Edition, Narosa/Addison Wesley, 1988.

[2] S. Mathew and M. S. Sunitha, Types of arcs in a fuzzy graph, Information Science 179(11) (2009), 1760-1768.

[3] A. Nagoorgani and D. Rajalaxmi, Fuzzy Labeling tree, International Journal of Pure and Applied Mathematics 90(2) (2014), 131-141.

[4] Narsingh Deo, Graph Theory with applications to Engineering and Computer Science, Englewood Cliffs, NJ: Prentice-Hall, 1974.

[5] K. Ranganathan and R. Balakrishnan, A Text Book of Graph Theory, Berlin/Heidelberg: Springer; 2012.

[6] A. Rosenfeld, Fuzzy graphs, Fuzzy Sets and their Applications to Cognitive and Decision Process, L. A. Zadeh, K. S. Fu, K. Tanaka and M. Shimura, eds., Academic Press, New York, 1975, pp. 75-95.

[7] M. S. Sunitha and A. Vijayakumar, A characterization of fuzzy trees, Information Science 113 (1999), 293-300.

[8] S. Yahya Mohamad and S. Suganthi, Matching in Fuzzy Labeling Graph, International Journal of Fuzzy Mathematical Archive 14(1) (2017), 155-161.

[9] S. Yahya Mohamad and S. Suganthi, Properties of Fuzzy Matching in Set Theory, Journal of Emerging Technologies and Innovative Research 5(2) 2018.

[10] S. Yahya Mohamad and S. Suganthi, Operations in Fuzzy Labeling Graph through Matching and Complete Matching, International Journal of Applied Engineering Research 13(12) (2018). PP10389-10393.

[11] S. Yahya Mohamad and S. Suganthi, Energy of Complete Fuzzy Labeling Graph through Fuzzy Complete Matching, International Journal of Mathematics Trends and Technology 58(3) 2018.

[12] S. Yahya Mohamad and S. Suganthi, Matching and Complete Matching Domination in Fuzzy Labelling Graph, Journal of Applied Science and Computations 5(10) 2018.

[13] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.

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D-Eccentric Domination in Graphs

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ABSTRACT

In a dominating set $D \subseteq V$ of a graph $G(V, E)$ if there exists at least one D-eccentric vertex u of v in D for every $v \in V - D$ then it is called a D-eccentric dominating set. In this article, the D-eccentric dominating set, minimal D-eccentric dominating set and D-eccentric domination number $(D\ ed)\ \gamma$ in graphs are determined. The D-eccentric domination numbers for some standard graphs are established. Some theorems related to D-eccentric domination in graphs are declared and verified.

1. INTRODUCTION

In 1962 O. Ore proposed a new idea dominating set and domination number [9]. In 1998 T. W. Haynes et al., deliberated various dominating parameters [5]. In 2010 T. N. Janakiraman et al., illustrated eccentric domination in graphs [6]. In 2011 M. Bhanumathi et al., detailed eccentric domination in trees and various bounds of eccentric domination in graph [1]. In 2013 L. N. Varma et al., determined D-Distance in graphs [10]. In 2019 A. Mohamed Ismayil et al., developed Detour eccentric domination in graphs [8]. Article [8, 10] inspired us to consider the D-eccentric domination in graphs.

2. PRELIMINARIES

Definition 2.1 [10]. The D-length of a $r - s$ path t is defined as $l^D(t) = d(r, s) + \deg(r) + \deg(s) + \sum \deg(w)$ where sum runs over all intermediate vertices w of t. The D-distance $d^D(r, s) = \min\{l^D(t)\}$, where the minimum is taken over all $r - s$ paths in G.

Definition 2.2 [10]. The D-radius, defined and denoted by $r^D(G) = \min\{e^D(s) : s \in V\}$. The D-diameter, defined and denoted by $d^D(G) = \max\{e^D(s) : s \in V\}$.

Definition 2.3. For a vertex s, each vertex at a D-distance $e^D(s)$ from s is a D-eccentric vertex of s. D-eccentric set of a vertex s is defined as $E^D(s) = \{r \in V / d^D(s) = e^D(s)\}$ or any vertex r for which $d^D(r, s) = e^D(s)$ is called D-eccentric vertex of s.

Definition 2.4. The D-eccentricity of a vertex s is defined by $e^D(s) = \max\{d^D(r, s)/r \in V\}$.

Definition 2.5. The vertex s in G is a D-central vertex if $r^D(G) = e^D(s)$ and the D-center $C^D(G)$ is the set of all central vertices.

Definition 2.6. The D-peripheral of G, $p^D(G) = e^{Dd}(G)$. V is a D-peripheral vertex if $e^D(s) = d^D(G)$. The D-periphery $P^D(G)$ is the set of all peripheral vertices.

Definition 2.7. A sub graph that has the same vertex set as G is called linear factor the degree of all vertices is one.

In this paper, as it were nontrivial basic associated undirected graphs are considered and for all the other vague terms one can allude [2, 3].

3. D-ECCENTRIC DOMINATING SET

Definition 3.1. Let $P \subseteq V(G)$ be a set of vertices in a graph $G,(V, E)$. Then P is said to be a D-eccentric vertex set of G if for every vertex $s \in V - P$ has at least one vertex r such that $r \in E^D(s)$. A D-eccentric vertex set P of G is called minimal D-eccentric vertex set. If no proper subset P' of P is a D-eccentric vertex set of G. The minimum cardinality of a minimal D-eccentric vertex set of P is called the D-eccentric number and is denoted by $e^D(G)$ and simply denoted by e^D. The maximum cardinality of a minimal D-eccentric vertex set is called the upper D-eccentric number and is denoted by $E^D(G)$ and simply denoted by E^D.

Example 3.1. The D-eccentric vertex set and its numbers are defined in a graph $G(V, E)$h suitable example as given below

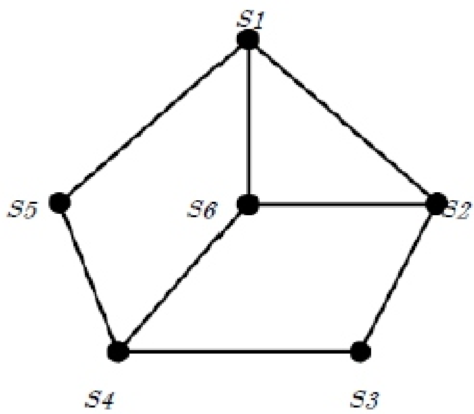

Figure 3.1

In a graph $G(V, E)$ as given figure 3.1, the D-eccentricity of s_1, s_2, s_3, s_4, s_5 and s_6 are respectively $e^D(s_1) = e^D(s_2) = e^D(s_3) = e^D(s_4) = e^D(s_5) = e^D(s_6) = 10$ and the D-eccentric set of s_1, s_2, s_3, s_4, s_5 and s_6 are $E^D(s_1) = \{s_3, s_4\}, E^D(s_2) = \{s_4, s_5\}$ $E^D(s_3) = \{s_1, s_5\}, E^D(s_4) = \{s_1, s_2\}, E^D(s_5) = \{s_2, s_6\}, E^D(s_6) = \{s_3, s_5\}$ respectively. Then the sets $P_1 = \{s_4, s_5\}, P_2 = \{s_1, s_2, s_6\}$ etc., are some D-eccentric vertex sets of $G(V, E)$ and D-eccentric number $e^D = 2$ and upper D-eccentric number $E^D = 3$.

Note: 3.1. r is a D-eccentric vertex of s, then $r \in E^D(s)$.

Observations 3.1.
(1) Every superset of a D-eccentric set is a D-eccentric vertex set.
(2) The subset of a D-eccentric vertex set need not be a D-eccentric vertex set.

(3) In a graph $G(V, E)$, $e^D(G) \leq E^D(G)$.

Definition 3.2. A dominating set $D \subseteq V$ of a graph $G(V, E)$ is said to be a D-eccentric dominating set if for every vertex $s \in V - D$, there exists at least one D-eccentric vertex r of s in D. A D-eccentric dominating set D is a minimal D-eccentric dominating set if there exists a subset $D' \subset D$ which is not a D-eccentric dominating set. The minimum cardinality of a minimal D-eccentric domination set of D is called the D-eccentric domination number and is denoted by γ_{ed}^D. The maximum cardinality of a minimal D-eccentric dominating set of D is called the upper D-eccentric dominating set and is denoted by $\Gamma_{ed}^D(G)$.

Remark 3.1. If P be a minimum D-eccentric vertex set of G then $D \cup S$ is a D-eccentric dominating set of G.

Example 3.2. The D-eccentric dominating set and its numbers are defined in a graph G (V , E) with suitable example as given below

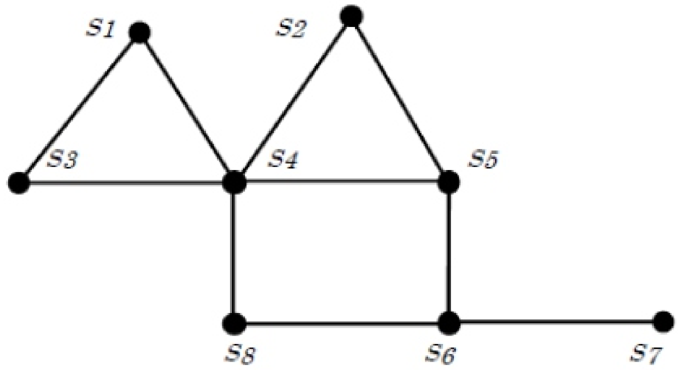

Figure 3.2

In this graph, $E^D(s_1) = \{s_8\}$, $E^D(s_2) = \{s_8\}$, $E^D(s_3) = \{s_8\}$ $E^D(s_4) = \{s_8\}$, $E^D(s_5) = \{s_1, s_3\}$, $E^D(s_6) = \{s_1, s_2, s_3\}$, $E^D(s_7) = \{s_1, s_3\}$, and $E^D(s_8) = \{s_1, s_3\}$. Here $P_1 = \{s_3, s_8\}$, $P_2 = \{s_1, s_8\}$ etc., are some D-eccentric vertex sets and $D_1 = \{s_3, s_4, s_8\}$, $D_2 = \{s_1, s_2, s_6, s_8\}$, etc., are some D-eccentric dominating sets. The D-eccentric domination number is $\gamma_{ed}^D = 3$ and upper D-eccentric domination number is $\Gamma_{ed}^D = 4$.

Results 3.1. (i) For any connected graph G, $\gamma(G) \leq \gamma_{ed}^D(G) \leq \Gamma_{ed}^D$.

(ii) Every D-eccentric dominating set is a dominating set but the converse is not true.

(iii) If $r^D(G) = d^D(G)$, then $\gamma(G) = \gamma_{ed}^D(G)$.

Observation 3.2. For any connected graph, $\gamma_{ed}^D(G) \leq \gamma(G) + e^D(G)$.

Observation 3.3. If G is disconnected then $\gamma(G) = \gamma_{ed}^D(G)$, since vertices from different components are D-eccentric to each other and if G is disconnected graph and r, s are in different components then $d^D(r, s) = \infty$.

Observation 3.4. For any graph $1 \leq \gamma_{ed}^D(G) \leq n$. The bounds are sharp, since $\gamma_{ed}^D(G) = 1$ iff $G = K_n$ and $\gamma_{ed}^D(G) = n$ iff $G = \overline{K_n}$.

4. BOUND ON D-ECCENTRIC DOMINATION

Observations 4.1

(i) $\gamma_{ed}^{D}(K_n) = 1$

(ii) $\gamma_{ed}^{D}(K_{1,n}) = 2,\ n \geq 2$

(iii) $\gamma_{ed}^{D}(K_{m,n}) = 2.$

(iv)

$$\gamma_{ed}^{D}(C_n) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor, & \text{for } n \geq 3 \text{ and } n \neq 5 \text{ where } \lfloor n \rfloor \text{ is a greatest integer less than } n \\ 3, & n = 5. \end{cases}$$

Theorem 4.1.

$$\gamma_{ed}^{D}(W_n) = \begin{cases} 1, & \textit{for } n = 3. \\ \left\lceil \frac{n}{2} \right\rceil, & \textit{for } n = 4, 5, \textit{where } \lceil n \rceil \textit{ is a least integer greatest than } n. \\ \left\lfloor \frac{n}{2} \right\rfloor, & \textit{for } n \geq 6, \textit{ where } \lfloor n \rfloor \textit{ is a greatest integer less than } n. \end{cases}$$

Proof. $G = W_3 = K_4$. Hence $\gamma_{ed}^{D}(W_3) = 1$. When $G = W_4$, consider $D = \{r, s\}$, where r and s are any two adjacent non central vertices. D is a minimum D-eccentric dominating set. Therefore $\gamma_{ed}^{D}(W_4) = 2$. In a graph $G = W_5$, $D = \{r, s, w\}$, where r and s are any two adjacent non central vertices and w is the central vertex. If $G = W_{n,n} \geq 6$ $D = \{r, s, w\}$, when r and s are any two adjacent vertices of w is a central vertex, then D is a minimum D-eccentric dominating set of G. Therefore $\gamma_{ed}^{D}(W_n) = \left\lfloor \frac{n}{2} \right\rfloor$ for $n \geq 6$.

Theorem 4.2. *If the graph* K_n *by deleting edges of a linear factor then* $\gamma_{ed}^{D}(G) = \frac{n}{2}$ *(n = even integer).*

Proof. Let G be graph create from a non-trivial K_n has minimum two components. By the result 3.4, $\gamma(D) = \gamma_{ed}^{D}(G)$. Therefore $\gamma(G) = \frac{n}{2}$ where

G has an even number of vertices. That is $\gamma(G) = \gamma_{ed}^{D}(G) = \frac{n}{2}$.

Proposition 4.1. The domination number of path with four vertices is equal to the D-eccentric domination number of path with four vertices.

Theorem 4.3. *In a path* (P_n) *of order* $n > 2$, $m = 1, 2, \ldots, \frac{n-2}{3}$

$$\gamma_{ed}^{D}(P_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil + 1, \text{ if } n = 3m, \\ \left\lceil \frac{n}{3} \right\rceil, \text{ if } n = 3m+1, \\ \left\lceil \frac{n}{3} \right\rceil + 1, \text{ if } n = 3m+2. \end{cases}$$

Proof. Case (i) $n = 3m$.

Let $s_1, s_2, s_3, \ldots, s_{3m}$ represent the path P_n and has all the peripheral vertices. $D = \{s_2, s_5, s_8, \ldots, s_{3m-1}\}$ is the only γ-set of P_n but not $\gamma_{ed}^{D}(P_n)$. That is $\gamma_{ed}^{D}(P_n)$ is $D' = \{s_1, s_4, s_7, \ldots, s_{3m}\}$ where $|D'| = m + 1 = \gamma(P_n) + 1$. Therefore, $\gamma_{ed}^{D}(P_{3m}). + 1 = \left\lceil \frac{n}{3} \right\rceil + 1$.

Case (ii) $n = 3m + 2$.

$D = \{s_1, s_4, s_7, \ldots, s_{3m+2}, s_{3m+1}\}$ is the least dominating set P_n has two peripheral vertices. Hence, $\gamma_D(P_n) = \gamma_{ed}^{D}(P_n) = \left\lceil \frac{n}{3} \right\rceil$.

Case (iii) $n = 3m + 2$.

$D = \{s_2, s_5, s_8, \ldots, s_{3m+2}\}$ has end vertices s_{3m+2} and it is not a D-eccentric dominating set. Hence, $D \cup \{s_1\}$ is a minimum D-eccentric dominating set. Therefore $\gamma_{ed}^{D}(P_n) = \gamma_D(P_n) + 1 = \left\lceil \frac{n}{3} \right\rceil + 1$.

Remark 4.1. In a path (P_n) of order $n = 2$, $\gamma_{ed}^{D}(P_n) = 1$.

REFERENCES

[1] *M. Bhanumathi and S. Muthammai, On eccentric domination in trees, International Journal of Engineering Science, Advanced Computing and Bio Technology 2(1) (2011), 38-46.*

[2] *J. A. Bondy and U. S. R. Murthy, Graph Theory with Applications, The Macmillan Press Ltd., Great Britain, 1976.*

[3] *F. Buckly and F. Harary, Distance in graphs, Addition-Wesley Publishing Company, Redwood City, San Francisco, Peninsula, 1990.*

[4] *F. Harary, Graph Theory, Addition-Wesley Publishing Company, Reading, Mass., 1972.*

[5] *T. W. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, Inc., New York, 1998.*

[6] *T. N. Janakiraman, M. Bhanumathi and S. Muthammai, Eccentric domination in graphs, International Journal of Engineering Science, Advanced Computing and Bio Technology 1(2) (2010), 55-70.*

[7] *V. R. Kulli, Theory of domination in graphs, Vishwa International Publications, 2010.*

[8] *A. Mohamed Ismayil and R. Priyadharshini, Detour eccentric domination in graphs, Bulletin of Pure and Applied Sciences, Vol.38 E (math & Stat), P.342-347, (2019).*

[9] *O. Ore, Theory of Graphs, Amer. Math. Soc. Colloq. Publ., 38, (Amer. Math. Soc., Providence, RI), (1962).*

[10] *D. Reddy Babu and L. N. Varma, D-Distance in graphs, Golden Research Thoughts 2(9) (2013).*

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Alpha, Beta and Gamma Strong Vertices

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ABSTRACT

In this paper, boundary nodes are easily found with the help of α, β *and* γ *strong nodes from fuzzy vertex order colouring. Some of the properties are discussed.*

1. INTRODUCTION

The concepts of boundary nodes and interior nodes using sum distance are introduced by Tom and Sunitha [6]. The fuzzy vertex order colouring using α , β and γ-strong nodes are introduced by A. Nagoor Gani and B. Fathima Kani [7]. In this paper we connect these three α , β and γ–strong nodes to boundary nodes. In Section 2 we discussed the basic definitions.

Definitions of Boundary nodes, eccentricity nodes, fuzzy radius and fuzzy diameter are given in section 3 and also properties related to boundary nodes and α , β and γ strong nodes are also discussed.

2. BASIC DEFINITIONS

A fuzzy graph $G:(\sigma,\mu)$ is said to be complete fuzzy graph if $\mu(u,v)=\sigma(u)\wedge\sigma(v)$ for all $u, v\in V$. Let $G^*:(V,E)$ be a graph. Length of $u-v$ path P as the sum of the weights of the arcs in P, $L(P)=\sum_{i=1}^{n}\mu(u_{i-1},u_i)$. The sum distance between u and v denoted by $d_s(u,v)=\min\{L(P_i):P_i\in P, i=1,2,\ldots\}$. $d_s(u,v)$ is a metric on V. A node v is strictly α-strong if $d(v)>d(u), \forall u\in A(v)$. A γ-strong node is strict always. A β-strong vertex is strong as well as weak.

3. Relation among Boundary Nodes and α, β and γ Strong Vertices

Definition 3.1 [6]. A boundary node in a connected fuzzy graph satisfies $d_s(u,v)\geq d_s(u,w)$ for each neighbour w of v. Collection of all boundary nodes denoted by u^b.

Example 3.2. Here $d(u)=1.5, d(v)=0.9, d(w)=1.7, d(x)=1.3$. $S_\alpha(V)=\{w\}, \beta(V)=\{u\}, W_\gamma(V)=\{v,x\}$. In example 3.2 $u^b=\{x\}$, $v^b=\{x\}$, $w^b=\{v,x\}$, $x^b=\{v\}$. The boundary nodes of G are $\{v,x\}$.

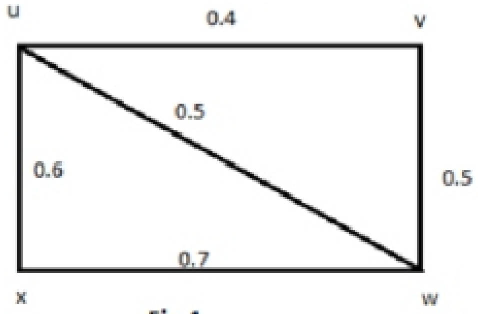

Fig.1

Property 3.3. In a fuzzy-graph all α, γ-strong nodes are the boundary nodes.

Proof. If $x \in u^b$, (i.e.) $u^b = \{x\}$, then $d_s(u, x) \geq d_s(u, w)$ for each neighbour w of x. That is $d_s(u, x) \geq d_s(u, A(x))$. A vertex is a weak or γ-strong vertex if $d(x) < d(A(x)) \, \forall x \in A(x)$. Combining these two equations we get $d(A(x)) > d(x) > d_s(u_i, x) \geq d_s(u_i, A(x))$ or $d(A(x)) > d_s(u_i, x) > d(x)$ $\geq d_s(u_i, A(x))$ for every vertices $u_i \in V$. We know that if a node v is α-strong if $\{\forall v \in V / d(v) \geq d(A(v))\}$. It is clear that $d(v) \geq d(A(v)) \geq d_s(u_i, v)$ $\geq d_s(u_i, A(v))$. Thus α-strong nodes are also serve as a boundary nodes.

Example 3.4. Consider fuzzy graph G. Here $u^b = \{x\}$, $v^b = \{u, x\}$, $w^b = \{u, x\}$, $x^b = \{u, y\}$, $y^b = \{u, x\}$. Boundary nodes of G are $\{u, x, y\}$.

Here $d(u) = 1.4$, $d(v) = 2.0$, $d(w) = 1.8$, $d(x) = 2.2$, $d(y) = 1.0$, $S_\alpha(V) = \{x\}$, $\beta(V) = \{v, w\}$, $W_\gamma(V) = \{u, y\}$.

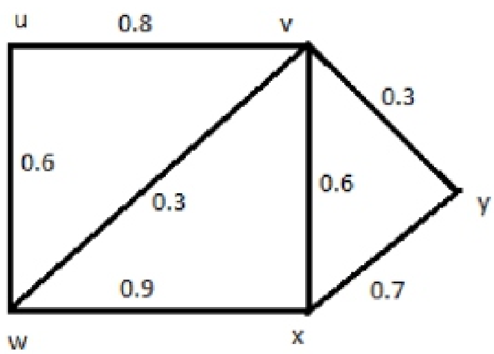

Fig. 2

Thus we clear that α-strong node $\{x\}$ and γ-strong nodes $\{u, y\}$ are boundary nodes.

Definition 3.5 [5]. The fuzzy eccentricity of a vertex u is $e_w(u) = \max_{v \in V} d_s(u, v)$.

Definition 3.6 [5]. The min of the fuzzy eccentricities of all vertices is the fuzzy radius say, $\gamma_s(G) = \min_{u \in V} e_w(u)$.

Definition 3.7 [5]. The max of the fuzzy eccentricities of all the vertices is called the fuzzy diameter say, $d_s(G) = \max_{u \in V} e_w(u)$.

Property 3.8. In a fuzzy-graph β-strong node is a fuzzy radius and γ- strong nodes are fuzzy diameter of a fuzzy graph.

Proof. Property 3.3 states that, fuzzy graph G have boundary nodes which are α and γ-strong nodes. Thus the sum distance of γ-strong nodes are greater than or equal to its sum distance of their neighbours. We know that fuzzy diameter is the max of all eccentricities. Thus γ-strong nodes are fuzzy diameter. Only β-strong nodes have the min eccentricity. Hence β-strong nodes are fuzzy radius.

Example 3.9. Here $(u) = 1.2$, $d(v) = 1.7$, $d(w) = 1.1$, $d(x) = 1.8$. $S_\alpha(V) = \{x\}$, $\beta(V) = \{v\}$, $W_\gamma(V) = \{u, w\}$.

Property 3.8. In a fuzzy-graph β-strong node is a fuzzy radius and γ- strong nodes are fuzzy diameter of a fuzzy graph.

Proof. Property 3.3 states that, fuzzy graph G have boundary nodes which are α and γ-strong nodes. Thus the sum distance of γ-strong nodes are greater than or equal to its sum distance of their neighbours. We know that fuzzy diameter is the max of all eccentricities. Thus γ-strong nodes are fuzzy diameter. Only β-strong nodes have the min eccentricity. Hence β-strong nodes are fuzzy radius.

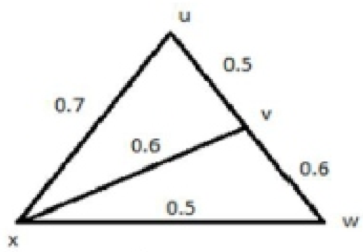

Fig.3

In this example $u^b = \{w\}$, $v^b = \{w, x\}$, $w^b = \{u\}$, $x^b = \{u\}$. Hence the boundary nodes of G are $\{u, x, w\}$. It is clear that α-strong node and γ-strong nodes are the boundary nodes of G.

Here $e_w(u) = 1.1$, $e_w(v) = 0.6$, $e_w(w) = 1.1$, $e_w = 0.7$.

We know that $\gamma_s(G) = \min_{u \in V} e_w(u) = e_w(v) = 0.6$. Thus β-strong node '$v$' is fuzzy radius. And $d_s(G) = \max_{u \in V} e_w(u) = e_w(u) = e_w(u) = 1.1$. Hence γ-strong nodes are fuzzy diameter of G.

Property 3.10. Let G be a fuzzy graph and its underlying crisp graph is complete. Then α-strong and β-strong nodes are the boundary nodes.

Example 3.11. Here $d(u) = 2.0,\ d(v) = 1.5,\ d(w) = 1.8,\ d(x) = 1.9.$ $S_\alpha(V) = \{u\},\ \beta(V) = \{x, w\}, W_\gamma(V) = \{v\}.$ In this example 3.11, $u^b = \{w\},$ $v^b = \{x\}, w^b = \{u\}, x^b = \{u\}.$ Hence the boundary nodes are $\{u, x, w\}.$ It is clear that Then α-strong and β-strong nodes are the boundary nodes.

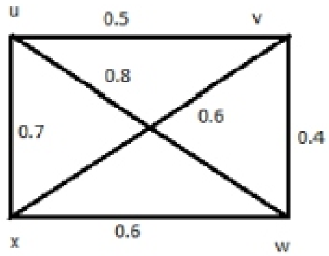

Fig.4

Property 3.12. Let G be a fuzzy-graph with its underlying crisp graph is complete. γ-strong nodes are the fuzzy radius and α-strong and β-strong nodes are the fuzzy diameter.

In example 3.11 $e_w(u) = 0.8,\ e_w(v) = 0.6,\ e_w(w) = 0.8,\ e_w(x) = 0.7.$

We know that $\gamma_s(G) = \min_{u \in V} e_w(u) = e_w(v) = 0.6.$ Thus γ-strong node 'v' is fuzzy radius. And $d_s(G) = \max_{u \in V} e_w(u) = e_w(u) = e_w(w).$ Hence α-strong and β-strong nodes are fuzzy diameter of G.

Property 3.13. If a fuzzy-graph G have more than one β-strong nodes then the highest degree β-strong node is fuzzy radius.

Example 3.14. Here $e_w = 1.4,\ e_w(v) = 0.8,\ e_w(w) = 1.6,\ e_w(x) = 1.4,$ $e_w(y) = 1.6.$ The vertex 'v' has the min fuzzy eccentricity. Thus the fuzzy radius is 'v', (i.e.) β-strong node. But in this example we have two β-strong nodes, say $\{w, v\}.$ Degree of these nodes are $d(w) = 1.8,\ d(v) = 2.0.$ Thus the highest degree node belong to fuzzy radius.

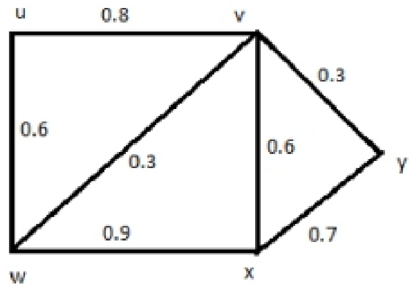

Fig.5

4. CONCLUSION

The graphs without vertices are meaningless. In the similar way in a fuzzy graph nodes are the most important for finding many applications in more areas like networking. Finding the nodes on the network boundary is the need for correct operation in wireless applications. Focusing on these applications a research under boundary nodes in a fuzzy graph are carried out. But finding boundary nodes in small order fuzzy graphs is very tedious way. For simplifying this process of finding boundary nodes, we can use the three strong nodes α-strong and β-strong, γ-strong. We can find these three nodes very easily as per the definitions. It is observed and verified that α- strong and γ-strong nodes are served as a boundary nodes. But if we consider a fuzzy graph G with its underlying crisp graph is complete then α-strong and β-strong nodes are served as a boundary nodes.

REFERENCES

[1] K. R. Bhutani and A. Rosenfeld, Fuzzy end nodes in fuzzy graphs, Information Sciences 152 (2003), 323-326.

[2] G. Chartr and P. Zang, Introduction to Graph Theory, Tata McGraw Hill Edition, 2006.

[3] F. Harary, Graph Theory, Addition-Wesley Publishing Company, Inc., 1969.

[4] J. N. Mordeson and P. S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica-Verlag, 2000.

[5] J. K. Mathew and Sunil Mathew, A characterization of fuzzy self centered graphs, Annals of Fuzzy Mathematics and Informatics 10(2) (2015), 249-260.

[6] Mini Tom and M. S. Sunitha, Boundary and interior nodes in a fuzzy graph using sum distance, Fuzzy Inf. Eng. 8 (2016), 75-85.

[7] A. Nagoor Gani and B. Fathima Kani, Fuzzy vertex order colouring, International Journal of Pure and Applied Mathematics 107(3) (2016), 601-614.

[8] A. Rosenfeld, Fuzzy Graphs, In: L. A. Zadeh, K. S. Fu, M. Shimura (Eds.), Fuzzy Sets and Their Application to Cognitive and Decision Processes, Academic Press, New York, 1975, pp77-95.

[9] R. T. Yeh and S. Y. Bang, Fuzzy Relations, Fuzzy Graphs and Their Application to Clustering Analysis, Academic Press, New York (1975), 125-149.

[10] L. A. Zadeh, Fuzzy Sets, Information and Control 8 (1965), 338-353.

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Finding Optimal Solution of the Transportation Problem with Modern Zero Suffix Method

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ABSTRACT

In this paper a new procedure namely Modern Zero Suffix (MOZES) method is proposed to find the IFBS it meets optimal solution for the transportation problem. A new algorithm is generated to find the optimal solution for the transportation problem with the aid of above said notion. The relevant numerical illustrations are given to justify the above proposed notion.

1. INTRODUCTION

In operations research transportation problem is a modern class of linear programming problem. Transportation problems used in various fields such as scheduling, assignment, and product mix problems and so on. Dantzing G. B [2] solved linear programming and extensions. Nagoor Gani and Stephen Dinagar [3] proposed a note on linear programming in fuzzy environment. Abdul Quddoos et al. [1] finding a new method of an optimal solution for transportation problems. Stephen Dinagar and Keerthivasan [4, 5, 6] suggested some new algorithm for transportation problem.

In this effort, a new method is recommended to examine optimality of the TP. Also, the new algorithm provided now to find the optimality. A relative study is too carried out by solving transportation problems.

The organization of this paper is arranged as follows. In section 2, the proposed Modern Zero Suffix method is lead with its algorithm illuminated step by step. The numerical illustrations are obtainable in section 3. A relative analysis is carried out in section 4. Finally the conclusion part is in section 5.

2. ALGORITHM OF MODERN ZERO SUFFIX (MOZES) METHOD

Step 1: Build the cost tables from the certain problem. Inspect whether sum of the supply equal to sum of the demand, if it stable then go to step 2. If not introduce a dummy row or dummy column.

Step 2: In a cost matrix, deduct each row by the least element of this row. From the concentrated matrix deduct each column by the least element of its column. From the concentrated matrix, every row and every column has no less than one zero.

Step 3: Select one zero and computation the number of zeros in the corresponding row and column expect the selected zero, and mark the sum of the number of zeros in suffix.

Step 4: Select every zero and mark the suffix as the way of step 3.

Step 5: Select the lowest suffix and allocate the conforming cell. Each allocation is in rising order of suffices.

Step 6: Sometime suffix values are identical; select the minimum cost cell of the conforming suffix values.

Step 7: When the allocation if we ensure less than m+n-1 cells, reprise the process from step 2 to step 6.

Step 8: Remain the process all rim necessities are fulfilled.

3. NUMERICAL ILLUSTRATIONS

Illustration 3.1. Solve the optimal cost of TP with three factories and three markets:

Table-3.1.1

	M_1	M_2	M_3	Supply
F_1	3	3	5	9
F_2	6	5	4	8
F_3	6	10	7	10
Demand	7	12	8	

Solution

Table-3.1.2

	M_1	M_2	M_3	Supply
F_1	0_2	0_1	2	9
F_2	2	1	0_0	8
F_3	0_1	4	1	10
Demand	7	12	8	

Here minimum suffix value is 0 and the conforming supply and demand are equal so we have a choice the next minimum suffix is 1.

Table-3.1.3

	M_1	M_2	M_3	Supply
F_1	3	3_9	5	9
F_2	6	5_3	4_5	8
F_3	6_7	10	7_3	10
Demand	7	12	8	

Minimum Cost =125.

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Illustration 3.2. Solve the optimal cost of TP with three factories and four markets:

Table-3.2.1

	M_1	M_2	M_3	M_4	Supply
F_1	9	8	5	7	12
F_2	4	6	8	7	14
F_3	5	8	9	5	16
Demand	8	18	13	3	

Solution:

Table-3.2.2

	M_1	M_2	M_3	M_4	Supply
F_1	4	1	0_0	2	12
F_2	0_2	0_1	4	3	14
F_3	0_2	1	4	0_1	16
Demand	8	18	13	3	

Table-3.2.3

	M_1	M_2	M_3	M_4	Supply
F_1	9	8	5_{12}	7	12
F_2	4	6_{14}	8	7	14
F_3	5_8	8_4	9_1	5_3	16
Demand	8	18	13	3	

Minimum cost = 240.

Illustration 3.3. Solve the optimal cost of TP with four factories and three markets:

Table-3.3.1

	M_1	M_2	M_3	Supply
F_1	3	2	8	250
F_2	1	6	3	350
F_3	7	5	3	400
F_4	5	9	2	200
Demand	300	400	500	

Solution:

Table-3.3.2

	M_1	M_2	M_3	Supply
F_1	2	0_0	6	250
F_2	0_0	4	1	350
F_3	5	2	0_1	400
F_4	3	7	0_1	200
Demand	300	400	500	

Table-3.3.3

	M_1	M_2	M_3	Supply
F_1	3	2_{250}	8	250
F_2	1_{300}	6	3_{50}	350
F_3	7	5_{150}	3_{250}	400
F_4	5	9	2_{200}	200
Demand	300	400	500	

Minimum Cost = 2850.

4. Result Analysis

Above table, it is evidently noted that our suggested method "Modern Zero Suffix Method" is meet to MODI method.

METHOD	Total Transportation Cost		
	Illustration-1	Illustration-2	Illustration-3
Least Cost Method	159	248	2850
VAM	143	248	2850
MODI- Method	125	240	2850
Modern Zero Suffix (MOZES)Method	125	240	2850

5. CONCLUSION

A novel approach is proposed and termed it as modern zero suffix (MOZES) method to inspect the optimal solution for the TP. The leading improvement of the proposed algorithm is very calm to realize and performs stress-free calculation and provides the optimal solution with least steps. This new Proposed method is more real to find out the minimum cost when associate with additional existing methods for decision makers.

REFERENCES

[1] Abdul Quddoos, Shakeel Javaid and M. M. Khalid, A New Method for Finding an Optimal Solution for Transportation Problems, International Journal on Computer Science and Engineering (IJCSE), ISSN : 0975-3397 ,Vol. 4 No. 07 July 2012.

[2] G. B. Dantzing, Linear Programming and Extensions, New Jersey: Princeton University Press. (1963).

[3] A. Nagoor Gani and D. Stephen Dinagar, A note on linear programming in fuzzy environment, Proc. Nat. Aca. Sci. India, (2009) Sec A, Part-I.

[4] D. Stephen Dinagar and R. Keerthivasan, Optimal solution of transportation problem by Ragupathy method, The International Journal of analytical and experimental modal analysis, ISSN : 0886-9367, Vol. XI, Issue X, October/2019, PP. 115-119.

[5] D. Stephen Dinagar and R. Keerthivasan, Solving transportation problem using modified best candidate method, Journal of Computer and Mathematical Sciences, ISSN: 0976-5727.

[6] D. Stephen Dinagar and R. Keerthivasan, Solving fuzzy transportation problem with modified zero suffix method, American International Journal of Research in Science, Technology, Engineering & Mathematics, ISSN: 2328-3491,PP: 68-72,2019.

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On Interval Valued Neutrosophic Fuzzy Matrices

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ABSTRACT

Matrices play significant roles in various areas in science and engineering. The problems involving various types of uncertainties cannot be solved by the classical matrix theory. Neutrosophic sets theory was proposed by Florentin Smarandache in 1999, where each element had three associated essential functions, namely the membership function (T), the non-membership (F) function and the indeterminacy function (I) defined on the universe of discourse X, the three functions are entirely independent. In this paper, the interval-valued neutrosophic fuzzy matrix (IVNFM) is introduced. Some fundamental operations are also presented. The need of the interval-valued neutrosophic fuzzy matrix (IVNFM) is explained by an illustration. Illumination of some of the operators are given with the help of the example.

1. INTRODUCTION

Academics in economics, sociology, medical science, industrial, atmosphere science and many other numerous fields agree with the vague, imprecise and infrequently lacking information of exhibiting inexact data. As a result, fuzzy set theory was introduced by L. A. Zadeh [15]. Then, the intuitionistic fuzzy sets was developed by K. A. Atanassov [1, 2]. Estimation of non-membership values is also not constantly possible for the identical reason as in case of membership values and so, there exists an indeterministic part upon which hesitation persists. As a result, Smarandache et al. [8, 9] has introduced the concept of Neutrosophic Set (NS) which is a generalization of conventional sets, fuzzy set, intuitionistic fuzzy set etc.

The problems concerning various types of hesitations cannot solved by the classical matrix theory. That type of problems are solved by using fuzzy matrix [13, 14]. Fuzzy matrix deals with only membership values. These matrices cannot deal non membership values. Intuitionistic fuzzy matrices (IFMs) introduced first time by Khan, Shyamal and Pal [12]. But, essentially it is difficult to measure the membership or non membership value as a point. So, we consider the membership value as an interval and also in the case of non membership values, it is not nominated as a point, it can be considered as an interval. Interval valued Intuitionistic fuzzy matrices was considered by Madhumangal pal et al [13]. But, the indeterminate values cannot be considered by the Intuitionistic fuzzy matrices. Hence, the concept is extended to interval valued neutrosophic fuzzy matrices (IVNFMs) and some basic operators on IVNFMs are introduced. The interval-valued neutrosophic fuzzy determinant (IVNFD) is also defined. A real life problem on IVNFM is presented. Interpretation of some of the operators are given with the help of this example.

In this work, some definitions are discussed in section 2. Section 3 dealt with the operations of interval valued neutrosophic matrices. Properties of interval valued neutrosophic matrices are given in section 4. The importance of IVNFM is discussed in section 5. Concluding remarks are given in section 6.

2. DEFINITION AND PRELIMINARIES

In this section, we first define the neutrosophic fuzzy matrix (NFM) based on the definition of neutrosophic fuzzy sets introduced by Smarandache [8, 9]. The intuitionistic fuzzy matrices are introduced by M. Pal et al. [13]. The same concept is extended to neutrosophic fuzzy matrices here.

Definition 1. Neutrosophic fuzzy matrix (NFM): An neutrosophic fuzzy matrix (NFM) A of order $m \times n$ is defined as $A = [X_{ij}, \langle a_{ij\mu}, a_{ij\lambda}, a_{ijv} \rangle]_{m \times n}$, where $a_{ij\mu}, a_{ij\lambda}, a_{ijv}$ are called truth, indeterminacy and falsity of X_{ij} in A, which maintaining the condition $0 \leq a_{ij\mu} + a_{ij\lambda} + a_{ijv} \leq 3$. For simplicity, we write $A = [X_{ij}, a_{ij}]_{m \times n}$ or simply $[a_{ij}]_{m \times n}$ where $a_{ij} = \langle a_{ij\mu}, a_{ij\lambda}, a_{ijv} \rangle$.

Using the concept of neutrosophic fuzzy sets and interval valued fuzzy sets, we define interval valued neutrosophic fuzzy matrices as follows:

Definition 2. Interval-valued neutrosophic fuzzy matrix (IVNFM): An interval valued neutrosophic fuzzy matrix (IVNFM) A of order $m \times n$ is defined as $A = [X_{ij}, \langle a_{ij\mu}, a_{ij\lambda}, a_{ijv} \rangle]_{m \times n}$, where $a_{ij\mu}, a_{ij\lambda}$ and a_{ijv} are the subsets of $[0, 1]$ which are denoted by $a_{ij\mu} = [a_{ij\mu L}, a_{ij\lambda U}]$, $a_{ij\lambda} = [a_{ij\lambda L}, a_{ij\lambda U}]$ and $a_{ijv} = [a_{ij\mu L}, a_{ijvU}]$ which maintaining the condition $0 \leq \sup a_{ij\mu} + \sup a_{ij\lambda} + \sup a_{ijv} \leq 3$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Definition 3. Interval-valued neutrosophic fuzzy determinant (IVIFD): An interval valued neutrosophic fuzzy determinant (IVNFD) function $f : M \to F$ is a function on the set M (of all $n \times n$ IVNFMs) to the set F, where F is the set of elements of the form $\langle [a_{\mu L}, a_{\mu U}], [a_{\lambda L}, a_{\lambda U}], [a_{vL}, a_{vU}] \rangle$, maintaining the condition $0 \leq a_{\mu U} + a_{\lambda U} + a_{vU} \leq 3$, $0 \leq a_{\mu L} \leq a_{\mu U} \leq 1$ and $0 \leq a_{\lambda L} + a_{\lambda U} \leq 1$, $0 \leq a_{vL} \leq a_{vU} \leq 1$ such that $A \subset M$ then $f(A)$ or $|A|$ or $\det(A)$ belongs to F and is given by

$$|A| = \sum_{\sigma \in s_n} \prod_{i=1}^{n} \langle [a_{i\sigma(i)\mu L}, a_{i\sigma(i)\mu U}], [a_{i\sigma(i)\lambda L}, a_{i\sigma(i)\lambda U}], [a_{i\sigma(i)vL}, a_{i\sigma(i)vU}] \rangle$$

and s_n denotes the symmetric group of all permutations of the symbols $\{1, 2, \ldots, n\}$.

Definition 4. The adjoint IVNFM of an IVNFM: The adjoint IVNFM of an IVNFM A of order $n \times n$, is denoted by adj. A and is defined by adj. $A = [A_{ji}]$ where A_{ji} is the determinant of the IVNFM A of order

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$(n-1)\times(n-1)$ formed by suppressing row j and column i of the IVNFM A. In other words, A_{ji} can be written in the form

$$\sum_{\sigma\in s_{n_i n_j}}\prod_{t\in n_j}\langle[a_{t\sigma(t)\mu L},\ a_{t\sigma(t)\mu U}],\ [a_{t\sigma(t)\lambda L},\ a_{t\sigma(t)\lambda U}],\ [a_{t\sigma(t)\nu L},\ a_{t\sigma(t)\nu U}]\rangle$$

where, $n_j = \{1, 2, \ldots, n\}\setminus\{j\}$ and $s_{n_i n_j}$ is the set of all permutations of set n_j over the set n_i. Depending on the values of diagonal elements, the unit IVNFM are classified into two types: (i) a-unit IVNFM and (ii) r-unit IV NFM.

Definition 5. Acceptance unit IVNFM (a-unit IVNFM): A square IVNFM is a-unit IVNFM if all diagonal elements are $\langle[1, 1], [0, 0], [0, 0]\rangle$ and all remaining elements are $\langle[0, 0], [1, 1], [1, 1]\rangle$ and it is denoted by $I_{\langle[0,\,0],\,[1,\,1],\,[1,\,1]\rangle}$.

Definition 6. Rejection unit IVNFM (r-unit IVNFM): A square IVNFM is a r-unit IVNFM if all diagonal elements are $\langle[0, 0], [1, 1], [1, 1]\rangle$ and all remaining elements are $\langle[1, 1], [0, 0], [0, 0]\rangle$ and it is denoted by $I_{\langle[1,\,1],\,[0,\,0],\,[0,\,0]\rangle}$. Similarly, three types of null IVNFMs are defined on its elements.

Definition 7. Complete null IVNFM (c-null IVNFM): An IVNFM is a c-null IVNFM if all the elements are $\langle[0, 0], [0, 0], [0, 0]\rangle$.

Definition 8. Acceptance null IVNFM (a-null IVNFM): An IVNFM is a a-null IVNFM if all the elements are $\langle[0, 0], [1, 1], [1, 1]\rangle$.

Definition 9. Rejection null IVNFM (r-null IVNFM): An IVNFM is a r-null IVNFM if all the elements are $\langle[1, 1], [0, 0], [0, 0]\rangle$.

3. SOME OPERATIONS ON IVNFMS

Let $A = [\langle[a_{ij\mu L}, a_{ij\mu U}], [a_{ij\lambda L}, a_{ij\lambda U}], [a_{ij\nu L}, a_{ij\nu U}]\rangle]$, $B = [\langle[b_{ij\mu L}, b_{ij\mu U}], [b_{ij\lambda L}, b_{ij\lambda U}], [b_{ij\nu L}, b_{ij\nu U}]\rangle]$, be two IVNFMs. Then,

(i) $\langle[a_{ij\mu L}, a_{ij\mu U}], [a_{ij\lambda L}, a_{ij\lambda U}], [a_{ij\nu L}, a_{ij\nu U}]\rangle + \langle[b_{ij\mu L}, b_{ij\mu U}], [b_{ij\lambda L}, b_{ij\lambda U}], [b_{ij\nu L}, b_{ij\nu U}]\rangle = [\max(a_{ij\mu L}, a_{ij\mu L}), \max(a_{ij\mu U}, a_{ij\mu U})]$

$\langle[\min(a_{ij\lambda L}, b_{ij\lambda L}), \min(a_{ij\lambda U}, b_{ij\lambda U})]\rangle[\min(a_{ij\nu L}, b_{ij\nu L}), \min(a_{ij\nu U}, b_{ij\nu U})]$

(ii) $\langle[a_{ij\mu L}, a_{ij\mu U}], [a_{ij\lambda L}, a_{ij\lambda U}], [a_{ij\nu L}, a_{ij\nu U}]\rangle \cdot \langle[b_{ij\mu L}, b_{ij\mu U}], [b_{ij\lambda L}, b_{ij\lambda U}],$

$[b_{ij\nu L}, b_{ij\nu U}]\rangle = [\max(a_{ij\mu L}, b_{ij\mu L}), \max(a_{ij\mu U}, b_{ij\mu U})]$

$\langle[\min(a_{ij\lambda L}, b_{ij\lambda L}), \min(a_{ij\lambda U}, b_{ij\lambda U})]\rangle [\min(a_{ij\nu L}, a_{ij\lambda L}), \min(a_{ij\nu U}, a_{ij\nu U})]$

$[\max(a_{ij\mu L}, b_{ij\mu L}), \max(a_{ij\mu U}, b_{ij\mu U})],$

(iii) $A + B = \langle[\min(a_{ij\mu L}, b_{ij\mu L}), \min(a_{ij\lambda U}, b_{ij\lambda U})]\rangle, [\min(a_{ij\nu L}, b_{ij\lambda L}),$ $\min(a_{ij\nu U}, b_{ij\nu U})], [\max(a_{ij\mu L}, b_{ij\mu L}), \max(a_{ij\mu U}, b_{ij\mu U})]$

(iv) $A \cdot B = \langle[\min(a_{ij\lambda L}, b_{ij\lambda L}), \min(a_{ij\lambda U}, b_{ij\lambda U})]\rangle, [\min(a_{ij\nu L}, b_{ij\lambda L}),$ $\min(a_{ij\nu U}, b_{ij\nu U})],$

(v) $\overline{A} = [\langle[a_{ij\nu L}, a_{ij\nu U}], [1 - a_{ij\lambda L}, 1 - a_{ij\lambda U}], [a_{ij\mu L}, a_{ij\mu U}]\rangle]$. (complement of A)

(vi) $A^T = \langle[a_{ji\mu L}, a_{ji\mu U}], [a_{ji\lambda L}, a_{ji\lambda U}], [a_{ji\nu L}, a_{ji\nu U}]\rangle_{n\times m}$ (Transpose of A)

(vii) $A \oplus B = [[a_{ji\mu L} + b_{ji\mu L} - a_{ij\mu L} \cdot b_{ij\mu L}, a_{ij\mu U} + b_{ij\mu U} - a_{ij\mu U} \cdot b_{ij\mu U}],$ $\langle[a_{ij\lambda L} \cdot b_{ij\lambda L}, a_{ij\lambda U} \cdot b_{ij\lambda U}]\rangle [a_{ij\nu L} \cdot b_{ij\nu L}, a_{ij\nu U} \cdot b_{ij\nu U}]]$

$$\text{(viii)}\ A \odot B = \begin{bmatrix} [a_{ij\mu L} \cdot b_{ij\mu L}, a_{ij\mu U} \cdot b_{ij\mu U}] \\ \langle[a_{ij\lambda L} + b_{ij\lambda L} - a_{ij\lambda L} \cdot b_{ij\lambda L}, a_{ij\lambda U} + b_{ij\lambda U} - a_{ij\lambda U} \cdot b_{ij\lambda U}]\rangle \\ \langle[a_{ij\nu L} + b_{ij\nu L} - a_{ij\nu L} \cdot b_{ij\nu L}, a_{ij\nu U} + b_{ij\nu U} - a_{ij\nu U} \cdot b_{ij\nu U}]\rangle \end{bmatrix}$$

$$\text{(ix)}\ A @ B = \begin{bmatrix} \left[\dfrac{a_{ij\mu L} + b_{ij\mu L}}{2}, \dfrac{a_{ij\mu U} + b_{ij\mu U}}{2}\right] \\ \left\langle\left[\dfrac{a_{ij\lambda L} + b_{ij\lambda L}}{2}, \dfrac{a_{ij\lambda U} + b_{ij\lambda U}}{2}\right]\right\rangle \\ \left[\dfrac{a_{ij\nu L} + b_{ij\nu L}}{2}, \dfrac{a_{ij\nu U} + b_{ij\nu U}}{2}\right] \end{bmatrix}$$

$$\text{(x)}\ A \$ B = \begin{bmatrix} [\sqrt{a_{ij\mu L} + b_{ij\mu L}}, \sqrt{a_{ij\mu U} + b_{ij\mu U}}] \\ \langle[\sqrt{a_{ij\lambda L} + b_{ij\lambda L}}, \sqrt{a_{ij\lambda U} + b_{ij\lambda U}}]\rangle \\ [\sqrt{a_{ij\nu L} + b_{ij\nu L}}, \sqrt{a_{ij\nu U} + b_{ij\nu U}}] \end{bmatrix}$$

$$\text{(xi)}\quad A \# B = \begin{bmatrix} \left[\dfrac{2a_{ij\mu L}\cdot b_{ij\mu L}}{a_{ij\mu L}+b_{ij\mu L}}, \dfrac{2a_{ij\mu U}\cdot b_{ij\mu U}}{a_{ij\mu U}+b_{ij\mu U}}\right] \\ \left\langle\left[\dfrac{2a_{ij\lambda L}\cdot b_{ij\lambda L}}{a_{ij\lambda L}+b_{ij\lambda L}}, \dfrac{2a_{ij\lambda U}\cdot b_{ij\lambda U}}{a_{ij\lambda U}+b_{ij\lambda U}}\right]\right\rangle \\ \left[\dfrac{2a_{ij\upsilon L}\cdot b_{ij\upsilon L}}{a_{ij\upsilon L}+b_{ij\upsilon L}}, \dfrac{2a_{ij\upsilon U}\cdot b_{ij\upsilon U}}{a_{ij\upsilon U}+b_{ij\upsilon U}}\right] \end{bmatrix}$$

$$\text{(xii)}\quad A * B = \begin{bmatrix} \left[\dfrac{a_{ij\mu L}+b_{ij\mu L}}{2(a_{ij\mu L}\cdot b_{ij\mu L}+1)}, \dfrac{a_{ij\mu U}+b_{ij\mu U}}{2(a_{ij\mu U}\cdot b_{ij\mu U}+1)}\right] \\ \left[\dfrac{a_{ij\lambda L}+b_{ij\lambda L}}{2(a_{ij\lambda L}\cdot b_{ij\lambda L}+1)}, \dfrac{a_{ij\lambda U}+b_{ij\lambda U}}{2(a_{ij\lambda U}\cdot b_{ij\lambda U}+1)}\right] \\ \left[\dfrac{a_{ij\upsilon L}+b_{ij\upsilon L}}{2(a_{ij\upsilon L}\cdot b_{ij\upsilon L}+1)}, \dfrac{a_{ij\upsilon U}+b_{ij\upsilon U}}{2(a_{ij\upsilon U}\cdot b_{ij\upsilon U}+1)}\right] \end{bmatrix}$$

(xiii) $A \le B$ iff $a_{ij\mu L} \le b_{ij\mu L}$, $a_{ij\mu U} \le b_{ij\mu U}$, $a_{ij\lambda L} \ge b_{ij\lambda L}$, $a_{ij\lambda U} \ge b_{ij\lambda U}$, $a_{ij\upsilon L} \ge b_{ij\upsilon L}$, $a_{ij\upsilon U} \ge b_{ij\upsilon U}$

(xiv) $A = B$ iff $A \le B$ and $B \le A$.

In the following section, we consider a daily life problem which can be studied using IVNFMs in better way.

4. IMPORTANCE OF INTERVAL VALUED NEUTROSOPHIC FUZZY MATRICES (IVNFMS)

A network consisting of four important cities (vertices) in a country is considered. They are connected by highways (edges). The number neighboring to an edge characterizes the distance between the cities (vertices). The above network can be represented with the help of a classical matrix $A = [a_{ij}]$, $i, j = 1, 2, \ldots, n$, where, n is the total number of nodes. The ij^{th} element a_{ij} of A is defined as

$$a_{ij} = \begin{cases} 0, \text{ if } i = j \\ \infty, \text{the vertices } i \text{ and } j \text{ are not directly connected by an adge} \\ w_{ij}, w_{ij} \text{ is the distance of the road connecting } i \text{ and } j \end{cases}$$

Thus the adjacent matrix of the network of is

$$\begin{bmatrix} 0 & 15 & 20 & 35 \\ 15 & 0 & 55 & 40 \\ 20 & 55 & 0 & 75 \\ 35 & 40 & 75 & 0 \end{bmatrix}$$

Since the distance between two vertices is identified, precisely, so the above matrix is obviously a conventional matrix. Generally, the distance between two cities are crisp value, so the corresponding matrix is crisp matrix. Now, we study the crowdness of the roads connecting cities. It is clear that the crowdness of a road clearly, is a fuzzy quantity. The amount of crowdness depends on the decision makers temperament, practices, environments, etc. i.e., finally depends on the decision maker. The measurement of crowdness as a point is a difficult task for the decision maker. So, here we consider the amount of crowdness as an interval instead of a point. The aloneness is considered as an interval and also the indeterminacy is considered as an interval. The crowdness, indeterminacy and the aloneness of a network cannot be represented as a crisp matrix, it can be represented appropriately by a matrix which we designate by interval-valued neutrosophic fuzzy matrices (IVNFMs). The matrix representation of the traffic crowdness, indeterminacy and aloneness of the network of is shown in the following IVNFM.

$$\begin{bmatrix} \langle[0,0],[1,1],[1,1]\rangle & \langle[.1,.3],[.2,.4],[.2,.5]\rangle & \langle[.2,.4][.3,.5],[.1,.5]\rangle & \langle[.3,.4],[.2,.5],[.5,.6]\rangle \\ \langle[.1,.3],[.2,.4],[.2,.5]\rangle & \langle[0,0],[1,1],[1,1]\rangle & \langle[.7,.8],[.2,.4],[0,.1]\rangle & \langle[.3,.5],[.3,.6],[.4,.5]\rangle \\ \langle[.2,.4],[.3,.5],[.1,.5]\rangle & \langle[.7,.8],[.2,.4],[0,.1]\rangle & \langle[0,0],[1,1],[1,1]\rangle & \langle[.5,.6],[.1,.3],[.2,.3]\rangle \\ \langle[.3,.4],[.2,.5],[.5,.6]\rangle & \langle[.3,.5],[.3,.6],[.4,.5]\rangle & \langle[.5,.6],[.1,.3],[.2,.3]\rangle & \langle[0,0],[1,1],[1,1]\rangle \end{bmatrix}$$

To explain the meaning of the operators defined earlier we consider two IVNFMs A and B. Let A and B represent respectively the crowdness, indeterminacy and loneliness of the network at two time instances t_1 and t_2. Now, the IVNFM $A + B$ represents the maximum amount of traffic crowdness, the minimum of indeterminacy and the minimum amount of aloneness of the network between the time instances t_1 and t_2. $A \cdot B$ represents the minimum amount of traffic crowdness, minimum amount of indeterminacy and the maximum amount of loneliness of the network. $\overline{A}$ matrix represents the aloneness, confidence and crowdness of the network. $A @ B$, $A \$ B$ and $A \# B$ reveals the arithmetic mean, geometric mean and harmonic mean of the crowdness, indeterminacy and aloneness in between the two time instances t_1 and t_2 of the network. To illustrate the operators $A \cdot B$, $A + B$ and $|A|$, we consider a network consisting two vertices and two edges. The crowdness, indeterminacy and aloneness of the network are observed at two different time instances t_1 and t_2. The matrices A_{t_1} and A_{t_2} represent the status of the network at t_1 and at t_2. The number adjacent to the sides represents the crowdness, indeterminacy and aloneness of the roads at two different instances of the same network.

Let

$$A_{t_1} = \begin{bmatrix} \langle[0, 0], [1, 1], [1, 1]\rangle & \langle[.1, .3], [.2, .4], [.2, .5]\rangle \\ \langle[.1, .3], [.2, .4], [.2, .5]\rangle & \langle[0, 0], [1, 1], [1, 1]\rangle \end{bmatrix}$$

$$A_{t_2} = \begin{bmatrix} \langle [0,0],[1,1],[1,1] \rangle & \langle [.2,.4],[.3,.5],[.1,.5] \rangle \\ \langle [.2,.4],[.3,5],[.1,.5] \rangle & \langle [0,0],[1,1],[1,1] \rangle \end{bmatrix}.$$

Then $A_{t_1} + A_{t_2} = \begin{bmatrix} \langle [0,0],[1,1],[1,1] \rangle & \langle [.2,.4],[.2,.4],[.1,.5] \rangle \\ \langle [.2,.4],[.2,.4],[.1,.5] \rangle & \langle [0,0],[1,1],[1,1] \rangle \end{bmatrix}$ and

$$A_{t_1} \cdot A_{t_2} = \begin{bmatrix} \langle [0,0],[1,1],[1,1] \rangle & \langle [.1,.3],[.3,.5],[.2,.5] \rangle \\ \langle [.1,.3],[.3,.5],[.2,.5] \rangle & \langle [0,0],[1,1],[1,1] \rangle \end{bmatrix}$$

$$|A_{t_1}| = \langle [0,0],[1,1],[1,1] \rangle \cdot \langle [0,0],[1,1],[1,1] \rangle + \langle [.1,.3],[.2,.4],[.2,.5] \rangle$$

$$\langle [.1,.3],[.2,.4],[.2,.5] \rangle = \langle [0,0],[1,1],[1,1] \rangle + \langle [.1,.3],[.2,.4],[.2,.5] \rangle = \langle [.1,.3],[.2,.4],[.2,.5] \rangle.$$

5. PROPERTIES OF INTERVAL VALUED NEUTROSOPHIC FUZZY MATRICES (IVNFMS)

In this section some properties of IVNFMs are presented. IVNFMs satisfy the commutative and associative properties over the operators $+, \cdot \oplus,$ and $\odot$. The operator '.' is distributed over '+' in left and right but the left and right distribution laws do not hold for the operators $\oplus$ and $\odot$.|

(1) $A + B = B + A$

(2) $A + (B + C) = (A + B) + C$

(3) $A \cdot B = B \cdot A$

(4) $A \cdot (B \cdot C) = (A \cdot B) \cdot C$

(5) (i) $A \cdot (B + C) = A \cdot B + A \cdot C$

(ii) $(B + C) \cdot A = B \cdot A + A \cdot C$

(6) $A \oplus B = B \oplus A$

(7) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

(8) $A \odot B = B \odot A$

(9) $A \odot (B \odot C) = (A \odot B) \odot C$

(10) (i) $A \odot (B \oplus C) \neq (A \odot B) \oplus (A \odot C)$

(ii) $(B \oplus C) \odot A \neq (B \odot A) \oplus (C \odot A)$

Proof.

(1) Let $A = [\langle [a_{ij\mu L}, a_{ij\mu U}], [a_{ij\lambda L}, a_{ij\lambda U}], [a_{ij\nu L}, a_{ij\nu U}]\rangle]$,

$B = [\langle [b_{ij\mu L}, b_{ij\mu U}], [b_{ij\lambda L}, b_{ij\lambda U}], [b_{ij\nu L}, b_{ij\nu U}]\rangle]$ and

$C = [\langle [c_{ij\mu L}, c_{ij\mu U}], [c_{ij\lambda L}, c_{ij\lambda U}], [c_{ij\nu L}, c_{ij\nu U}]\rangle]$.

$$A + B = \begin{matrix} [\max(a_{ij\mu L}, b_{ij\mu L}), \max(a_{ij\mu U}, b_{ij\mu U})], \\ \langle [\min(a_{ij\lambda L}, b_{ij\lambda L}), \min(a_{ij\nu U}, b_{ij\nu U})]\rangle \\ [\min(a_{ij\nu L}, b_{ij\lambda L}), \min(a_{ij\nu U}, b_{ij\nu U})] \end{matrix}$$

$$B + A = \begin{matrix} [\max(b_{ij\mu L}, a_{ij\mu L}), \max(b_{ij\mu U}, a_{ij\mu U})], \\ \langle [\min(b_{ij\lambda L}, a_{ij\lambda L}), \min(b_{ij\lambda U}, a_{ij\lambda U})]\rangle, \\ [\min(b_{ij\nu L}, a_{ij\lambda L}), \min(b_{ij\nu U}, a_{ij\nu U})] \end{matrix}$$

Therefore, $A + B = B + A$. Similarly, (2), (3), (4), (5), (6), (7), (8) and (9) can be proved.

$$(10)\ B \oplus C = \begin{bmatrix} [b_{ij\mu L} + c_{ij\mu L} - b_{ij\mu L} \cdot c_{ij\mu L}, b_{ij\mu U} + c_{ij\mu U} - b_{ij\mu U} \cdot c_{ij\mu U}], \\ \langle [b_{ij\lambda L} \cdot c_{ij\lambda L}, b_{ij\lambda U} \cdot c_{ij\lambda U}]\rangle, \\ [b_{ij\nu L} \cdot c_{ij\nu L}, b_{ij\nu U} \cdot c_{ij\nu U}] \end{bmatrix}$$

$$A \odot (B \oplus C) = \begin{bmatrix} [a_{ij\mu L} \cdot (b_{ij\mu L} + c_{ij\mu L} - b_{ij\mu L} \cdot c_{ij\mu L}), a_{ij\mu U} \cdot (b_{ij\mu U} + c_{ij\mu U} - b_{ij\mu U} \cdot c_{ij\mu U})] \\ \langle [a_{ij\lambda L} + b_{ij\lambda L} \cdot c_{ij\lambda L} - a_{ij\lambda L} \cdot b_{ij\lambda L} \cdot c_{ij\lambda L}, a_{ij\lambda U} + b_{ij\lambda U} \cdot c_{ij\lambda U} - a_{ij\lambda U} \cdot b_{ij\lambda U} \cdot c_{ij\lambda U})]\rangle \\ [a_{ij\nu L} + b_{ij\lambda U} \cdot c_{ij\lambda U} - a_{ij\nu L} \cdot b_{ij\nu L}, a_{ij\nu U} + b_{ij\nu U} \cdot c_{ij\nu U} - a_{ij\nu U} \cdot b_{ij\nu U} \cdot c_{ij\nu U}] \end{bmatrix}$$

$$A \odot B = \begin{bmatrix} [a_{ij\mu L} \cdot b_{ij\mu L}, a_{ij\mu U} \cdot b_{ij\mu U}], \\ \langle [a_{ij\lambda L} + b_{ij\lambda L} - a_{ij\lambda L} \cdot b_{ij\lambda L}, a_{ij\lambda U} + b_{ij\lambda U} - a_{ij\lambda U} \cdot b_{ij\lambda U}]\rangle \\ [a_{ij\nu L} + b_{ij\nu L} - a_{ij\nu L} \cdot b_{ij\nu L}, a_{ij\nu U} + b_{ij\nu U} - a_{ij\nu U} \cdot b_{ij\nu U}] \end{bmatrix},$$

$$A \odot C = \begin{bmatrix} [a_{ij\mu L} \cdot c_{ij\mu L}, a_{ij\mu U} \cdot c_{ij\mu U}], \\ \langle [a_{ij\lambda L} + c_{ij\lambda L} - a_{ij\lambda L} \cdot c_{ij\lambda L}, a_{ij\lambda U} + c_{ij\lambda U} - a_{ij\lambda U} \cdot c_{ij\lambda U}]\rangle \\ [a_{ij\nu L} + c_{ij\nu L} - a_{ij\nu L} \cdot c_{ij\nu L} + c_{ij\nu U} - a_{ij\nu U} \cdot c_{ij\nu U}] \end{bmatrix}.$$

Now,

$$(A \odot B) \oplus (A \odot C) = \begin{bmatrix} [a_{ij\mu L}(b_{ij\mu L} + c_{ij\mu L}) - a_{ij\mu L}^2 \cdot b_{ij\mu L} \cdot c_{ij\mu L}, \\ a_{ij\mu U}(b_{ij\mu U} + c_{ij\mu U}) - a_{ij\mu U}^2 \cdot b_{ij\mu U} \cdot c_{ij\mu U}], \\ \langle [(a_{ij\nu L} + b_{ij\nu L} - a_{ij\nu L} \cdot b_{ij\nu L}) \cdot (a_{ij\lambda L} + c_{ij\lambda L} - a_{ij\lambda L} \cdot c_{ij\lambda L}), \\ (a_{ij\lambda U} + b_{ij\lambda U} - a_{ij\lambda U} \cdot b_{ij\lambda U}) \cdot (a_{ij\lambda U} + c_{ij\lambda U} - a_{ij\lambda U} \cdot c_{ij\lambda U})] \rangle \\ [(a_{ij\nu L} + b_{ij\nu L} - a_{ij\nu L} \cdot b_{ij\nu L}) \cdot (a_{ij\nu L} + c_{ij\nu L} - a_{ij\nu L} \cdot c_{ij\nu L}), \\ (a_{ij\nu U} + b_{ij\nu U} - a_{ij\nu U} \cdot b_{ij\nu U}) \cdot (a_{ij\nu U} + b_{ij\nu U} - a_{ij\nu U} \cdot b_{ij\nu U})] \end{bmatrix}.$$

So, $A \odot (B \oplus C) \neq (A \odot B) \oplus (A \odot C)$. Similarly, $(B \oplus C) \odot A \neq (B \odot A) \oplus (C \odot A)$ can be proved.

Property 1. Let A be an IVNFM of any order then, $A + A = A$.

Proof. Let $A = [\langle [a_{ij\mu L}, a_{ij\mu U}], [a_{ij\lambda L}, a_{ij\lambda U}], [a_{ij\nu L}, a_{ij\nu U}] \rangle]$.

$$[\max(a_{ij\mu L}, a_{ij\mu L}), \max(a_{ij\mu U}, a_{ij\mu U})],$$

Then, $A + B = \langle [\min(a_{ij\lambda L}, a_{ij\lambda L}), \min(a_{ij\lambda U}, a_{ij\lambda U})] \rangle = [\langle [a_{ij\mu L}, a_{ij\mu U}], [a_{ij\lambda L}, a_{ij\lambda U}], [a_{ij\nu L}, a_{ij\nu U}] \rangle] = A$

$$[\min(a_{ij\nu L}, a_{ij\lambda L}), \min(a_{ij\nu U}, a_{ij\nu U})].$$

Property 2 If A be an IVNFM of any order then, $A + I_{\langle [0,0],[0,0],[0,0] \rangle} \geq A$ where, $I_{\langle [0,0],[0,0],[0,0] \rangle}$ is the null IVNFM of same order.

Proof. Let $A = [\langle [a_{ij\mu L}, a_{ij\mu U}], [a_{ij\lambda L}, a_{ij\lambda U}], [a_{ij\nu L}, a_{ij\nu U}] \rangle]$ and $I_{\langle [0,0],[0,0],[0,0] \rangle} = \langle [0, 0], [0, 0], [0, 0] \rangle$.

$$[\max(a_{ij\mu L}, 0), \max(a_{ij\mu U}, 0)],$$

Then, $A + I_{\langle [0,0],[0,0],[0,0] \rangle} = \langle [\min(a_{ij\lambda L}, 0), \min(a_{ij\lambda U}, 0)] \rangle = [\langle [a_{ij\mu L}, a_{ij\mu U}][0,0],[0,0] \rangle]$ $[\min(a_{ij\nu L}, 0), \min(a_{ij\nu U}, 0)]$.

Therefore, $A + I_{\langle [0,0],[0,0],[0,0] \rangle} \geq A$.

Some more properties on determinant and adjoint of IVNFM are presented below.

Proof. Let $A = [\langle [a_{ij\mu L}, a_{ij\mu U}], [a_{ij\lambda L}, a_{ij\lambda U}], [a_{ijvL}, a_{ijvU}]\rangle]$. Then $A^T = [\langle [a_{ji\mu L}, a_{ji\mu U}], [a_{ji\lambda L}, a_{ji\lambda U}], [a_{jivL}, a_{jivU}]\rangle]$.

$$\langle [a_{\sigma(1)1\mu L}, a_{\sigma(1)1\mu U}], [a_{\sigma(1)1\lambda L}, a_{\sigma(1)1\lambda U}], [a_{\sigma(1)1vL}, a_{\sigma(1)1vU}]\rangle.$$

Now,

$$|A^T| = \sum_{\sigma \in s_n} \langle [a_{\sigma(2)2\mu L}, a_{\sigma(2)2\mu U}], [a_{\sigma(2)2\lambda L}, a_{\sigma(2)2\lambda U}], [a_{\sigma(2)2vL}, a_{\sigma(2)2vU}]\rangle$$

$$\ldots \langle [a_{\sigma(n)n\mu L}, a_{\sigma(n)n\mu U}], [a_{\sigma(n)n\lambda L}, a_{\sigma(n)n\lambda U}], [a_{\sigma(n)nvL}, a_{\sigma(n)nvU}]\rangle.$$

Let ψ be the permutation of $\{1, 2, \ldots, n\}$ such that $\psi\sigma = I$, the identity permutation. Then $\psi = \sigma - 1$. As σ runs over the whole set of permutations, so does ψ. Let $\sigma(i) = j$, $i = \sigma^{-1}(j) = \psi(j)$.

Therefore, $a_{\sigma(i)i\mu L} = a_{j\psi(j)\mu L}$, $a_{\sigma(i)i\mu U} = a_{\sigma(i)i\mu U}$, $a_{\sigma(i)i\lambda L} = a_{j\psi(j)\lambda L}$, $a_{\sigma(i)i\lambda L} = a_{j\psi(j)\lambda L}$, $a_{\sigma(i)ivL} = a_{j\psi(j)vL}$, $a_{\sigma(i)ivL} = a_{j\psi(j)vL}$. As i goes over the set $\{1, 2, \ldots, n\}$, j does so.

Now, $\langle [a_{\sigma(1)1\mu L}, a_{\sigma(1)1\mu U}], [a_{\sigma(1)1\lambda L}, a_{\sigma(1)1\lambda U}], [a_{\sigma(1)1vL}, a_{\sigma(1)1vU}]\rangle$

$$\langle [a_{\sigma(2)2\mu L}, a_{\sigma(2)2\mu U}], [a_{\sigma(2)2\lambda L}, a_{\sigma(2)2\lambda U}], [a_{\sigma(2)2vL}, a_{\sigma(2)2vU}]\rangle \ldots$$

$$\langle [a_{\sigma(n)n\mu L}, a_{\sigma(n)n\mu U}], [a_{\sigma(n)n\lambda L}, a_{\sigma(n)n\lambda U}], [a_{\sigma(n)nvL}, a_{\sigma(n)nvU}]\rangle$$

$$\langle [a_{1\psi(1)\mu L}, a_{1\psi(1)1\mu U}], [a_{1\psi(1)\lambda L}, a_{\psi(1)1\lambda U}], [a_{1\psi(1)vL}, a_{1\psi(1)vU}]\rangle$$

$$= \langle [a_{2\psi(2)\mu L}, a_{2\psi(2)\mu U}], [a_{2\psi(2)\lambda L}, a_{2\psi(2)\lambda U}], [a_{2\psi(2)vL}, a_{2\psi(2)vU}]\rangle \ldots$$

$$\langle [a_{n\psi(n)\mu L}, a_{n\psi(n)\mu U}], [a_{n\psi(n)\lambda L}, a_{n\psi(n)\lambda U}], [a_{n\psi(n)vL}, a_{n\psi(n)vU}]\rangle$$

$$\langle [a_{\sigma(1)1\mu L}, a_{\sigma(1)1\mu U}], [a_{\sigma(1)1\lambda L}, a_{\sigma(1)1\lambda U}], [a_{\sigma(1)1vL}, a_{\sigma(1)1vU}]\rangle$$

Therefore, $|A^T| = \sum_{\sigma \in s_n} \langle [a_{\sigma(2)2\mu L}, a_{\sigma(2)2\mu U}], [a_{\sigma(2)2\lambda L}, a_{\sigma(2)2\lambda U}],$ $[a_{\sigma(2)2vL}, a_{\sigma(2)2vU}]\rangle \ldots = \langle [a_{\sigma(n)n\mu L}, a_{\sigma(n)n\mu U}], [a_{\sigma(n)n\lambda L}, a_{\sigma(n)n\lambda U}], [a_{\sigma(n)nvL}, a_{\sigma(n)nvU}]\rangle$

$$\langle [a_{1\psi(1)\mu L}, a_{1\psi(1)1\mu U}], [a_{1\psi(1)\lambda L}, a_{1\psi(1)1\lambda U}], [a_{1\psi(1)vL}, a_{1\psi(1)vU}]\rangle$$

$$\sum_{\mu \in s_n} \langle [a_{2\psi(2)\mu L}, a_{2\psi(2)\mu U}], [a_{2\psi(2)\lambda L}, a_{2\psi(2)\lambda U}], [a_{2\psi(2)vL}, a_{2\psi(2)vU}]\rangle \ldots$$

$$|A| = \langle [a_{n\psi(n)\mu L}, a_{n\psi(n)\mu U}], [a_{n\psi(n)\lambda L}, a_{n\psi(n)\lambda U}], [a_{n\psi(n)vL}, a_{n\psi(n)vU}]\rangle.$$

Property 4. If A and B be two square IVNFMs and $A \leq B$, then, adj $A \leq adj\ B$.

Proof. Let $C = [\langle [c_{ij\mu L}, c_{ij\mu U}], [c_{ij\lambda L}, c_{ij\lambda U}], [c_{ijvL}, c_{ijvU}]\rangle] = A$, $D = [\langle [d_{ij\mu L}, d_{ij\mu U}], [d_{ij\lambda L}, d_{ij\lambda U}], [d_{ijvL}, d_{ijvU}]\rangle] = adj\ B$.

where,

$\langle [c_{ij\mu L}, c_{ij\mu U}], [c_{ij\lambda L}, c_{ij\lambda U}], [c_{ijvL}, c_{ijvU}]\rangle = \sum_{\sigma \in s_{n_i n_j}} \prod_{t \in n_j} \langle [a_{t\sigma(t)\mu L}, a_{t\sigma(t)\mu U}]$,

$[a_{t\sigma(t)\lambda L}, a_{t\sigma(t)\lambda U}], [a_{t\sigma(t)vL}, a_{t\sigma(t)vU}]\rangle$

and

$\langle [d_{ij\mu L}, d_{ij\mu U}], [d_{ij\lambda L}, d_{ij\lambda U}], [d_{ijvL}, d_{ijvU}]\rangle = \sum_{\sigma \in s_{n_i n_j}} \prod_{t \in n_j} \langle [b_{t\sigma(t)\mu L}, b_{t\sigma(t)\mu U}]$,

$[b_{t\sigma(t)\lambda L}, b_{t\sigma(t)\lambda U}], [b_{t\sigma(t)vL}, b_{t\sigma(t)vU}]\rangle$

It is clear that $\langle [c_{ij\mu L}, c_{ij\mu U}], [c_{ij\lambda L}, c_{ij\lambda U}], [c_{ijvL}, c_{ijvU}]\rangle \leq \langle [d_{ij\mu L}, d_{ij\mu U}]$, $[d_{ij\lambda L}, d_{ij\lambda U}], [d_{ijvL}, d_{ijvU}]\rangle$. Since, $a_{t\sigma(t)\mu L} \leq b_{t\sigma(t)\mu L}$, $a_{t\sigma(t)\mu L} \leq b_{t\sigma(t)\mu U}$, $a_{t\sigma(t)\lambda L} \geq b_{t\sigma(t)\lambda L}$, $a_{t\sigma(t)\lambda L} \geq a_{t\sigma(t)\lambda U}$ and $a_{t\sigma(t)vL} \geq b_{t\sigma(t)vU}$ for all

Therefore $C \leq D$, i.e., $adj\ A \leq adj\ B$.

Property 5. For a square IVNFM A, $adj\ (A^T) = (adj\ A)^T$.

Proof. Let $B = adj\ A$, $C = adj\ A^T$.

Therefore,

$\langle [b_{ij\mu L}, b_{ij\mu U}], [b_{ij\lambda L}, b_{ij\lambda U}], [b_{ijvL}, b_{ijvU}]\rangle = \sum_{\sigma \in s_{n_i n_j}} \prod_{t \in n_j} \langle [a_{t\sigma(t)\mu L}, a_{t\sigma(t)\mu U}]$,

$[a_{t\sigma(t)\lambda L}, a_{t\sigma(t)\lambda U}], [a_{t\sigma(t)vL}, a_{t\sigma(t)vU}]\rangle$ and

$\langle [c_{ij\mu L}, c_{ij\mu U}], [c_{ij\lambda L}, c_{ij\lambda U}], [c_{ijvL}, c_{ijvU}]\rangle = \sum_{\sigma \in s_{n_i n_j}} \prod_{t \in n_j} \langle [a_{t\sigma(t)\mu L}, a_{t\sigma(t)\mu U}]$,

$[a_{t\sigma(t)\lambda L}, a_{t\sigma(t)\lambda U}], [a_{t\sigma(t)nL}, a_{t\sigma(t)vU}]\rangle = \langle [b_{ij\mu L}, b_{ij\mu U}], [b_{ij\lambda L}, b_{ij\lambda U}], [b_{ijvL}, b_{ijvU}]\rangle$.

Therefore, $adj\ (A^T) = (adj\ A)^T$.

Property 6. For an IVNFM A, $|A| = |adj\ A|$.

Proof. $A = [\langle [A_{ij\mu L}, A_{ij\mu U}], [A_{ij\lambda L}, A_{ij\lambda U}], [A_{ij\nu L}, A_{ij\nu U}]\rangle]$ where, $[\langle [A_{ij\mu L}, A_{ij\mu U}], [A_{ij\lambda L}, A_{ij\lambda U}], [A_{ij\nu L}, A_{ij\nu U}]\rangle]$ is the cofactor of the element $[\langle [a_{ij\mu L}, a_{ij\mu U}], [a_{ij\lambda L}, a_{ij\lambda U}], [a_{ij\nu L}, a_{ij\nu U}]\rangle]$ in the IVNFM A. Therefore,

$$\langle [A_{1\sigma(1)\mu L}, A_{1\sigma(1)\mu U}], [A_{1\sigma(1)\lambda L}, A_{1\sigma(1)\lambda U}], [A_{1\sigma(1)\nu L}, A_{1\sigma(1)\nu U}]\rangle$$

$$|adj\ A| = \sum_{\sigma \in s_n} \langle [A_{2\sigma(2)\mu L}, A_{2\sigma(2)\mu U}], [A_{2\sigma(2)\lambda L}, A_{2\sigma(2)\lambda U}], [A_{2\sigma(2)\nu L}, A_{2\sigma(2)\nu U}]\rangle.$$
$$\langle [A_{n\sigma(n)\mu L}, A_{n\sigma(n)\mu U}], [A_{n\sigma(n)\lambda L}, A_{n\sigma(n)\lambda U}], [A_{n\sigma(n)\nu L}, A_{n\sigma(n)\nu U}]\rangle$$

$$= \sum_{\sigma \in s_n} \prod_{i=1}^{n} \langle [A_{i\sigma(i)\mu L}, A_{i\sigma(i)\mu U}], [A_{i\sigma(i)\lambda L}, A_{i\sigma(i)\lambda U}], [A_{i\sigma(i)\nu L}, A_{i\sigma(i)\nu U}]\rangle$$

$$= \sum_{\sigma e s_n} \prod_{i=1}^{n} \sum_{\theta e s_{n_i n_{\sigma(i)}}} \prod_{t e n_j} \langle [a_{t\theta(t)\mu L}, a_{t\theta(t)\mu U}], [a_{t\theta(t)\lambda L}, a_{t\theta(t)\lambda U}], [a_{t\theta(t)\nu L}, a_{t\theta(t)\nu U}]\rangle$$

$$= \sum_{\sigma e s_n} [(\prod_{t \in n_1} \langle [a_{t\theta_1(t)\mu L}, a_{t\theta_1(t)\mu U}], [a_{t\theta_1(t)\lambda L}, a_{t\theta_1(t)\lambda U}], [a_{t\theta_1(t)\nu L}, a_{t\theta_1(t)\nu U}]\rangle)$$

$$(\prod_{t \in n_2} \langle [a_{t\theta_2(t)\mu L}, a_{t\theta_2(t)\mu U}], [a_{t\theta_2(t)\lambda L}, a_{t\theta_2(t)\lambda U}], [a_{t\theta_2(t)\nu L}, a_{t\theta_2(t)\nu U}]\rangle)\cdots$$
$$(\prod_{t \in n_n} \langle [a_{t\theta_n(t)\mu L}, a_{t\theta_n(t)\mu U}], [a_{t\theta_n(t)\lambda L}, a_{t\theta_n(t)\lambda U}], [a_{t\theta_n(t)\nu L}, a_{t\theta_n(t)\nu U}]\rangle)]$$

$$= \sum_{\sigma \in s_n} [(\langle [a_{2\theta_1(2)\mu L}, a_{2\theta_1(2)\mu U}], [a_{2\theta_1(2)\lambda L}, a_{2\theta_1(2)\lambda U}], [a_{2\theta_1(2)\nu L}, a_{2\theta_1(2)\nu U}]\rangle$$

$$\langle [a_{3\theta_1(3)\mu L}, a_{3\theta_1(3)\mu U}], [a_{3\theta_1(3)\lambda L}, a_{3\theta_1(3)\lambda U}], [a_{3\theta_1(3)\nu L}, a_{3\theta_1(3)\nu U}]\rangle$$

$$\langle [a_{n\theta_1(n)\mu L}, a_{n\theta_1(n)\mu U}], [a_{n\theta_1(n)\lambda L}, a_{n\theta_1(n)\lambda U}], [a_{n\theta_1(n)\nu L}, a_{n\theta_1(n)\nu U}]\rangle)$$

$$\langle [a_{1\theta_2(1)\mu L}, a_{1\theta_2(1)\mu U}], [a_{1\theta_2(1)\lambda L}, a_{1\theta_2(1)\lambda U}], [a_{1\theta_2(1)\nu L}, a_{1\theta_2(1)\nu U}]\rangle$$

$$\langle [a_{3\theta_2(3)\mu L}, a_{3\theta_2(3)\mu U}], [a_{3\theta_2(3)\lambda L}, a_{3\theta_2(3)\lambda U}], [a_{3\theta_2(3)\nu L}, a_{3\theta_2(3)\nu U}]\rangle$$

$$\langle [a_{n\theta_2(n)\mu L}, a_{n\theta_2(n)\mu U}], [a_{n\theta_2(n)\lambda L}, a_{n\theta_2(n)\lambda U}], [a_{n\theta_2(n)\nu L}, a_{n\theta_2(n)\nu U}]\rangle)\cdots$$

$$(\langle [a_{1\theta_n(1)\mu L}, a_{1\theta_n(1)\mu U}], [a_{1\theta_n(1)\lambda L}, a_{1\theta_n(1)\lambda U}], [a_{1\theta_n(1)\nu L}, a_{1\theta_n(1)\nu U}]\rangle\cdots$$

$$\langle [a_{2\theta_n(2)\mu L}, a_{2\theta_2(2)\mu U}], [a_{2\theta_n(2)\lambda L}, a_{2\theta_n(2)\lambda U}], [a_{2\theta_n(2)\nu L}, a_{2\theta_n(2)\nu U}]\rangle$$

$$\langle [a_{n-1\theta_2(n-1)\mu L}, a_{n-1\theta_2(n-1)\mu U}], [a_{n-1\theta_2(n-1)\lambda L}, a_{n-1\theta_2(n-1)\lambda U}], [a_{n-1\theta_2(n-1)\nu L},$$
$a_{n-1\theta_2(n-1)\nu U}]\rangle)]$ where, $f_{\hat{\theta}\theta} \in [1, 2, \ldots, n] \setminus \{\hat{\theta}\}$, $\hat{\theta} = 1, 2, \ldots, n$.

But since,

$$\left\langle [a_{\hat{\theta}\theta f_{\hat{\theta}}(\hat{\theta})\mu L}, a_{\hat{\theta}\theta f_{\hat{\theta}}(\hat{\theta})\mu U}], [a_{\hat{\theta}\theta f_{\hat{\theta}}(\hat{\theta})\lambda L}, a_{\hat{\theta}\theta f_{\hat{\theta}}(\hat{\theta})\lambda U}], [a_{\hat{\theta}\theta f_{\hat{\theta}}(\hat{\theta})\upsilon L}, a_{\hat{\theta}\theta f_{\hat{\theta}}(\hat{\theta})\upsilon U}] \right\rangle$$

$$= \left\langle [a_{n\sigma(n)\mu L}, a_{n\sigma(n)\mu U}], [a_{n\sigma(n)\lambda L}, a_{n\sigma(n)\lambda U}], [a_{n\sigma(n)\upsilon L}, a_{n\sigma(n)\upsilon U}] \right\rangle.$$

Therefore

$$\left\langle [a_{1\sigma(1)\mu L}, a_{1\sigma(1)\mu U}], [a_{1\sigma(1)\lambda L}, a_{1\sigma(1)\lambda U}], [a_{1\sigma(1)\upsilon L}, a_{1\sigma(1)\upsilon U}] \right\rangle$$

$$|adj\ A| = \sum_{\sigma \in s_n} \left\langle [a_{2\sigma(2)\mu L}, a_{2\sigma(2)\mu U}], [a_{2\sigma(2)\lambda L}, a_{2\sigma(2)\lambda U}], [a_{2\sigma(2)\upsilon L}, a_{2\sigma(2)\upsilon U}] \right\rangle$$

$$\left\langle [a_{n\sigma(n)\mu L}, a_{n\sigma(n)\mu U}], [a_{n\sigma(n)\lambda L}, a_{n\sigma(n)\lambda U}], [a_{n\sigma(n)\upsilon L}, a_{n\sigma(n)\upsilon U}] \right\rangle = |A|.$$

CONCLUSION

In this work, Interval valued neutrosophic fuzzy matrices are introduced based on M. Pal et al. [13]. Then the operations on Interval valued neutrosophic fuzzy matrices are discussed. Some properties on them are conferred with the real time example. The same operations and properties can be extended to various Neutrosophic numbers.

REFERENCES

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986), 87-96.

[2] K. Atanassov, Operations over interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 64 (1994), 159-174.

[3] H. Hashimoto, Canonical form of a transitive matrix, Fuzzy Sets and Systems 11 (1983), 157-162.

[4] R. Hemasinha, N. R. Pal and J. C. Bezdek, Iterates of fuzzy circulant matrices, Fuzzy Sets and Systems 60 (1993), 199-206.

[5] S. K. Khan and M. Pal, Some operations on intuitionistic fuzzy matrices, Acta Ciencia Indica, XXXII M (2) (2006), 515-524.

[6] K. H. Kim and F. W. Roush, Generalised fuzzy matrices, Fuzzy Sets and Systems 4 (1980), 293-315.

[7] W. Kolodziejczyk, Canonical form of a strongly transitive fuzzy matrix, Fuzzy Sets and Systems 22 (1987), 292-302.

[8] S. Broumi, F. Smarandache and M. Dhar, On fuzzy soft matrix based on reference function, Information Engineering and Electronic Business 2 (2013), 52-59.

[9] F. Smarandache, Neutrosophic set, A generalisation of the intuitionistic fuzzy sets, Inter. J. Pure Appl. Math. 24 (2005), 287-297.

[10] J. B. Kim, Determinant theory for fuzzy and boolean matrices, Congressus Numerantium (1988), 273-276.

[11] M. Pal, Intuitionistic fuzzy determinant, V. U. J. Physical Sciences 7 (2001), 87-93.

[12] M. Pal, S. K. Khan and A. K. Shyamal, Intuitionistic fuzzy matrices, Notes on Intuitionistic Fuzzy Sets 8(2) (2002), 51-62.

[13] Madhumangal Pal and Susanta K. Khan, Interval valued intuitionistic fuzzy matrices, NIFS 11 (2005), 16-27.

[14] M. Z. Ragab and E. G. Emam, The determinant and adjoint of a square fuzzy matrix, Fuzzy Sets and Systems 61 (1994), 297-307.

[15] M. G. Ragab and E. G. Emam, On the min-max composition of fuzzy matrices, Fuzzy Sets and Systems 75 (1995), 83-92.

[16] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.

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